RBF-based decision feedback aided turbo equalisation of convolutional and space-time trellis-coded systems

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A reduced-complexity Jacobian radial basis function aided turbo equalisation (TEQ) scheme is proposed, which was found to provide a bit error ratio performance similar to that of the conventional trellis-based TEQ benchmarker at a 14-fold reduced complexity for a serially concatenated systematic convolutional coded and systematic space-time trellis-coded arrangement.

Introduction: Bauch *et al.* [1] proposed a joint channel equalisation and space-time trellis (STT) decoding scheme, which yielded an improved performance in contrast to separate channel equalisation and STT coding by iteratively exploiting the soft-decision-based information fed back from the STT decoder's output to the channel equaliser's input. In [2], the performance of the STT encoded system was further improved by employing additional channel encoding in conjunction with turbo equalisation. However, due to the associated computational complexity, the employment of this scheme was limited to low-order modulation modes, such as 4PSK. In this work we aimed to reduce the complexity associated with the CC-SSTTC system of [2] by invoking a reduced-complexity radial basis function (RBF) turbo equalisation (TEQ) [3], which we will refer to as the RBF-TEQ-STTC scheme.

RBF aided channel equaliser for space-time coding: Channel equalisation encountered in a p-transmitter space-time coded system can be considered as a geometric classification problem [4], namely that of classifying the received phasor into one of the M^p classes that represent the transmitted symbol vector $\mathbf{x}_k^p = [x_{1,k} \ldots x_{p,k}]$ during each signalling instant k. The channel output at instant k is given by $y_k = \Sigma_{i=1}^{p} \mathbf{h}_i \mathbf{x}_{i,k}^T + \eta_k$, where the ith channel impulse response (CIR) $\mathbf{h}_i = [h_{0,i}\ h_{1,i} \ldots h_L]$ $i = 1, \ldots, p$, having a memory of L symbols, is convolved with a sequence of $L + 1$ transmitted symbols, namely with $\mathbf{x}_{i,k} = [x_{i,k}\ x_{i,k-1} \ldots x_{i,k-L}]$ while η_k is the additive Gaussian noise term having a variance of σ_η. For a p-transmitter system using an m-tap equaliser at the receiver and communicating over a channel having a CIR memory of L (assuming that all of the p CIRs have the same memory), there are $n_s = M^{(m+L)\cdot p}$ number of possible received phasor combinations due to the transmitted sequence. Hence there are n_s number of different possible channel output vectors in the absence of channel noise, which are expressed as $\tilde{\mathbf{y}}_k = [\tilde{y}_k\ \tilde{y}_{k-1} \ldots \tilde{y}_{k-m+1}]$, where m is the equaliser order. Upon adding the noise we have the m-symbol received channel output vector $\mathbf{y}_k = \tilde{\mathbf{y}}_k + \eta_k$. Expounding further, we denote each of the n_s number of different possible combinations of the channel's input sequence $\bar{\mathbf{x}}_k = [\mathbf{x}_k^p \ldots \mathbf{x}_{k-L-m+1}^p]$ of length $(L + m) \times p$ symbols as $\mathbf{s}_i$, where the channel's input state $\mathbf{s}_i$ determines the desired channel output state $\mathbf{r}_i$, $i = 1, \ldots, n_s$. This is formulated as $\tilde{\mathbf{y}}_k = \mathbf{r}_i$, if $\bar{\mathbf{x}}_k = \mathbf{s}_i$, $i = 1, \ldots, n_s$. The equaliser has to provide the associated nonlinear decision boundaries for the classification algorithm.

The RBF equaliser provides the so-called optimal Bayesian equalisation solution [4] and generates the conditional probability density functions of all the M^p number of possible transmitted symbols $\mathbf{x}_{k-\tau}^p$ emitted by the transmitters at instant $k - \tau$ in the form of

$$P(\mathbf{y}_k|\mathbf{x}_{k-\tau}^p = \mathbf{I}_j) = \sum_{i=1}^{n_s^j} p_{i,j}(2\pi\sigma_\eta^2)^{-m/2} \exp\left\{-\frac{1}{2\sigma_\eta^2}\|\mathbf{y}_k - r_{i,j}\|^2\right\} \quad j = 1, \ldots, M^p \qquad (1)$$

The term n_s^j is the number of possible channel states $\mathbf{r}_{i,j}$ corresponding to the jth transmitted symbol sequence $\mathbf{I}_j$, $j = 1, \ldots, M^p$ of the p-antenna SSTTC scheme. The term $p_{i,j}$ is the *a priori* probability of occurance of the channel state $r_{i,j}$. The RBF equaliser of [4] has an excessive complexity. Hence, here we advocate the reduced-complexity Jacobian RBF equaliser introduced in [5]. The *a posteriori* probability of the transmitted symbols $\mathbf{x}_{k-\tau}^p$ in eqn. 1 provides the *a posteriori* log-likelihood ratio (LLR) values of the convolutionally-coded symbols, which can then be fed to the STT decoder. The *a priori* probability of occurrence of the ith channel state $\mathbf{r}_{i,j}$ corresponding to the transmitted symbol sequence $\mathbf{I}_j$, $p_{i,j}$, can be evaluated from the LLRs generated by the STT decoder as described below.

