Blind FIR Equalisation for High-Order QAM Signalling

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Overview

Low-complexity affordable blind equalisation for high-order QAM

◯ Constant modulus algorithm

◯ Concurrent CMA and decision-directed scheme (De Castro *et al*, ICC'2001)

◯ Multi-stage blind clustering or bootstrap maximum *a posteriori* probability scheme (Chen *et al*, ICC'1993)

Comparative study

Blind Equalisation

○ Baseband channel model

$$r(k) = \sum_{i=0}^{n-1} a_i s(k-i) + e(k)$$

$a_i = a_{iR} + j a_{iI}$: channel taps; $s(k) = s_R(k) + j s_I(k) \in M$-QAM set:

$$\mathcal{S} = \{s_{il} = (2i - Q - 1) + j(2l - Q - 1), \quad 1 \leq i, l \leq Q\}$$

with $Q = \sqrt{M} = 2^L$ and L an integer; $e(k) = e_R(k) + j e_I(k)$: Gaussian white with $\mathrm{E}[e_R^2(k)] = \mathrm{E}[e_I^2(k)] = \sigma_e^2$

○ Equaliser

$$y(k) = \sum_{i=0}^{m-1} w_i r(k-i) = \mathbf{w}^T \mathbf{r}(k)$$

$\mathbf{w}(k) = [w_0 \ w_1 \cdots w_{m-1}]^T$: equaliser weight vector with $w_i = w_{iR} + j w_{iI}$;
$\mathbf{r}(k) = [r(k) \ r(k-1) \cdots r(k-m+1)]^T$: equaliser input vector

Concurrent CMA and DD Scheme

De Castro *et al* (ICC'2001): $\mathbf{w} = \mathbf{w}_c + \mathbf{w}_d$

○ CMA equaliser $\mathbf{w}_c$ minimises

$$\bar{J}_{\mathrm{CMA}}(\mathbf{w}) = \mathsf{E}\left[\left(|y(k)|^2 - \Delta_2\right)^2\right]$$

$$\left.\begin{aligned} \epsilon(k) &= y(k)\left(\Delta_2 - |y(k)|^2\right) \\ \mathbf{w}_c(k+1) &= \mathbf{w}_c(k) + \mu_c \epsilon(k) \mathbf{r}^*(k) \end{aligned}\right\}$$

○ DD equaliser $\mathbf{w}_d$ minimises

$$\bar{J}_{\mathrm{DD}}(\mathbf{w}) = \frac{1}{2}\mathsf{E}\left[|\mathcal{Q}[y(k)] - y(k)|^2\right]$$

with quantized equalizer output defined by

$$\mathcal{Q}[y(k)] = \arg \min_{s_{il} \in \mathcal{S}} |y(k) - s_{il}|^2$$

DD adaptation follows CMA adaptation:

$$\mathbf{w}_d(k+1) = \mathbf{w}_d(k) + \mu_d \delta(\mathcal{Q}[\tilde{y}(k)] - \mathcal{Q}[y(k)])(\mathcal{Q}[y(k)] - y(k))\mathbf{r}^*(k)$$

where $\mathcal{Q}[\tilde{y}(k)]$ and $\mathcal{Q}[y(k)]$ are equaliser hard decisions after and before CMA adaptation, and indicator function

$$\delta(x) = \begin{cases} 1, & x = 0 + j0 \\ 0, & x \neq 0 + j0 \end{cases}$$

- $\mathbf{w}_d$ is updated only if equaliser hard decisions before and after CMA adaptation are the same

- Hard decision directed adaptation: if decision is wrong $\Rightarrow$ error propagation

- If equaliser hard decisions before and after CMA adaptation are the same, decision is probably right one

Bootstrap MAP Scheme

After correct equalisation, equaliser output

$$y(k) \approx s(k - k_d) + v(k)$$

with

$$\left[\begin{array}{cc} \mathsf{E}[v_R^2(k)] & \mathsf{E}[v_R(k)v_I(k)] \\ \mathsf{E}[v_I(k)v_R(k)] & \mathsf{E}[v_I^2(k)] \end{array} \right] \approx \left[\begin{array}{cc} \rho & 0 \\ 0 & \rho \end{array} \right]$$

Thus, the *a posteriori* PDF of $y(k)$ is approximately

$$p(\mathbf{w}, y(k)) \approx \sum_{q=1}^{Q} \sum_{l=1}^{Q} \frac{p_{ql}}{2\pi\rho} \exp\left(- \frac{|y(k) - s_{ql}|^2}{2\rho} \right)$$

where p_{ql} are the *a priori* probabilities of symbols s_{ql}, $1 \le q, l \le Q$.

