Optimum Mode-Switching Assisted Adaptive Modulation

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Abstract – **Adaptive modulation techniques combat fading by employing a suitable modulation mode depending on the instantaneous channel conditions for improving the Bit Error Rate (BER) performance or the average throughput. Based on a generic model of constant-power adaptive modulation, exact closed form expressions of the average BER and the average throughput are derived, when employing square Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) as the constituent modulation modes, operating over a Nakagami fading channel. The optimum modulation-mode switching levels, achieving the highest possible throughput under the constraint of the average BER, are obtained using the Lagrangian optimisation method. Adaptive modulation employing the optimum switching levels shows a superior performance, while maintaining a constant average BER.**

1. INTRODUCTION

Mobile communications channels typically exhibit time-variant channel quality fluctuations [1] and hence conventional fixed-mode modems suffer from bursts of transmission errors, even if the system was designed to provide a high link margin. An efficient approach to mitigating these detrimental effects is to adaptively adjust the transmission format based on the near-instantaneous channel quality perceived by the receiver, which is fed back to the transmitter with the aid of a feedback channel [2]. *Hayes* [3] proposed transmission power adaptation, while *Cavers* [4] suggested invoking a variable symbol duration scheme in response to the perceived channel quality at the expense of a variable bandwidth requirement. Since a variable-power scheme increases both the transmitted power requirements and the level of co-channel interference [5], variable-rate Adaptive Quadrature Amplitude Modulation (AQAM) was proposed by *Steele* and *Webb* as an alternative, employing various star-QAM constellations [5, 6]. With the advent of Pilot Symbol Assisted Modulation (PSAM), *Otsuki*, *Sampei* and *Morinaga* [7] employed square constellations instead of star constellations for AQAM, as a practical fading counter measure. Analysing the channel capacity of Rayleigh fading channels [8, 9], *Goldsmith* and *Varaiya* showed that variable-power, variable-rate adaptive schemes are optimum in terms of approaching the channel capacity and they characterised the average throughput performance of variable-power AQAM [9] . However, *Goldsmith* and *Varaiya* also found that the extra channel capacity achieved by variable-power assisted adaptation over the constant-power, variable-rate scheme is a fraction of a dB for most types of fading channels [9, 10].

Our interest here is the determination of the modulation-mode switching levels for the constant-power adaptive modulation scheme. The first serious attempt of finding the optimum switching levels satisfying various requirements was made by *Webb* and *Steele* [5]. They used the Bit Error Rate (BER) curves of each constituent modulation mode, obtained from simulations over an AWGN channel, in order to find the Signal-to-Noise Ratio (SNR) values, where each modulation mode satisfies the target BER requirement [2]. This intuitive concept of determining the switching levels has been widely used by many researchers [7, 10] since then. *Torrance* and *Hanzo* introduced the average BER of AQAM experienced over fading channels as the constraint and optimised the switching levels for achieving as high an average throughput as possible [11]. Since the corresponding average BER showed a good agreement with the target average BER, *Torrance*'s switching levels have been used for numerous simulation studies [12, 13, 14]. However, since the switching levels are constant across the entire range of SNR values, the average BER varies slightly and the SNR range over which the average BER remains constant is limited. Recently, we proposed a new set of SNR-dependent switching levels optimised at each SNR value in order to maximise the average throughput, while maintaining the target average BER up to the *avalanche SNR* point, beyond which the highest-order constituent modulation mode is activated [15]. Even though this technique renders the adaptive modulation scheme a constant-BER, variable-throughput arrangement, the *Powell* multi-dimensional optimisation technique [16] employed is often trapped in local minima and the heuristic cost function used has to be fine-tuned depending on the target BER and the average SNR values. The aim of this contribution is to derive the globally optimum switching levels using the Lagrangian multiplier technique [17].

In the next section, we present a generic model of constant-power adaptive modulation schemes and derive the closed form expressions of the average throughput and the average BER over Nakagami fading channels. In Section 3, we invoke the Lagrangian multiplier technique in order to obtain the globally optimised switching levels achieving the highest possible average throughput, while maintaining a constant target average BER.