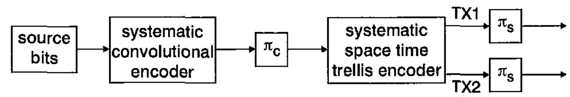

Fig. 1 *Transmitter of serially concatenated systematic convolutional coded and systematic STT coded system*

π_c channel bit interleaver
π_s space-time symbol interleaver

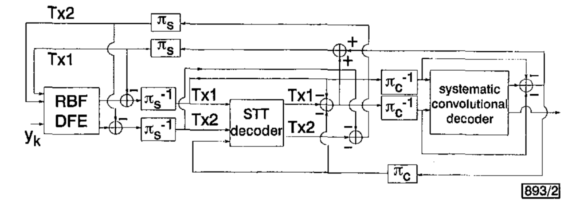

Fig. 2 *Receiver of serially concatenated systematic convolutional coded and systematic STTC system using RBF DFE assisted turbo equalisation*

π_c channel bit interleaver
π_c^{-1} channel bit deinterleaver
π_s space-time symbol interleaver
π_s^{-1} space-time symbol deinterleaver

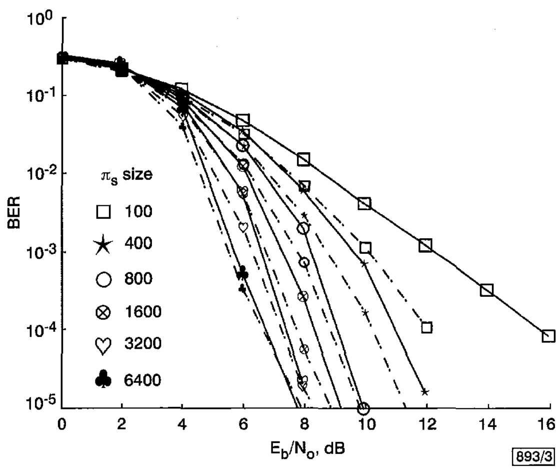

Fig. 3 *BER performance of RBF DFE (m = 2, n = 1, τ = 1) assisted turbo-equalised serially concatenated convolutional coded and STTC system using various STTC interleaver sizes, after eight turbo equalisation iterations*

Performance of CT-TEQ-SSTTC system is also shown as a benchmark
——— RBF-TEQ-SSTTC
—·—·— TTEQ-SSTTC

System overview: Fig. 1 is the schematic diagram of the CC-SSTTC transmitter which consists of a serially concatenated systematic convolutional encoder and a systematic STT encoder. The convolutional encoder, denoted as CC (2, 1, 3), is a half-rate recursive systematic convolutional (RSC) coding scheme having a constraint length of $K = 3$ and octal generator polynomials of $G_0 = 7$ and $G_1 = 5$. We denote the systematic STT encoder used as the SSTTC ($n = 4$, $m = 4$) scheme, since it is an $n = 4$-state, $m = 4$-PSK-based STT code [6]. Fig. 2 is the schematic diagram of the receiver. The channel equaliser of Fig. 2 computes the *a posteriori* LLR values for the systematic STT coded symbols of both transmitter TX1 and TX2. Subsequently, these LLR values are deinterleaved by the STT deinterleaver π_s^{-1} and passed to the SSTTC (4, 4) decoder. In the first iteration, the channel equaliser only evaluates the received signal y_k, since there is no *a priori* feedback information from the output of the RSC decoder. However, in subsequent iterations the channel equaliser will receive additional *a priori* information concerning the STT codeword from the other decoding stages. The extrinsic information is forwarded to the next component of the receiver as detailed in [2].

In our investigations the transmission burst structure consists of 100 data symbols. A two-path, symbol-spaced fading CIR of equal weights was used, where the Rayleigh fading statistics obeyed a normalised Doppler frequency of 3.3615×10^{-5}. The fading magnitude and phase was kept constant for the duration of a transmission burst. We have assumed that the CIR was perfectly estimated at the receiver in order to investigate the best-case performance of these systems. At the receiver, the systematic STT decoder and the RSC decoder employed the log-MAP algorithm [7]. The Jacobian RBF DFE has a feedforward order of $m = 2$, feedback order of $n = 1$ and decision delay of $\tau = 1$.

