Efficient Channel Coding and Interleaving Schemes for Mobile Radio Communications

L.Hanzo, K.H.H.Wong and R.Steele[1]

1 Introduction

Motivated by the increasing demand for high quality, robust yet bandwidth efficient data and speech communications in the hostile mobile radio environment [1] we embarked on the comparative study of a number of multilayer forward error correction(FEC) and interleaving scenarios. The basic attributes of speech and data communication channels are entirely different. Speech transmission channels must not have long coding delays, but their bit error rates(BER) are allowed to be higher than those for data transmission. However, the higher data integrity required for data transmission can be achieved by deploying longer coding/interleaving delays, which are tolerated in this application. To limit the plurality of scenarios and system parameters to a managable number we suppose an overall coding rate of one half. We postulate the memorylength of speech channels to around 450 bits, while that of data channels to around five times as long, i.e.2500 bits.

2 System Architecture

The system block diagram studied is depicted in Figure 1. Two forward error correction layers, as well as two interleaving layers are employed. The inner interleaver randomises the channel's bursty errors for the inner FEC codec. If the inner coding layer cannot combat the errors, they occur again in bursts and therefore another interleaver highly improves the system's performance. Minimum Shift Keying(MSK) modulation schemes are as robust as BPSK, but at double bandwidth efficiency. Hence we utilised in our experiments MSK modulation.

For both FEC layers convolutional and block codes can be implemented. Theoretically four combinations are possible, but only one of them is of practical merit. Namely, inner layer convolutional codes(CC) with Viterbi decoding(VD) that fully exploit the power of soft-decision using channel measurement information received from the MSK demodulator. The outer block coding layer can be used for reliable detection of uncorrectable errors. This is particularly important in activating post-enhancement algorithms in speech communications [2] or in automatic repeat request(ARQ) systems deployed in data communications. After presenting theoretical performance results for both Reed-Solomon block codes and convolutional codes over memoryless channels, we compare the performance of such concatenated coding/interleaving schemes with that of single layer block and convolutional coding/interleaving schemes.

3 Convolutional Codes

The BER performance of convolutional codes can be characterised with the assistence of their weight distribution [3] [4]. The determination of the weight distribution is based on the linearity of the code. Accordingly, all code properties are generalised from results deduced for the transmission of

[1]Department of Electronics and Computer Science, University of Southampton.

the all-zero codeword. The weight distribution itself can, for example, be computed by an exhaustive computer search by evaluating the weights of all the information sequences which produce a path of Hamming distance d from the all-zero codeword in the trellis diagram.

In our non-concatenated coding schemes we used the $CC(2,1,5)$ half-rate convolutional code with constraint length five. Although this constraint length gives a sligthly more modest performance over memoryless channels than its counterpart with constraint length seven (which is used in satellite systems), but it has approximately four times lower complexity. In our concatenated coding schemes the punctured convolutional code $PCC(3,1,5)$ was used, where the coding rate after puncturing was 2/3 and the constraint length was again five. For these codes we have found by computer search the following weigth distributions:

$$W_{CC215}(d) = 14d^5 + 62d^6 + 304d^7 + \ldots = \sum_{n=d_{min}}^{\infty} W_n d^n, \tag{1}$$

$$W_{PCC315}(d) = 4d^7 + 12d^8 + 20d^9 + \ldots = \sum_{n=d_{min}}^{\infty} W_n d^n. \tag{2}$$

With the weight distributions known the so-called union bound on the post-decoding bit error probability p_{bp} for codes of rate $R = k/n$ is given by [3]

$$p_{bp} \leq \frac{1}{k} \sum_{d=d_{min}}^{\infty} W_d P_{ICD}(d), \tag{3}$$

where $P_{ICD}(d)$ is the probability of incorrect decoding, i.e., the probability that the decoder selects a path at distance d from the correct all-zero path in the trellis. The union bound on the probability of incorrect decoding in case of hard decisions is:

$$P_{ICD}(d) = \begin{cases} \sum_{i=(d+1)/2}^{d} \binom{d}{i} p_b^i (1-p_b)^{d-i} & ; d \text{ is odd,} \\ \sum_{i=d/2+1}^{d} \binom{d}{i} p_b^i (1-p_b)^{d-i} + \frac{1}{2} \binom{d}{d/2} p_b^{d/2} (1-p_b)^{d/2} & ; d \text{ is even,} \end{cases} \tag{4}$$

where p_b is the probability of bit errors over the memoryless channel. The post-decoding bit error probability for hard-decision demodulation is found by substituting Equation (4) into Equation (3).