The financial support of LGE, Korea; The CEC, Brussels; EPSRC, UK; and that of the Mobile VCE, UK is gratefully ackowledged.

2. SYSTEM MODEL

A K-mode adaptive modulation scheme adjust its transmit mode to mode-k, where $k \in \{0, 1 \cdots K-1\}$, by employing m_k-ary modulation according to the channel quality ξ perceived at the receiver. The mode selection rule is given by :

$$\text{Choose mode } k, \quad \text{when} \quad s_k \le \xi < s_{k+1} , \tag{1}$$

where a switching level s_k belongs to the set $\mathbf{s} = \{s_k \mid k = 0, 1, \cdots, K\}$. The boundary switching levels are usually given as $s_0 = 0$ and $s_K = \infty$. The Bit Per Symbol (BPS) throughput b_k of a modulation mode k is given as $b_k = \log_2(m_k)$ if $m_k \neq 0$, otherwise $b_k = 0$. It is convenient to define the incremental BPS c_k as $c_k = b_k - b_{k-1}$, when $k > 0$ and as $c_0 = b_0$ provided that $k = 0$.

The channel quality measure ξ can be the instantaneous channel SNR, the Received Signal Strength Indicator (RSSI) output [5], the decoded BER [5], the Signal to Interference-plus-Noise Ratio (SINR) at the equaliser's output [12], or the SINR at the output of a joint detector [13]. However, we restrict our interest here to the adaptive modulation schemes employing the instantaneous SNR per symbol, namely γ, as the channel quality measure ξ. In the presence of both interferences as well as noise, our analysis can still be applied using the SINR instead of the SNR as the channel quality metric, provided that the interference plus noise exhibits a near-Gaussian distribution.

For example, a 5-mode AQAM scheme has been studied extensively due to the superior BER performance of Gray-mapped square QAM constellations in comparison to other m-ary techniques. The parameters of this 5-mode AQAM system are summarised in Table 1.

Wireless fading channels are often modeled as Nakagami fading channels. The Probability Density Function (PDF) of the instantaneous channel SNR γ over a Nakagami fading channel is given as [18]
$f(\gamma) = (m/\bar{\gamma})^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-m\gamma/\bar{\gamma}} , \quad \gamma \ge 0$, where the parameter m governs the severity of fading, $\bar{\gamma}$ is the average SNR and $\Gamma(m)$ is the Gamma function. When $m = 1$, the PDF corresponds to that of a Rayleigh fading channel. As m increases, the fading behaves like Rician fading, and it models the AWGN channel, when m increases to ∞. Here we restrict the value of m to positive integers.

The average throughput $B(\bar{\gamma}, \mathbf{s})$ of our adaptive modulation scheme operating over a Nakagami channel can be expressed in terms of BPS as :

$$B(\bar{\gamma}, \mathbf{s}) = \sum_{k=0}^{K-1} b_k \int_{s_k}^{s_{k+1}} f(\gamma)\, d\gamma = \sum_{k=0}^{K-1} c_k\, F_c(\gamma) , \tag{2}$$

where $F_c(\gamma)$ is the complementary Cumulative Distribution Function (CDF) of the instantaneous SNR γ, given as :

$$F_c(\gamma) \triangleq \int_{\gamma}^{\infty} f(x)\, dx = e^{-m\gamma/\bar{\gamma}} \sum_{i=0}^{m-1} \frac{(m\gamma/\bar{\gamma})^i}{\Gamma(i+1)} . \tag{3}$$