Bootstrap MAP equaliser maximises log of the *a posteriori* PDF criterion

$$\bar{J}_{\mathrm{MAP}}(\mathbf{w}) = \mathsf{E}[J_{\mathrm{MAP}}(\mathbf{w}, y(k))]$$

where

$$J_{\mathrm{MAP}}(\mathbf{w}, y(k)) = \rho \log\left(p(\mathbf{w}, y(k))\right)$$

using stochastic gradient algorithm

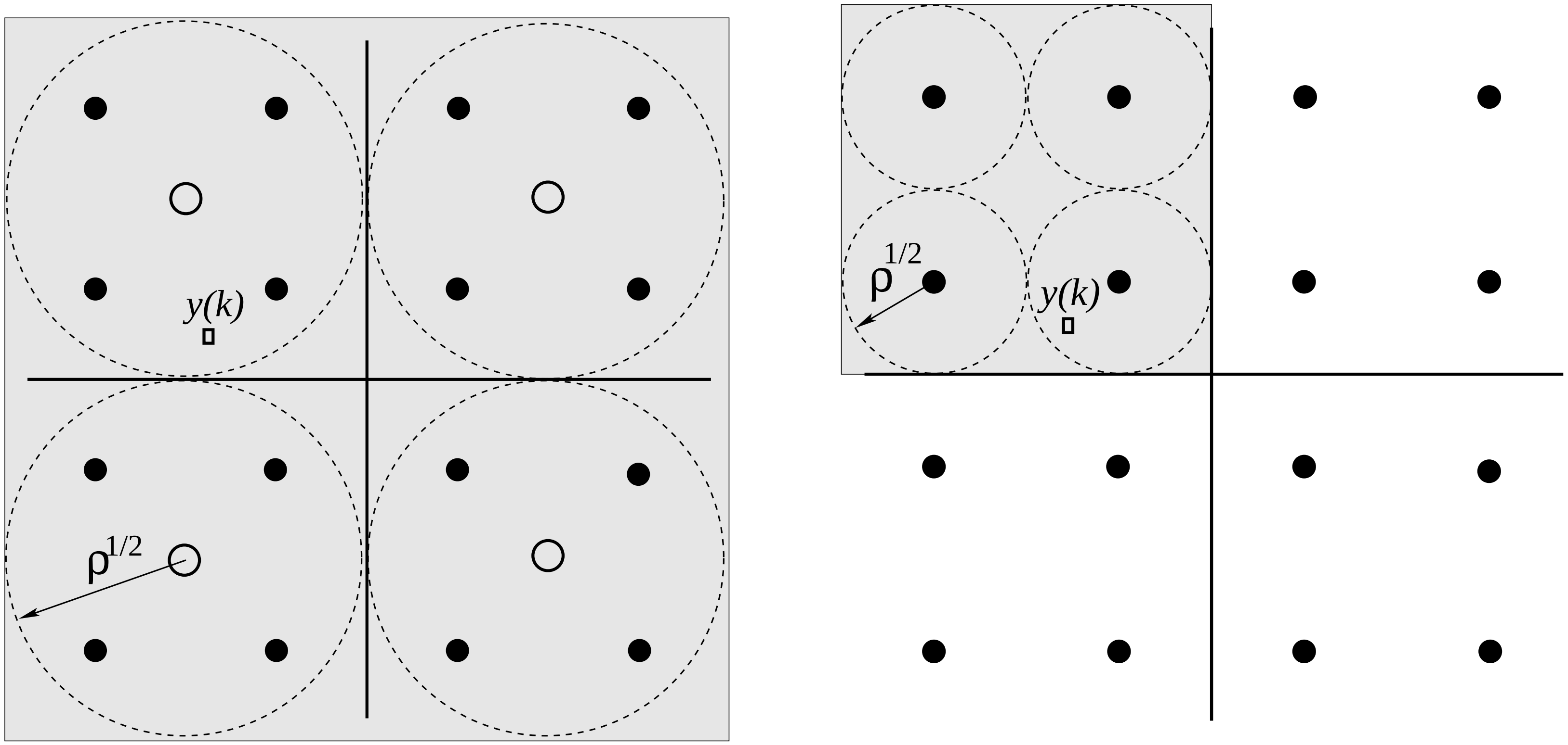
$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \frac{\partial J_{\mathrm{MAP}}(\mathbf{w}(k), y(k))}{\partial \mathbf{w}}$$

