PERFORMANCE OF RS CODED DS-CDMA USING NONCOHERENT M-ARY ORTHOGONAL MODULATION OVER MULTIPATH FADING CHANNELS

Lie-Liang Yang, Kai Yen, Markus Dillinger* and Lajos Hanzo

Dept. of ECS, University of Southampton, SO17 1BJ, UK.
Tel: +44-703-593 125, Fax: +44-703-594 508
* Siemens, Munich, Germany
Email: lh@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

ABSTRACT

The performance of Reed-Solomon (RS) coded direct-sequence code division multiple-access (DS - CDMA) systems using non-coherent M-ary orthogonal modulation is investigated over multipath Rayleigh fading channels. Equal gain combining (EGC) diversity reception is invoked and the related performance is evaluated for both uncoded and RS-coded DS-CDMA systems. 'Errors-and-erasures' decoding is considered, where the erasures are based on Viterbi's so-called ratio threshold test (RTT). The performance of the 'errors-and-erasures' decoding technique employing the RTT is compared to that of 'error-correction-only' decoding over multipath Rayleigh fading channels. The numerical results show that when using RTT-based 'errors-and-erasures' decoding, RS codes of a given code rate can achieve a higher coding gain than without erasure information.

1. INTRODUCTION

The ability of direct-sequence code division multiple-access (DS-CDMA) systems to benefit from the diversity effect of multipath fading and the option of mitigating the multiuser interference by multiuser detection [1] has rendered them popular in recent years. It has been argued that CDMA can offer a higher system capacity, than Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA) in cellular networks [2]. On the up-link of a DS-CDMA cellular system, due to the high complexity of coherent modulation/demodulation, which would require a pilot signal for each user, non-coherent M-ary orthogonal modulation using $M = 64$, i.e. 6-bit symbols, has been proposed for example for the reverse link of IS-95 [3]. The analysis of DS-CDMA systems using M-ary orthogonal signaling has been provided for example in [4]-[7] for both Additive White Gaussian noise (AWGN) and multipath fading channels.

Forward error-correction (FEC) is often used in cellular DS-CDMA for mitigating the effects of fading and interference. The so-called 'errors-and-erasures' decoding schemes [8] are often preferable to 'error-correction-only' decoding, since typically more erasures than errors can be corrected. Hence, it is beneficial to determine the reliability of the received symbols and to erase the low-reliability symbols prior to the decoding process. An erasure insertion scheme suitable for M-ary orthogonal modulation is the Ratio-Threshold Test (RTT), which was proposed by Viterbi [9]. Kim and Stark [10] have employed it for mitigating the effect of Rayleigh-fading and have analysed some of the performance limits of an RS-coded DS-CDMA system upon using 'errors-and-erasures' decoding.

In this contribution, we investigate the performance of an RS coded DS-CDMA system, when M-ary orthogonal signaling is employed in conjunction with an RTT-based erasure insertion scheme over multipath Rayleigh-fading channels. Equal gain combining (EGC) diversity reception is considered and its performance is evaluated in the context of the proposed RTT-based erasure insertion scheme. The probability density function (PDF) of the ratio defined in the context of RTT is investigated, when either the correct detection hypothesis (H_1) or the erroneous detection hypothesis (H_0) is satisfied by the received M-ary signals. These PDFs are then employed in order to quantify the performance of the M-ary DS-CDMA system investigated. Furthermore, with the aid of the PDFs, we can gain an insight into the basic characteristics of Viterbi's RTT and determine the optimum decision threshold for practical systems.

2. SYSTEM DESCRIPTION AND CHANNEL MODEL

2.1. The Transmitted Signals

The transmitter schematic of the coded M-ary orthogonal modulation based DS-CDMA system is shown in Fig.1. The information bits are first grouped into b-bit symbols, where $b = \log_2 M$. Then, $\mathcal{K}$ information symbols are encoded into an $\mathcal{N}$-symbol RS codeword. Finally, after the randomising symbol-interleaving each RS-coded symbol is M-ary modulated in the baseband, DS spread and carrier-modulated using the approach of Jalloul and Holtzman [6][1]. In a DS-CDMA system supporting K active users, the signal trans-

This work has been performed in the framework of the Pan-Europeapn IST project IST-1999-12070 (TRUST), which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues, although the views expressed are those of the authors.

The financial support of the EPSRC, Swindon, UK is also gratefully acknowledged.