The mode-specific average BER P_k is defined as :

$$P_k \triangleq \int_{s_k}^{s_{k+1}} p_{m_k}(\gamma)\, f(\gamma)\, d\gamma , \tag{4}$$

where $p_{m_k}(\gamma)$ is the BER of the m_k-ary modulation scheme over the AWGN channel. The BER of a Gray-coded square QAM scheme over AWGN channels can be expressed as [2, 19] :

$$p_{m_k}(\gamma) = \sum_i A_i\, Q(\sqrt{a_i \gamma}) , \tag{5}$$

where A_i and a_i are constants. The approximate BER of a Gray-coded m_k-ary coherent PSK ($k \ge 3$) scheme over an AWGN channel can also be expressed using (5) with the constants of $A_1 = A_2 = 2/k$, $a_1 = 2\sin^2(\pi/m_k))$ and $a_2 = 2\sin^2(3\pi/m_k))$ [20]. Upon substituting (5) into (4), we have :

$$\begin{aligned} P_k &= \int_{s_k}^{s_{k+1}} p_{m_k}(\gamma)\, f(\gamma)\, d\gamma \\ &= \sum_i A_i \int_{s_k}^{s_{k+1}} Q(\sqrt{a_i \gamma}) \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\, e^{-m\gamma/\bar{\gamma}}\, d\gamma \\ &= \sum_i A_i \left\{ \left[-e^{-m\,\gamma/\bar{\gamma}}\, Q(\sqrt{a_i \gamma}\,) \sum_{j=0}^{m-1} \frac{(m\,\gamma/\bar{\gamma})^j}{\Gamma(j+1)} \right]_{s_k}^{s_{k+1}} \right. \\ &\quad \left. + \left[\sum_{j=0}^{m-1} X_j(\gamma, a_i) \right]_{s_k}^{s_{k+1}} \right\} , \end{aligned} \tag{6}$$

where $[g(\gamma)]_{s_k}^{s_{k+1}} \triangleq g(s_{k+1}) - g(s_k)$ and X_j is given by :

$$\begin{aligned} &X_j(\gamma, a_i) = \\ &\frac{\mu^2\, \Gamma(j+\frac{1}{2})\, m^j}{\sqrt{2 a_i \pi}\, \Gamma(j+1)\, \bar{\gamma}^j} \sum_{k=1}^{j} \left(\frac{2\mu^2}{a_i}\right)^{j-k} \frac{\gamma^{k-\frac{1}{2}}}{\Gamma(k+\frac{1}{2})}\, e^{-a_i \gamma/(2\mu^2)} \\ &+ \left(\frac{2\mu^2\, m}{a_i \bar{\gamma}}\right)^j \frac{1}{\sqrt{\pi}} \frac{\Gamma(j+\frac{1}{2})}{\Gamma(j+1)}\, \mu\, Q\left(\sqrt{a_i \gamma}/\mu\right) , \end{aligned} \tag{7}$$

where $\mu \triangleq \sqrt{\frac{a_i \bar{\gamma}}{2m + a_i \bar{\gamma}}}$ and $\Gamma(x)$ is the Gamma function. Then, the average BER $P_{avg}(\bar{\gamma}, \mathbf{s})$ of our adaptive modulation scheme over slowly fading Nakagami fading channels can be represented as [21] :

$$P_{avg}(\bar{\gamma}, \mathbf{s}) = \frac{1}{B(\bar{\gamma}, \mathbf{s})} \sum_{k=0}^{K-1} b_k\, P_k . \tag{8}$$

k	0	1	2	3	4
m_k	0	2	4	16	64
b_k	0	1	2	4	6
c_k	0	1	1	2	2
mode	No Tx	BPSK	QPSK	16QAM	64QAM

Table 1: The parameters of 5-mode AQAM system

3. OPTIMUM SWITCHING LEVELS

Our aim is to optimise the set of switching levels, $\mathbf{s}$, so that the average BPS throughput $B(\bar{\gamma};\mathbf{s})$ can be maximised under the constraint of $P_{avg}(\bar{\gamma};\mathbf{s}) = P_{th}$, where P_{th} is the target average BER. Let us define P_R of a K-mode adaptive modulation scheme as the weighted sum of the mode-specific average BER, namely as $P_R(\bar{\gamma};\mathbf{s}) \triangleq \sum_{k=0}^{K-1} b_k\, P_k$, where b_k is the BPS of the k-th constituent fixed-mode modem and the mode-specific average BER P_k is given in (4). Then, with the aid of (8), the average BER constraint can also be written as :

$$P_{avg}(\bar{\gamma};\mathbf{s}) = P_{th} \iff P_R(\bar{\gamma};\mathbf{s}) = P_{th}\, B(\bar{\gamma};\mathbf{s}) \,. \quad (9)$$