Results and conclusions: Fig. 3 shows the performance of the proposed RBF-TEQ-STTC and that of the conventional trellis-based TEQ-STTC (CT-TEQ-STTC) scheme [2], using various STTC interleaving sizes and eight turbo equalisation iterations. It can be observed from Fig. 3 that by increasing the STTC interleaving size from 100 to 6400, the performance degradation of the RBF-TEQ-STTC scheme compared to the CT-TEQ-STTC arrangement of [2] expressed in terms of the excess signal-to-noise ratio (SNR) required for attaining a bit error rate (BER) of 10^{-4} decreases from 3.8 dB recorded for an STTC interleaver size of 100 symbols to 0 dB, as observed for the STTC interleaver size of 6400 symbols. This is because the error propagation of the RBF DFE component decreases, as the BER performance improves, when using a longer STTC interleaver. The performance difference of the two schemes is < 1 dB at a STTC interleaver length of 400 symbols, although the RBF-TEQ-STTC scheme has a substantially lower computational complexity, as it will be shown in the context of Table 1. The interleaving gain attained by the RBF-TEQ-STTC scheme was ~9 dB at a BER of 10^{-4}. Although higher interleaving gains can be achieved using longer STTC interleavers, the interleaver gain gradually saturates, when the STTC interleaver size is in excess of 1600 symbols. Following the approach of our computational complexity study in [3], Table 1 summarises the computational complexity of generating the *a posteriori* LLRs for each received signal at instant k in the context of a p-transmitter space-time coded system. The complexity imposed by the RBF-TEQ-STTC ($m = 2$, $n = 1$, $r = 1$) scheme was found to be a factor of 14 lower than that of the CT-TEQ-STTC scheme in the context of a two transmitter, one receiver system, e.g. if we used a higher-order modulation mode, such as the 8PSK mode used in [6] along with the same number of transmitters, as well as equaliser and channel parameters, the achievable computational complexity reduction is a factor of 55.

Table 1: Computational complexity of generating *a posteriori* LLRs for trellis-based equaliser and for Jacobian RBF equaliser [3]

	Trellis-based	Jacobian RBF
Subtract and add	$n_{s,f}(6M^p + 2) - 3$	$n_{s,f} + M^p n_s^i (m + 2) - 4$
Multiply and divide	$2n_{s,f}$	$2n_{s,f}$

Feedforward and feedback order of RBF equaliser denoted by m and n, respectively; number of RBF nodes is $n_i^s = M^{(m+L-n)\cdot p}/M$, $i = 1, ..., M^p$, where L is the CIR memory and p is the number of STTC transmitters; notation $n_{s,f} = M^{(L+1)\cdot p}$ indicates number of trellis transitions encountered in the trellis-based equaliser, also the number of possible different noise-free channel outputs $\tilde{y}_k$ of Jacobian RBF equaliser

In conclusion, we found that the Jacobian RBF equaliser-based TEQ constitutes a better design choice in STTC systems than the trellis-based SSTTC scheme of [2], especially in the context of STTC schemes using a high number of encoder states in an effort to improve the performance of the system, provided that a sufficiently high STTC interleaver length is affordable.

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Sliding window and interleaver design

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Terminating the trellis was considered essential to lower the bit error probability in a turbo coded system. When a sliding window algorithm is used, this issue becomes even more important. It is shown that trellis termination can be completely ignored by using an interleaver that takes into account the particular window size of the sliding window algorithm.

Introduction: It is well known that the decoding of convolutional codes relies on the memory of the code. A Viterbi decoder, at any moment, estimates the path metrics for all possible states and chooses accordingly the most likely path in the trellis. This means that if one goes back far enough on that path, say five times the memory of the code, the decoded data is the most likely transmitted data. However, the closer one gets to the last decoded bit, the more unreliable the decoded data is. The same thing happens to a maximum *a posteriori* (MAP) decoder. In the forward direction, the state metrics are highly reliable since the decoder starts from a known state, usually all zero state. In the backward direction, the situation is more complicated since the starting state is not known.

A histogram that plots the bit error rate (BER) function of the bit position in a frame would show an almost constant BER for the majority bits in a frame, except for the first and the last few bits in the frame. The first few bits are decoded given a well known starting state for the decoder, therefore the outcome is a lower BER. The opposite happens with the last bits in the frame which usually have two to five times higher the average BER. The size of the frame is not important in this case. To avoid the increase in BER in a convolutional coded system some dummy bits are usually added at the end of the frame to allow for a more reliable decoding. In a turbo coded system [1], with two encoders, there are different techniques to address trellis termination: add two tails to terminate both encoders in a known state [2], add only one tail and impose constraints on the interleaver such that both encoders still terminate in the same state [3], or frame-oriented convolutional turbo codes that do not need any tail [4]. However, a histogram for a turbo coded system will depend not only on the trellis termination but also on the frame size and interleaver design.

Sliding window algorithm: A sliding window (SW) algorithm was introduced in [5] and later in [6]. Since no tail can be appended for each window due to significant reduction in bandwidth efficiency,