If soft decision Viterbi decoding (SD-VD) is used, the corresponding formula for the probability of incorrect decoding is modulation dependent, and for MSK modulation it is:

$$P_{ICD}(d) = \frac{1}{2} erfc\left(\sqrt{d\Gamma}\right) = \frac{1}{2} erfc\left(\sqrt{dR\frac{E_b}{\eta_0}}\right) \tag{5}$$

where $\Gamma = A_s^2 T/\eta_0$ is the channel signal-to-noise ratio (SNR), E_b is the energy per information bit, $R = k/n$ is the coding rate and $\eta_0/2$ is the double sided power spectral density of the receiver's thermal noise. The post-decoding bit error probability for soft-decision demodulation is obtained by substituting Equation (5) into Equation (3).

4 Reed-Solomon Codes

Reed-Solomon codes constitute a non-binary, maximum minimum distance sub-class of Bose-Chaudhuri-Hocquenghem (BCH) codes. They are equally well suited to combat random and bursty errors. An $RS(n,k)$ code over Galois Field(GF) 2^m has a coding rate of $R = k/n$ and operates on non-binary symbols, constituted by m bits. It is capable of correcting $t = (n - k)/2$ number of symbol errors in a codeword [5]. We have found that the probability of correctly decoding an n symbol codeword is [6]:

$$P_{CD} = \sum_{i=0}^{t} \binom{n}{i} [1 - (1 - p_b)^m]^i \, [(1 - p_b)^m]^{n-i}, \tag{6}$$

where p_b is the bit error probability over the memoryless channel. The probability of incorrectly decoding a codeword can be described by the following formula [6]:

$$\begin{aligned} P_{ICD} = & \sum_{h=d}^{n} \left[\binom{n}{h} (q^m - 1) \sum_{j=0}^{h-d} (-1)^j \binom{h-1}{j} (q^m)^{h-d-j} \right] \\ & \sum_{s=0}^{t} \sum_{g=h-s}^{h+s} \left\{ \left[\sum_{z=z_{min}}^{z_{max}} \binom{h}{h-s+z} \binom{s-z}{g-h+s-2z} \binom{n-h}{z} \right. \right. \\ & \left. (2^m - 2)^{g-h+s-2z} (2^m - 1)^z \right] \left[\frac{1}{(2^m - 1)^g} \sum_{i_1=1}^{m} \sum_{i_2=1}^{m} \cdots \sum_{i_g=1}^{m} \right. \\ & \left. \left. \left[\binom{m}{i_1} \binom{m}{i_2} \cdots \binom{m}{i_g} p_b^{i_1+i_2+\cdots+i_g} (1 - p_b)^{mn-(i_1+i_2+\cdots+i_g)} \right] \right] \right\}. \end{aligned} \tag{7}$$

Observe that P_{ICD} tends to be zero if the SNR is high, as p_b is approximately zero. Armed with P_{CD} and P_{ICD} the probabilty of error detection P_{ED} and the relative error detection probability, P_{EDR} are determined using the following Equations:

$$P_{ED} = 1 - P_{CD} - P_{ICD} \tag{8}$$

$$P_{EDR} = \frac{P_{ED}}{P_{ED} + P_{ICD}}. \tag{9}$$

Clearly, P_{EDR} is the probability that an uncorrectable codeword can be detected.