As we discussed before, our goal is to maximise our objective function given by the average throughput of (2) $B(\bar{\gamma};\mathbf{s})$ under the constraint of (9). The set of switching levels $\mathbf{s}$ has $K+1$ elements in it. However, since we stipulate $s_0 = 0$ and $s_K = \infty$ in many adaptive modulation schemes, we have $K-1$ independent variables in $\mathbf{s}$. Hence, the optimisation task is a $K-1$ dimensional optimisation under an equality constraint [17]. A standard practice is to introduce a modified objective function using a Lagrangian multiplier and convert the problem into a set of one-dimensional optimisation problems. Hence the modified objective function Λ can be formulated employing a Lagrangian multiplier λ as [17] :

$$\Lambda(\mathbf{s};\bar{\gamma}) = B(\bar{\gamma};\mathbf{s}) + \lambda\left\{P_R(\bar{\gamma};\mathbf{s}) - P_{th}\, B(\bar{\gamma};\mathbf{s})\right\} \quad (10)$$
$$= (1-\lambda P_{th})\, B(\bar{\gamma};\mathbf{s}) + \lambda P_R(\bar{\gamma};\mathbf{s}) \,. \quad (11)$$

The optimum set of switching levels should satisfy :

$$\frac{\partial \Lambda}{\partial \mathbf{s}} = 0 \text{ and } P_R(\bar{\gamma};\mathbf{s}) - P_{th}\, B(\bar{\gamma};\mathbf{s}) = 0 \,. \quad (12)$$

The following relationships are readily derived, which are helpful in evaluating the partial differentiations in (12) :

$$\frac{\partial P_{k-1}}{\partial s_k} = \frac{\partial}{\partial s_k}\int_{s_{k-1}}^{s_k} p_{m_{k-1}}(\gamma)\, f(\gamma)\, d\gamma = p_{m_{k-1}}(s_k)\, f(s_k)$$
$$\frac{\partial P_k}{\partial s_k} = \frac{\partial}{\partial s_k}\int_{s_k}^{s_{k+1}} p_{m_k}(\gamma)\, f(\gamma)\, d\gamma = -p_{m_k}(s_k)\, f(s_k)$$
$$\frac{\partial}{\partial s_k} F_c(s_k) = \frac{\partial}{\partial s_k}\int_{s_k}^{\infty} f(\gamma)\, d\gamma = -f(s_k) \,.$$

Using these results, the partial derivative of P_R and B against s_k can be expressed as :

$$\frac{\partial P_R}{\partial s_k} = b_{k-1}\, p_{m_{k-1}}(s_k)\, f(s_k) - b_k\, p_{m_k}(s_k)\, f(s_k) \quad (13)$$
$$\frac{\partial B}{\partial s_k} = -c_k\, f(s_k) \,, \quad (14)$$

where b_k is the BPS throughput of an m_k-ary modem and c_k is the incremental BPS throughput defined as $c_k \triangleq b_k - b_{k-1}$ in Section 2. Hence, the first condition of $\partial\Lambda/\partial\mathbf{s} = 0$ in (12) results in :

$$-\, c_k(1-\lambda\, P_{th})\, f(s_k) + \lambda\left\{b_{k-1}\, p_{m_{k-1}}(s_k) - b_k p_{m_k}(s_k)\right\} f(s_k) = 0 \,. \quad (15)$$

A trivial solution of (15) is $f(s_k) = 0$. However, the corresponding s_k of either $s_k = \infty$ or $s_k = 0$ in conjunction with some $f(\gamma)$ does not satisfy the second condition given in (12). When $f(s_k) \neq 0$, Equation (15) can be simplified upon dividing both sides by $f(s_k)$, yielding :

$$c_k(\lambda\, P_{th} - 1) - \lambda\left\{b_k p_{m_k}(s_k) - b_{k-1}\, p_{m_{k-1}}(s_k)\right\} = 0 \,. \quad (16)$$