given stochastic gradient

$$\frac{\partial J_{\mathrm{MAP}}((\mathbf{w}(k), y(k))}{\partial \mathbf{w}} = \frac{\sum_{q=1}^{Q}\sum_{l=1}^{Q} \exp\left(-\frac{|y(k)-s_{ql}|^2}{2\rho}\right)(s_{ql} - y(k))}{\sum_{q=1}^{Q}\sum_{l=1}^{Q} \exp\left(-\frac{|y(k)-s_{ql}|^2}{2\rho}\right)} \mathbf{r}^*(k)$$

- $L = \log_2(M)/2$ stage procedure to achieve minimum complexity

Two-stage for 16-QAM



First Stage   Second Stage

Computational Complexity per Weight Update

equaliser	multiplications	additions	$\exp(\cdot)$ evaluations
CMA	$8 \times m + 6$	$8 \times m$	–
CMA+DD	$16 \times m + 8$	$20 \times m$	–
MAP	$8 \times m + 23$	$8 \times m + 19$	4

- m is equaliser order
- Four $\exp(\cdot)$ evaluations implemented through look up table
- Tuning of bootstrap MAP equaliser more complicated, as it involves L stage switchings, each having a set of different algorithm parameters

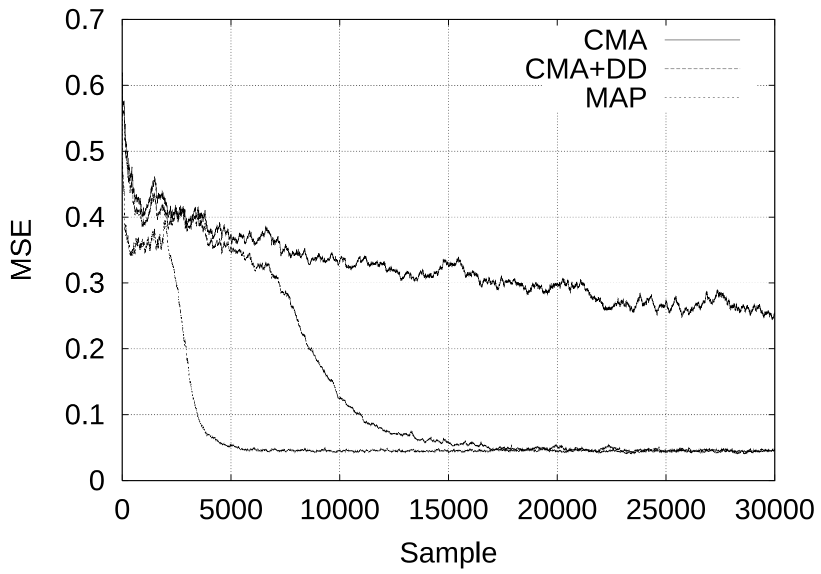
16-QAM Example

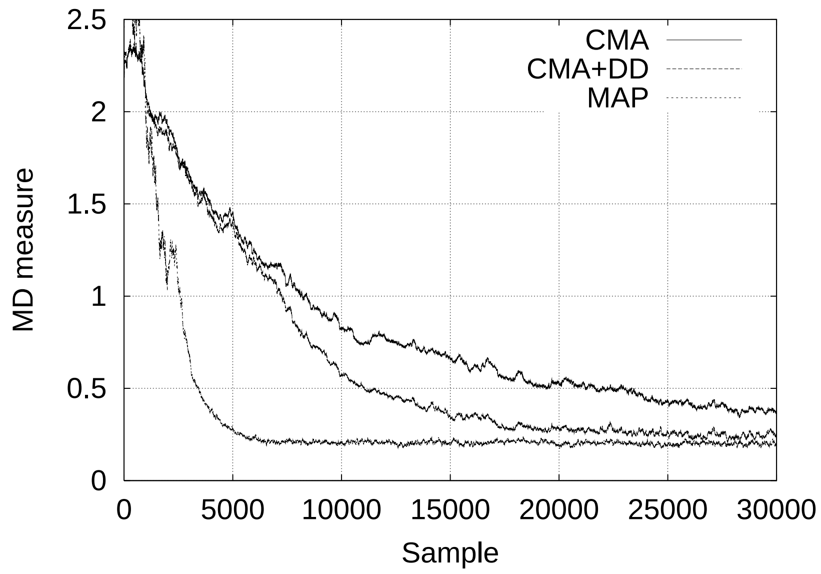
22-tap channel, 23-tap equaliser, channel SNR$=25$ dB

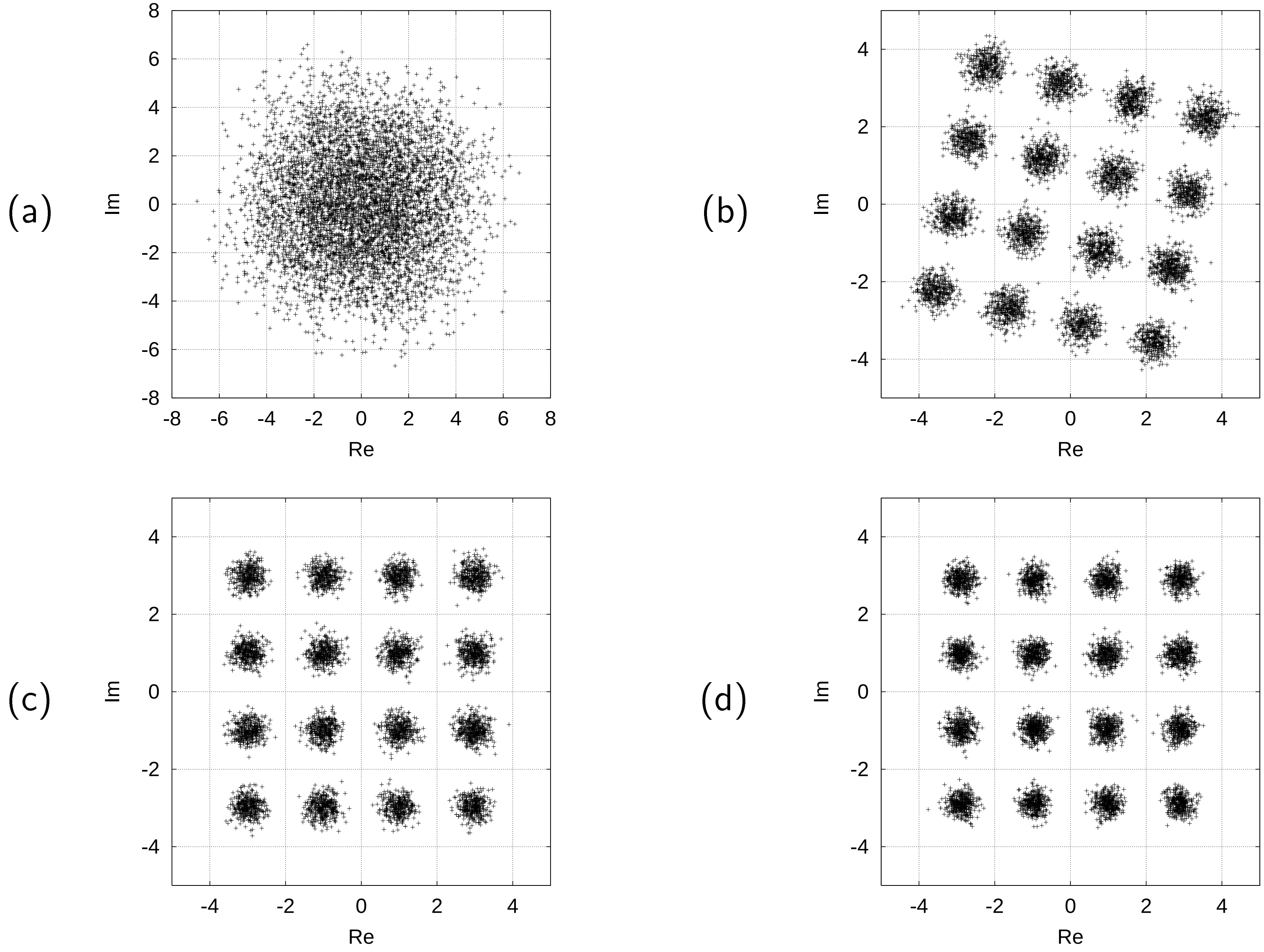
◯ Estimated MSE; and maximum distortion measure:

$$\mathrm{MD}=\frac{\sum_{i=0}^{n_c-1}|f_i|-|f_{i_{\max}}|}{|f_{i_{\max}}|}$$

with $\{f_i\}_{i=0}^{n_c-1}$ combined impulse response of channel and equaliser



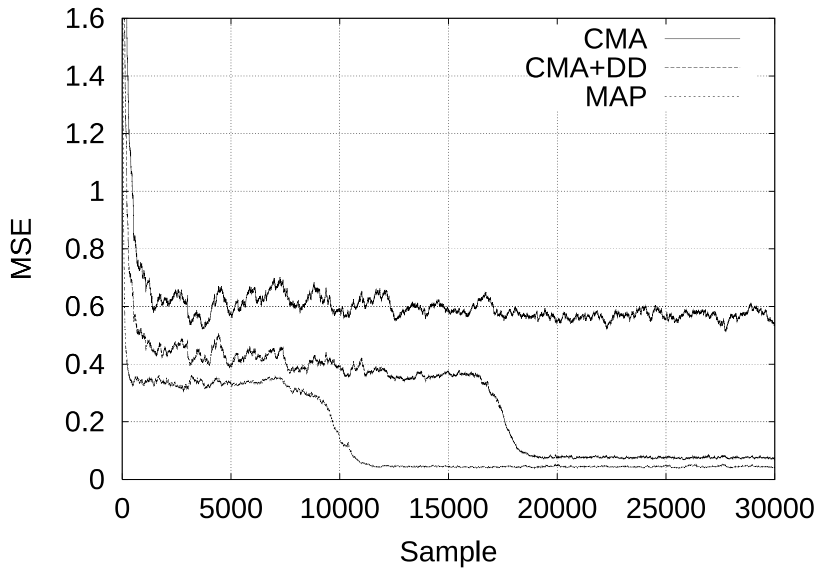


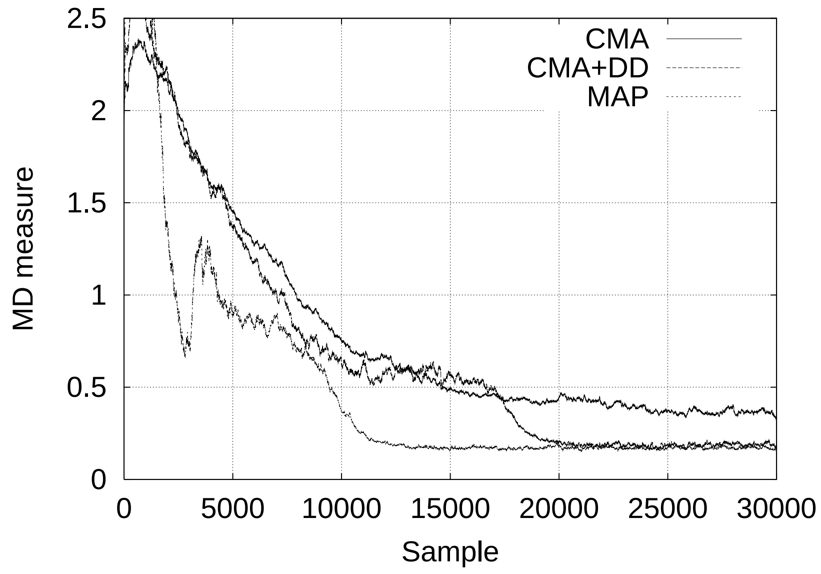


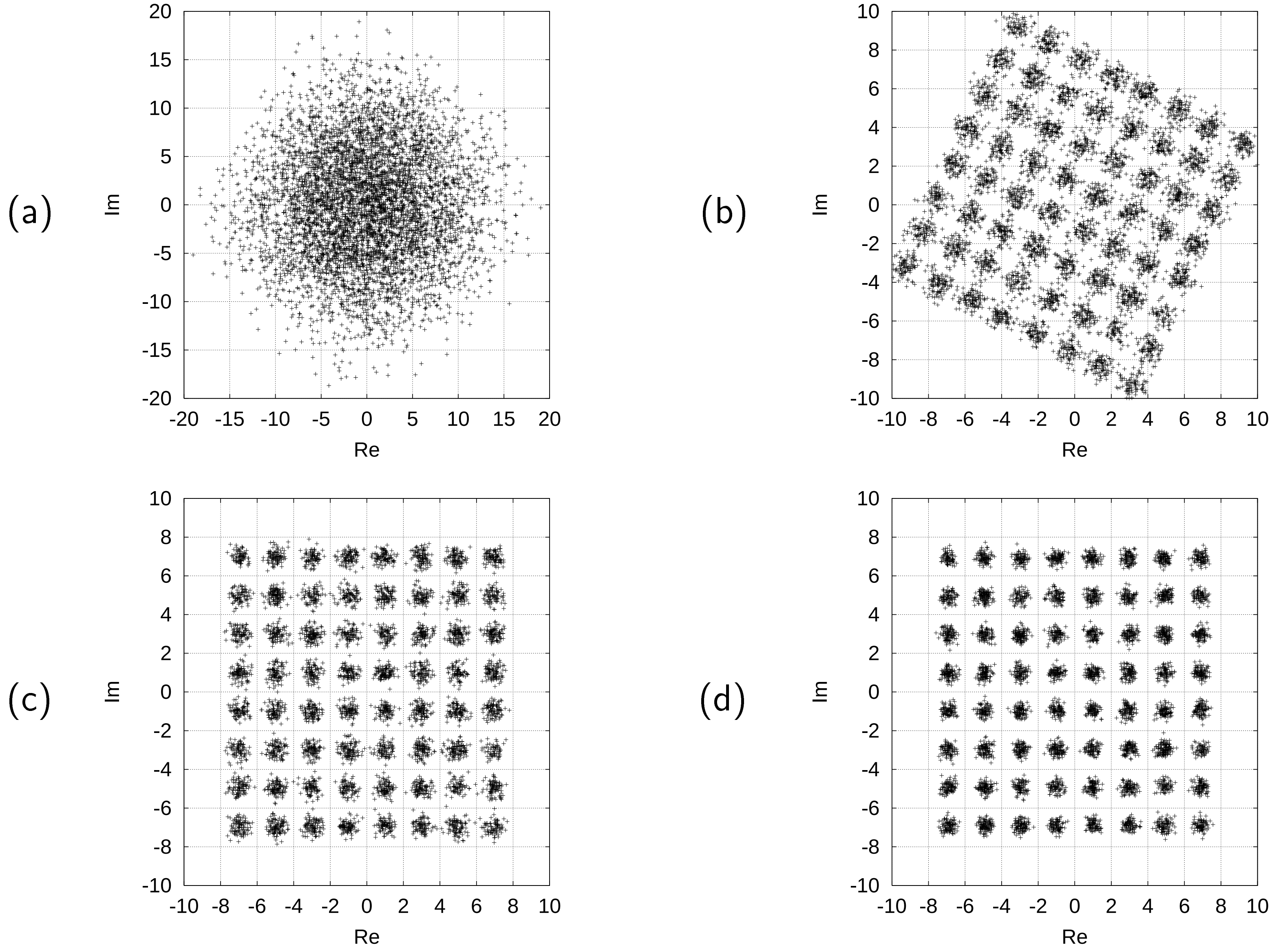
(a) unequalised; (b) CMA; (c) CMA+DD; (d) MAP