Based on the above results the post-decoding symbol and bit error probabilities were found as:

$$\begin{aligned} p_{sp} &= p_{sp1} + p_{sp2} \\ &= \sum_{i=1}^{m} \left[\binom{m}{i} (p_b P_{ED})^i (1 - p_b P_{ED})^{m-i} \right] + \frac{1}{n} \sum_{h=d}^{n} h P_{ICD}(h) \end{aligned} \tag{10}$$

and

$$\begin{aligned} p_{bp} &= p_{bp1} + p_{bp2} \\ &= p_b P_{ED} + \left[1 - e^{\frac{1}{m} ln\left(1 - \frac{1}{n} \sum_{h=d}^{n} h P_{ICD}(h)\right)} \right]. \end{aligned} \tag{11}$$

In Section 6 we present our findings for three non-concatenated RS codes. The short $RS(4,2)$ code over $GF(16)$ can be decoded by using soft-decision trellis decoding (SD-TD). The longer $RS(12,6)$ code over $GF(16)$, as well as the $RS(57,29)$ code over $GF(256)$ are decoded by using the Berlekamp-Massey-Forney (BMF) hard-decision decoding algorithm. We also report on the performance of a concatenated coding scheme, where an $RS(38,28)$ code over $GF(256)$ using the BMF hard-decision decoding method was employed as the outer encoder.

5 Interleaving Techniques

Interleaving is a process of rearranging the ordering of a sequence of symbols in some unique one-to-one deterministic manner. The reverse of this process is deinterleaving, which restores the sequence to its original ordering. Interleaving techniques [6] are generally deployed to disperse burst errors when the received signal level fades, and thereby reduce the concentration of errors that are applied to the channel decoder for correction. Before a sequence of symbols is transmitted the symbols from several codewords are interleaved. When an error burst occurs the errors on deinterleaving will be shared among numerous codewords, enabling a less powerful code to correct them. Thus interleaving effectively makes the bursty channel appear like a random error channel to the decoder. As the interleaving period increases, the error performance can be expected to improve in the sense that noise bursts are more dispersed. On the other hand, the delay due to interleaving and deinterleaving increases. Consequently, there is always a tradeoff between error performance and interleaving delay.

One of the simplest interleaving methods is reffered to as block interleaving, where the encoded symbols are written into a memory on a row-by-row basis and then read out on a column-by-column basis. The memory has an interleaving depth of D rows and a width of W columns. A bit or symbol block interleaving scheme with its associated parameters is designated as $BI/B(D,W)$ or $BI/S(D,W)$, respectively.

A more efficient scheme, reffered to as inter-block interleaving, takes an input block of NB symbols and disperses N symbols to each of the next B output blocks. The mapping from the m-th symbol of the i-th coded input block to the $(j+Bt)$-th interleaved symbol of the $(i+j)$-th output block is given by

$$y(i+j, j+Bt) = x(i,m), \quad \begin{array}{l} \text{for all } i, \\ \text{with } j = m \bmod B, \\ \text{and } \; t = m \bmod N. \end{array} \tag{12}$$

In our discussions we designate a bit or symbol interblock interleaving arrangement with its associated parameters as $IBI/B(B,N)$ or $IBI/S(B,N)$, respectively.

6 Results and Discussion

Our basic intention in this study was to find the most effective error coding schemes for mobile radio channels. In our simulations we have used a Rayleigh-fading envelope sampled at 16 ksamples/sec for a vehicular speed of 30 mph. Over this hostile bursty channel interleaving was crucial to render the channel memoryless for the FEC coding to be efficient. In Figure 2 we present the effect of interblock interleaving on the proposed $CC(2,1,5)$ code with different interleaving periods. The shortest interleaving period of 448 bits, characterised by the scheme $IBI/B(8,7)$ was suitable for speech channels having a considerable improvement compared to when no interleaving was used. The $IBI/B(24,5)$ scheme with an interleaving period of 2880 bits had nearly the same performance as the 4032 bits long $IBI/B(24,7)$ arrangement. Hence an interleaving period of the order of 2500 bits was sufficiently long for the Rayleigh-fading channel to closely approximate to the assumption of the memoryless channel model. In Figure 3 we analysed the effect of different block interleaving periods on the $RS(57,29)GF(256)$ code, which constituted the best representative of the block codes studied. The coding memory length was 456 bits with no interleaving, which gave an inferior performance when compared to the $CC(2,1,5), IBI/B(8,7)$ arrangement in Figure 2. Namely, the $CC(2,1,5)$ code guaranteed a $BER = 10^{-6}$ at $E_b/N_0 = 19.8dB$, while the $RS(57,29)$ code required $E_b/N_0 = 22.5dB$ to achive the same performance, although no interleaving had to be deployed. Observe that for this code an interleaving period of the order of 2500 bits was sufficiently high to randomise the bursty channel error statistics. Clearly, with an interleaving period of this size the simulation results closely approximated to the theoretical results derived for the memoryless channel and can be judiciously used for code design over even Rayleigh-fading channels.