Rearranging (16) for $k=1$ and assuming $c_1 \neq 0$, we have :

$$\lambda\, P_{th} - 1 = (\lambda/c_1)\left\{b_1 p_{m_1}(s_1) - b_0\, p_{m_0}(s_1)\right\} \,. \quad (17)$$

Substituting (17) into (16) and assuming $c_k \neq 0$ for $k \neq 0$, we have :

$$(\lambda/c_k)\{b_k p_{m_k}(s_k) - b_{k-1}\, p_{m_{k-1}}(s_k)\} = (\lambda/c_1)\{b_1 p_{m_1}(s_1) - b_0\, p_{m_0}(s_1)\} \,. \quad (18)$$

We note that the Lagrangian λ is not zero, because substitution of $\lambda = 0$ in (16) leads to $-c_k = 0$ for all k. Hence, we can eliminate the Lagrangian multiplier dividing both sides of (18) by λ. Then we have :

$$y_k(s_k) = y_1(s_1) \text{ for } k = 2, 3, \cdots K-1 \,, \quad (19)$$

where the Lagrangian-free function $y_k(s_k)$ is defined as :

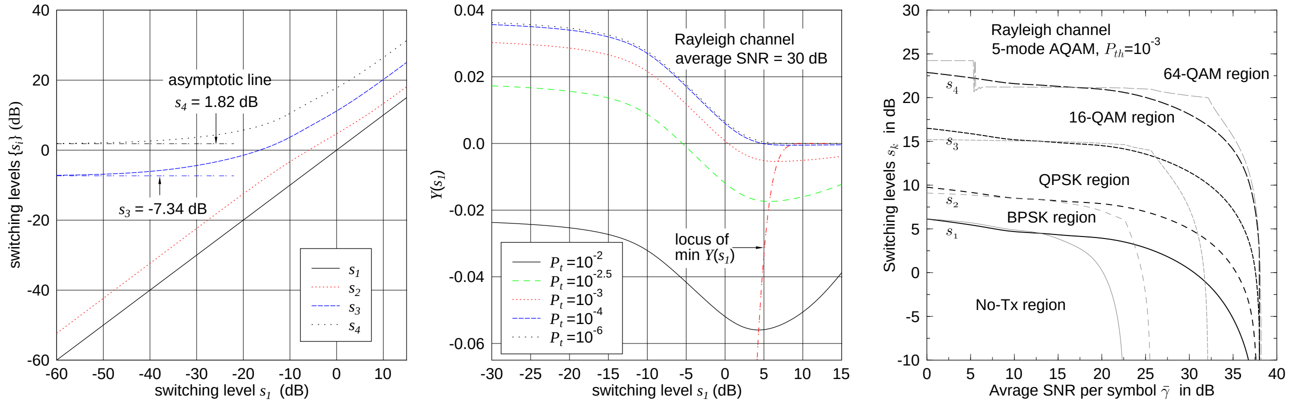
$$y_k(s_k) \triangleq (1/c_k)\left\{b_k p_{m_k}(s_k) - b_{k-1}\, p_{m_{k-1}}(s_k)\right\} \,. \quad (20)$$

The significance of (19) and (20) is that the relationship between the optimum switching level s_k , $k \neq 1$ and s_1 is independent of the underlying channel scenario. Only the constituent modulation mode related parameters, such as b_k, c_k and $p_{m_k}(\gamma)$, govern this relationship. Furthermore, since we made no assumptions concerning the modulation modes employed, (19) holds for generic adaptive modulation schemes.