PIMRC'2000, 18-22 Sept. 2000, London, UK

0-7803-6465-5/00 $10.00 © 2000 IEEE

[1]For convenience, we adopt notations similar to those used in [6].

mitted by user i, $i = 1, 2, \ldots, K$, can be expressed as [6]:

$$s_i(t) = \sqrt{P}W^j(t)a_I^i(t)\cos\omega_c t + \sqrt{P}W^j(t-T_0)a_Q^i(t-T_0)\sin\omega_c t, \; 0 \le t \le T_s, \quad (1)$$

where P is the transmitted power per symbol, T_s is the symbol duration, ω_c is the carrier's angular frequency, and T_0 is an offset time. Furthermore, $W^j(t)$ is the jth Walsh-Hadamard orthogonal function, which represents the jth orthogonal signal of the ith user's symbols, while $a_I^i(t)$ and $a_Q^i(t)$ represent the spreading waveforms of the in-phase (I) and quadrature (Q) phase channels, respectively. These quadrature components are expressed as:

$$a_I^i(t) = \sum_{h=-\infty}^{\infty} a_h^{I,i} p(t - hT_c) \quad (2)$$

$$a_Q^i(t) = \sum_{h=-\infty}^{\infty} a_h^{Q,i} p(t - hT_c), \quad (3)$$

where $a_h^{I,i}$, $a_h^{Q,i}$ are independent identically distributed (i.i.d) random variables assuming values of +1 and -1 with equal probability of 1/2. Furthermore, T_c represents the chip duration and $p(t)$ is assumed to be the rectangular chip waveform, which is defined over the interval $(0, T_c]$. Moreover, we assume that $N_s = T_s/T_c = bT/T_c = bN$, where T is the bit-duration and $N = T/T_c$.

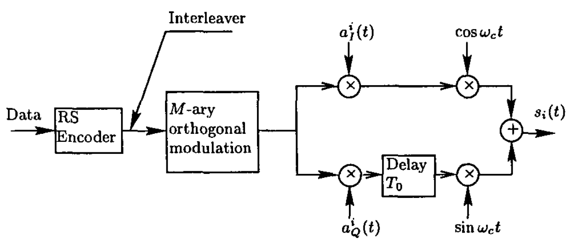

Figure 1: Transmitter block diagram of the DS-CDMA system using M-ary orthogonal modulation, RS encoding and symbol interleaving.

2.2. The Channel Model

We assume that the channel between the ith transmitter and the corresponding receiver is a multipath Rayleigh fading channel [11]. The complex lowpass equivalent representation of the impulse response experienced by user i is given by:

$$h_i(t) = \sum_{l_p=1}^{L_p} \alpha_{il_p}\delta(t - \tau_{il_p})\exp(-j\phi_{il_p}), \quad (4)$$

where α_{il_p}, τ_{il_p} and ϕ_{il_p} represent the attenuation factor, delay and phase-shift for the l_pth multipath component of the channel, respectively, while L_p is the total number of diversity paths and $\delta(t)$ is the Delta-function. We assume that the ith user's multipath attenuations $\{\alpha_{il_p}\}$ in Eq.(4) are independent Rayleigh-distributed random variables having a PDF given by [11]:

$$f_{\alpha_{il_p}}(R) = \frac{2R}{\Omega}\exp\left(-\frac{R^2}{\Omega}\right), \quad (5)$$

where $\Omega = E\left[\left(\alpha_{il_p}\right)^2\right]$. The phases $\{\phi_{il_p}\}$ of the different paths are assumed to be uniformly distributed random variables in $[0, 2\pi)$, while the ith user's path delays of $\{\tau_{il_p}\}$ are modelled as random variables that are mutually independent of each other and uniformly distributed in $[0, T_s)$. Upon assuming that ideal power control is employed, the signal received at the base station generated by the K users can be expressed as:

$$r(t) = \sum_{i=1}^{K} y_i(t) + n(t), \quad (6)$$

where

$$y_i(t) = \sum_{l_p=1}^{L_p} \sqrt{P}\alpha_{il_p}\left[W^j(t-\tau_{il_p})a_I^i(t-\tau_{il_p}) \cdot \cos\left(\omega_c(t-\tau_{il_p}) - \phi_{il_p}\right) + W^j(t-T_0-\tau_{il_p})a_Q^i(t-T_0-\tau_{il_p}) \cdot \sin\left(\omega_c(t-\tau_{il_p}) - \phi_{il_p}\right)\right], \quad (7)$$

while $n(t)$ represents the AWGN, which is modeled as a random variable with zero mean and double-sided power spectral density of $N_0/2$. After the receiver's bandpass filter, the noise $n(t)$ becomes a narrow-band noise process, which can be expressed as [6]:

$$n(t) = n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t, \quad (8)$$

where $n_c(t)$ and $n_s(t)$ represent lowpass-filtered Gaussian processes.

3. THE DETECTION MODEL

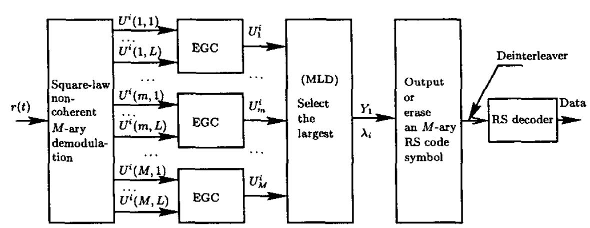

Figure 2: Receiver model of the DS-CDMA system investigated using non-coherent M-ary demodulation, multipath diversity combining, maximum-likelihood detection (MLD), erasure insertion based on the RTT, deinterleaving and RS 'errors-and-erasures' decoding.