We compared the performance of our coding arrangements for both speech and data channels and present our findings for coding memories of 456 bits and 2500 bits in Figure 4 and Figure 5, respectively. In Figure 4 the best arrangement is the concatenated $PCC(3,1,5)BI/B(19,24), RS(38,28)$ scheme, having a very reliable capability of detecting uncorrectable error patterns, the probability of which is on the order of $(1-10^{-8})$. A comparison with Figure 2 shows that the non-concatenated $CC(2,1,5), IBI/B(8,7)$ 448 bits long system has approximately the same performance, although the detection of uncorrectable error patterns is not possible, and this may be a major disadvantage. In case of the longer delay data channel the same conclusion can be drawn. Namely, the performances of the the concatenated $PCC(3,1,5), IBI/B(24,5), RS(38,28)GF(256)$ system and that of the $CC(2,1,5), IBI/B(24,5)$ arrangement are more or less equivalent. However, the concatenated scheme detects the occurrence of uncorrectable error patterns at the expense of sligthly higher complexity.

7 Conclusions

With the proviso that reliable error detection is not of prime importance and that efficient interblock interleaving and soft-decision Viterbi decoding is used, then convolutional codes can provide similar perfomance to that of concatenated schemes for both speech and data communications over mobile radio channels.

References

[1] **R.Steele**, "Towards a high-capacity digital cellular mobile radio system," IEE Proc., Pt F, 132, no5, pp.396-404, August 1985.

[2] **K.H.J.Wong, L.Hanzo** and **R.Steele**, "A sub-band codec with embedded Reed-Solomon coding for mobile radio speech communication," Proc. of ICCS 1988, Singapore, 31 October-3 November 1988.

[3] **A.J.Viterbi**, "Error bounds for convolutional codes and asymphtotically optimum decoding algorithm," IEEE Trans. Info. Theory, vol.IT-13, pp.260-269, April 1967.

[4] **A.J.Viterbi**, "Convolutional codes and their performance in communication systems," IEEE Trans. Commun. Technol., vol.COM-19, no.5, pp.751-772, October 1971.

[5] **R.E.Blahut**, "Theory and practice of error control codes," Addision-Wesley, 1983.

[6] **K.H.H.Wong, L.Hanzo** and **R.Steele**, "Channel coding for satellite mobile channels", submitted to Int. Journ. on Satellite Comm.

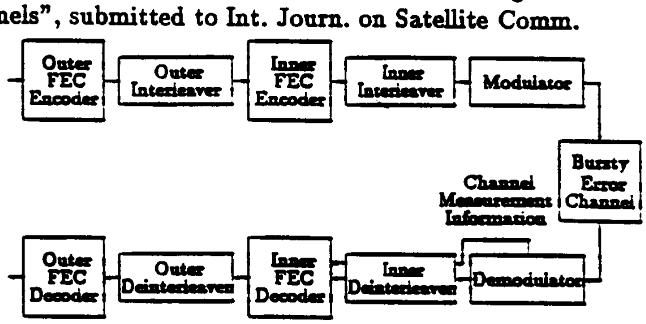

Figure 1 System block diagram.