64-QAM Example

5-tap channel, 23-tap equaliser, channel SNR$= 30$ dB



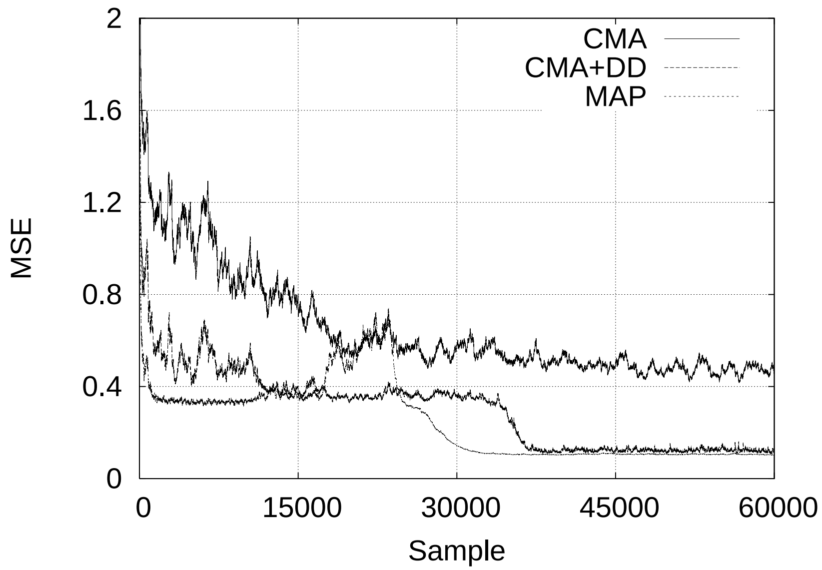


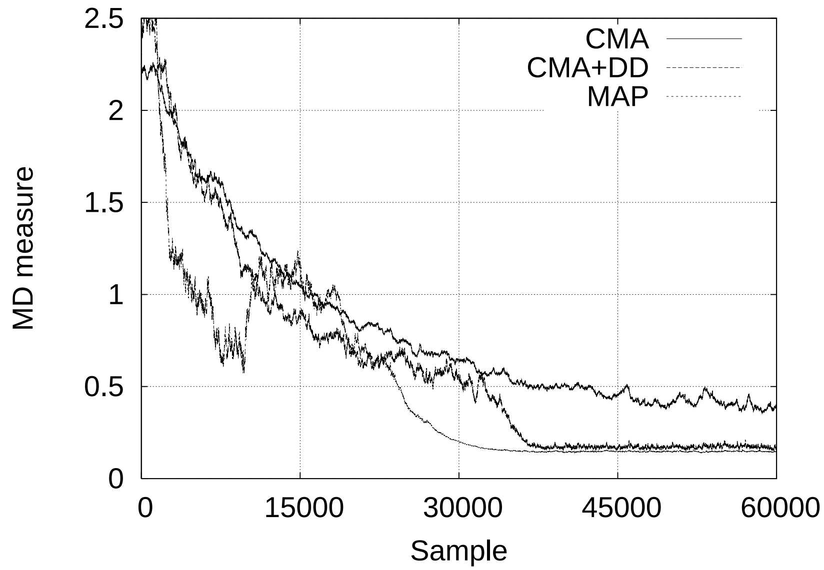


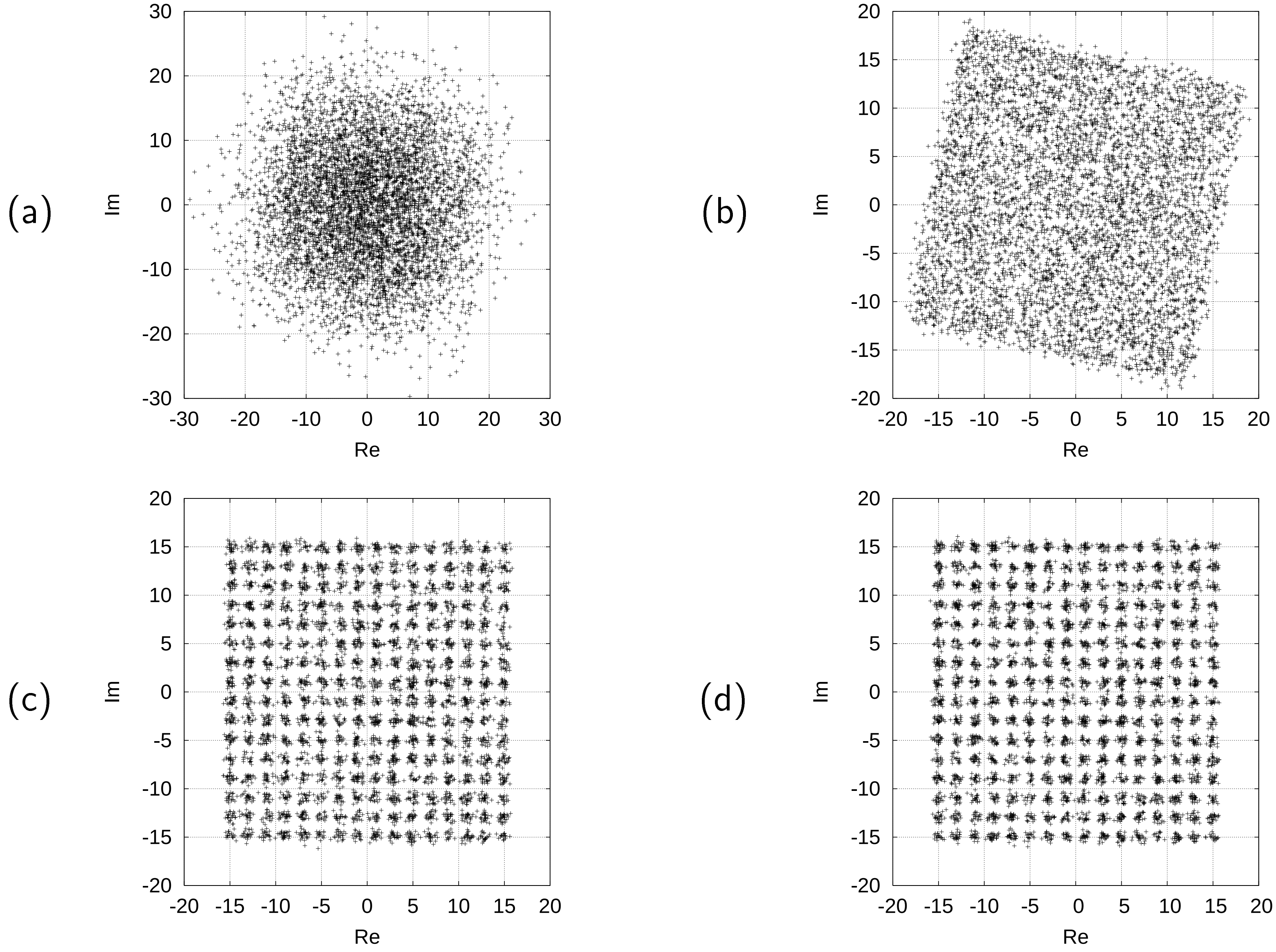
(a) unequalised; (b) CMA; (c) CMA+DD; (d) MAP

256-QAM Example

22-tap channel, 23-tap equaliser, channel SNR$= 80$ dB







(a) unequalised; (b) CMA; (c) CMA+DD; (d) MAP

Conclusions

Two novel low-complexity blind equalisers: concurrent CMA and DD scheme and bootstrap MAP scheme

$\Uparrow$ Bootstrap MAP blind equaliser has simpler computational complexity per weight update, faster convergence rate, and marginally better steady-state equalisation performance

$\Downarrow$ Tuning of bootstrap MAP blind equaliser is more complicated, as it involves L stage switchings, each having a set of different algorithm parameters

$\bigcirc$ Both offers significant improvement in equalisation performance over the very simple CMA