It is straightforward to solve (19) numerically, in order to find s_k as a function of s_1, since it is a one-dimensional root finding problem [16]. As an example, the relationship between the optimum switching levels of the 5-mode AQAM scheme is depicted in Figure 1(a) using the parameters summarised in Table 1,

Since we can relate the remaining switching levels to s_1, we have to determine the optimum value of s_1 for the given target BER P_{th} using the PDF of the instantaneous channel SNR $f(\gamma)$ and the second condition given in (12). This problem is also a one-dimensional root finding problem rather than a multi-dimensional optimisation problem, which was the case in [11, 15], where the latter sometimes settles in local minima. Let us define the constraint function $Y(\bar{\gamma};\mathbf{s}(s_1))$ as :

$$Y(\bar{\gamma};\mathbf{s}(s_1)) \triangleq P_R(\bar{\gamma};\mathbf{s}(s_1)) - P_t\, B(\bar{\gamma};\mathbf{s}(s_1)) \,, \quad (21)$$



(a) Relationship between s_k and s_1 (b) Constraint function Y (c) Optimised switching levels

Figure 1: Switching level optimisation for 5-mode AQAM. (a) There exists a fixed relationship between the optimum switching levels s_k and s_1, regardless of the underlying channel scenario. (b) The constraint equation of (9) has a unique solution, when $Y(\bar{\gamma}, \mathbf{s}(s_1 = 0)) < 0$. (c) The optimised switching levels for $P_{th} = 10^{-3}$. The switching levels of [15] are represented by corresponding thin grey lines for comparison.

where we used $\mathbf{s}(s_1)$ to emphasise that s_k is dictated by s_1 according to the relationships of (19) and (20). Even though the relationship implied by $\mathbf{s}(s_1)$ is independent of the channel conditions and of the signaling power, the constraint function Y of (21) and hence the actual values of the optimum switching levels are dependent on them through the PDF $f(\gamma)$ of the SNR per symbol and through the average SNR per symbol $\bar{\gamma}$.

The first derivative of $Y' = dY/ds_1$ can be expressed as :

$$Y' = (b_0\, p_{m_0}(s_1) - b_1\, p_{m_1}(s_1) + P_{th}) \sum_{k=1}^{K-1} \frac{c_k}{c_1} f(s_k) \frac{ds_k}{ds_1} \ . \quad (22)$$

A further study of Y and Y' revealed that Y has its first maximum at $s_1 = 0$, its minimum, when $b_1\, p_{m_1}(s_1) - b_0\, p_{m_0}(s_1) = P_{th}$ and its other asymptotic maximum of $Y = 0^-$ at $s_1 = \infty$. Hence, when satisfying the second condition in (12) expressed as $Y = 0$ we have a unique solution, provided that $Y(\bar{\gamma}; \mathbf{s}(0)) > 0$.

Figure 1(b) depicts the values of Y for 5-mode AQAM using various target BERs of P_{th}, when the average channel SNR is 30dB. We can observe that $Y = 0$ may have a root depending on the target BER P_{th}. When $s_k = 0$ for $k < 5$, the value of Y becomes :

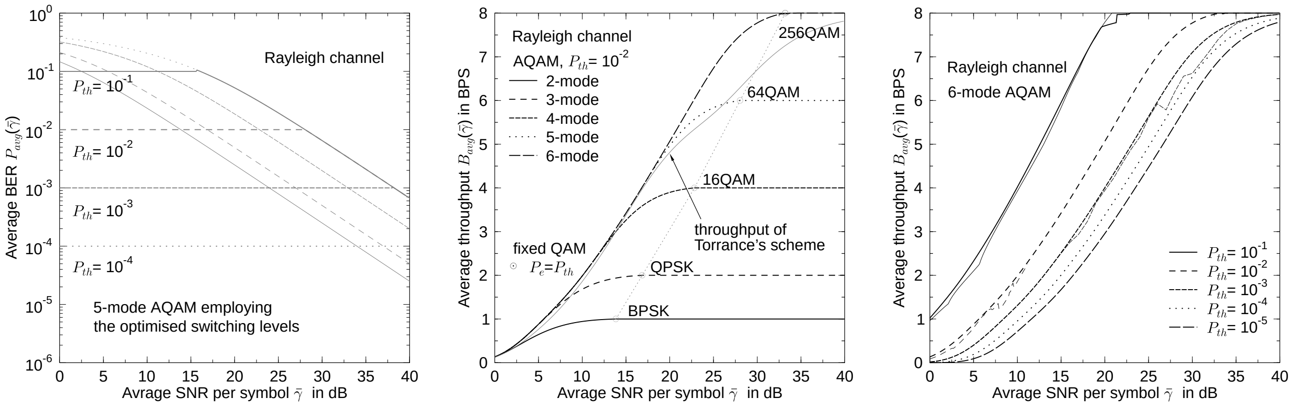
$$Y(\bar{\gamma}; 0) = 6(P_{64}(\bar{\gamma}) - P_{th}) \ , \quad (23)$$