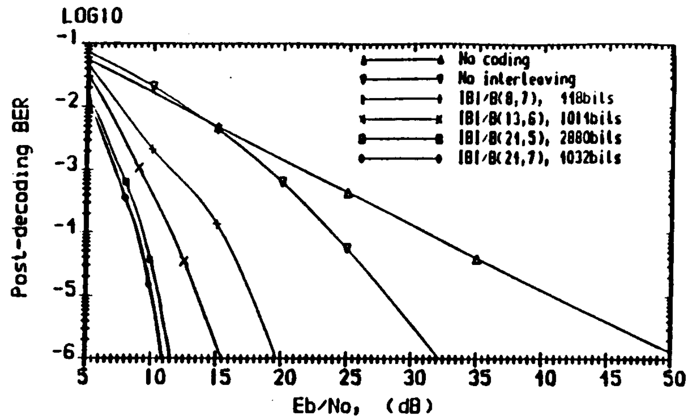

Figure 2. Effect of inter-block interleaving on post-decoding BER of CC(2,1,5) R=1/2 (VD-SD) over Rayleigh Fading channel.

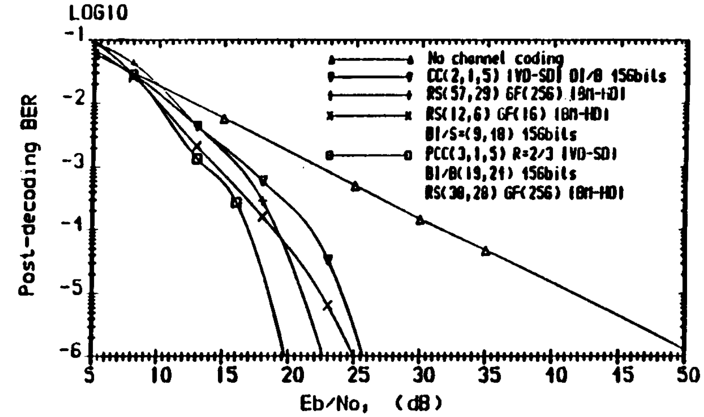

Figure 1. Different coding schemes for the speech channel with a fixed coding/interleaving delay of 156bits over Rayleigh Fading channel.

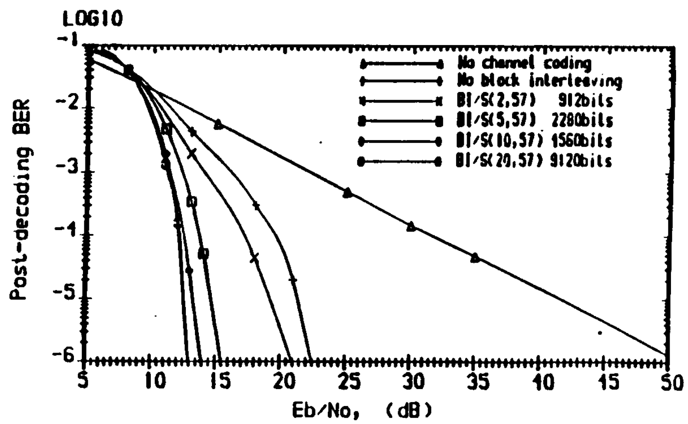

Figure 3. Effect of block interleaving on post-decoding BER of RS(57,29) GF(256) (BM-HD) over Rayleigh Fading channel.

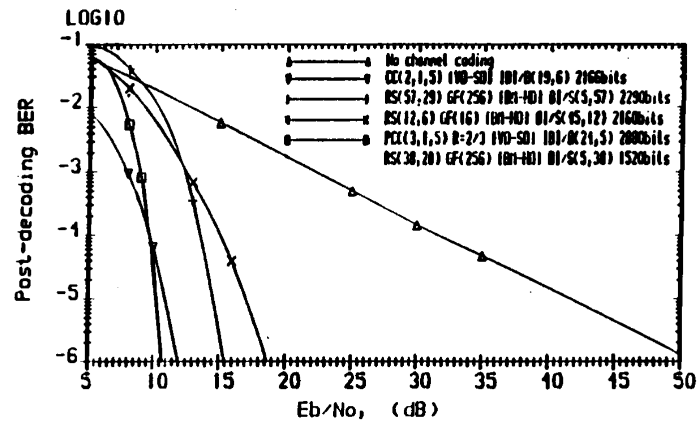

Figure 5. Different coding schemes for the data channel with coding/interleaving delay close to 2500bits over Rayleigh Fading channel.