where $P_{64}(\bar{\gamma})$ is the average BER of 64-QAM over a flat Rayleigh channel. The value of $Y(\bar{\gamma}; 0)$ in (23) can be negative or positive, depending on the target BER P_{th}. Figure 1(c) depicts the switching levels optimised in this manner for the 5-mode AQAM scheme achieving the target average BER of 10^{-3}. The optimised switching levels obtained using Powell's minimisation method for each SNR [15] are represented as thin grey lines in Figure 1(c) for comparison. When using the Lagrangian optimisation, the various modulation modes are not abandoned, until the average SNR reaches the *avalanche SNR* around 38dB, while Powell's minimisation required the AQAM regime to abandon the lower-order modulation modes one by one, as the average SNR increased.

Figure 2(a) depicts the average BER performance of 5-mode AQAM employing the Lagrangian optimised switching levels operating over a Rayleigh channel. The average BER remains constant until the average SNR reaches the *avalanche SNR*, where the average BER of the highest-order constituent modulation mode, *ie* 64-QAM, satisfies the target average BER constraint. Figure 2(b) depicts the average BPS throughput B of the AQAM scheme employing the optimised switching levels. The average throughput of 6-mode AQAM using *Torrance*'s scheme [11] is represented by the thin grey line. The Lagrangian based scheme showed SNR gains of 0.6dB, 0.5dB, 0.2dB and 3.9dB at a BPS throughput of 1, 2, 4 and 6, respectively, compared to *Torrance*'s scheme. The average throughput of 6-mode AQAM is depicted in Figure 2(c) for the several values of P_{th} are, where the BPS values of the AQAM scheme employing the switching levels optimised individually for each SNR value [15] using *Powell*'s method [16] are also represented as thin lines for $P_{th} = 10^{-1}$, 10^{-2} and 10^{-3}. Comparing the BPS curves, we conclude that the 'per-SNR' Powell optimisation method of [15] resulted in imperfect optimisation for some values of the average SNR. The schemes of [15] were unable to generate a reasonable set of switching levels leading to a monotonically increasing average BPS throughput for the target average value of $P_{th} = 10^{-4}$ and 10^{-5}.

4. CONCLUSION

We derived the optimum mode-switching levels for a generic constant-power adaptive modulation schemes achieving the highest possible average throughput, while maintaining a constant average BER. We found that there exists a fixed relationship between the switching levels regardless of the underlying channel model. Based on the closed form expressions of the aver-



(a) Average BER of the 5-mode AQAM (b) Average BPS for $P_{th} = 10^{-2}$ (c) Average BPS of the 6-mode AQAM

Figure 2: Performance of AQAM employing the optimised switching levels over a Rayleigh channel. (a) The average BER remains constant until the average SNR reaches the *avalanche SNR*, where the average BER of the highest modulation mode is down to P_{th}. (b) The average throughput of the 6-mode AQAM using Torrance's switching levels [11] is represented as a thin grey line for comparison. (c) The average throughputs of the 6-mode AQAM employing per-SNR optimised switching levels [15] represented in thin lines for $P_{th} = 10^{-1}$, 10^{-2} and 10^{-3} only.

age throughput and of the average BER derived for transmissions over Nakagami fading channels, we presented numerical results for AQAM operating over a Rayleigh fading channel as an example. Compared to other existing switching level design schemes, our technique exhibited superior performance. One of the advantages of our optimised constant-power adaptive modulation systems is that it can be readily combined with other power control schemes in cellular environments, since our scheme does not require a transmit-power adjustment for maintaining optimal operation, unlike the variable-power, variable-rate AQAM schemes of [10].

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