The receiver schematic of the studied DS-CDMA system including non-coherent M-ary demodulation, multipath diversity combining, maximum-likelihood detection (MLD), RTT-based erasure insertion, deinterleaving and RS 'errors-and-erasures' decoding is shown in Fig.2. The square-law-based non-coherent M-ary demodulation block is the same as that used in [6] and hence the interested readers are referred to [6] for further details. Multipath diversity combining invoking the EGC - where the input L-path signals are equally weighted and summed - was implemented in the 'EGC' block. The MLD block selects the largest from its input variables and computes the value of λ_i defined as the ratio of the largest to the second largest of the MLD's inputs. Following the MLD stage, the next block may output an M-ary RS code symbol or insert an erasure. Finally, after symbol-based deinterleaving, the RS decoder invokes 'errors-and-erasures' decoding and then outputs the received information bits.

Upon assuming that Gaussian approximation of the multipath interference and that of the multiple access interference can be employed, according to [6], $U^i(m,l)$ in Fig.2 is the sum of the square of two Gaussian random variables, each having a variance of [6] (Eq.(28)):

$$\sigma^2 = \frac{N_0}{2} + \frac{E_s\Omega}{3N_s}(KL_p - 1), \tag{9}$$

where Ω was defined in Eq.(5).

For a receiver using L-th order ($L \le L_p$) diversity reception and EGC the L branches are equally weighted and then summed in order to form the decision variables, which can be expressed as:

$$U_m^i = \sum_{l=1}^{L} U^i(m,l) \tag{10}$$

for $m = 1, 2, \ldots, M$ and $i = 1, 2, \ldots, \mathcal{N}$, where $\mathcal{N}$ is the length of the RS code. Assuming that the jth symbol was transmitted, then, since $U^i(m,l)$ is the sum of square of two Gaussian random variables, it can be shown that, for a given set of multipath attenuations $\{\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{iL}\}$, U_m^i is Chi-square distributed with $2L$ degrees of freedom. The normalized PDF can be derived and expressed as:

$$f_{U_j^i}(y|\gamma) = \left(\frac{y}{\gamma}\right)^{(L-1)/2} \exp\left(-[y+\gamma]\right) I_{L-1}\left(\sqrt{4y\gamma}\right), \tag{11}$$

$$f_{U_m^i}(y) = \frac{1}{(L-1)!} y^{L-1} \exp(-y), \qquad m \neq j, \tag{12}$$

where $I_L(\cdot)$ is the modified Bessel function of the Lth order [11], and

$$\gamma = \overline{\gamma}_c \cdot \sum_{l=1}^{L} \frac{\alpha_{il}^2}{\Omega}, \tag{13}$$

$$\overline{\gamma}_c = \left[\frac{2}{3N_s}(KL_p - 1) + \left(\frac{E_s\Omega}{N_0}\right)^{-1}\right]^{-1}. \tag{14}$$

We assume throughout that all the path gains are i.i.d random variables. Consequently, with the aid of Eq.(5), it can be shown that the PDF of γ can be expressed as:

$$f(\gamma) = \frac{1}{(L-1)!\overline{\gamma}_c^L} \gamma^{L-1} \exp\left(-\frac{\gamma}{\overline{\gamma}_c}\right). \tag{15}$$

Since the decision variables $\{U_1^i, U_2^i, \ldots, U_M^i\}$ are independent random variables, the conditioning in Eq.(11) can be removed with the aid of Eq.(15), yielding:

$$f_{U_j^i}(y) = \frac{1}{(L-1)!(1+\overline{\gamma}_c)^L} y^{L-1} \exp\left(-\frac{y}{1+\overline{\gamma}_c}\right). \tag{16}$$

4. AVERAGE ERROR PROBABILITY WITHOUT FEC

Let H_1 and H_0 represent the hypotheses of correct and erroneous decisions in the MLD block, respectively. Then the average erroneous symbol probability of $P(H_0)$ can be expressed as:

$$P(H_0) = 1 - P(H_1), \tag{17}$$

where $P(H_1)$ is the average correct symbol probability. With the aid of Eqs.(12) and (16), $P(H_1)$ can be expressed as:

$$P(H_1) = \int_0^\infty \frac{1}{(1+\overline{\gamma}_0)^L (L-1)!} y^{L-1} \exp\left(-\frac{y}{1+\overline{\gamma}_0}\right) \cdot \left[1 - \exp(-y)\sum_{k=0}^{L-1}\frac{y^k}{k!}\right]^{M-1} dy. \tag{18}$$

5. PERFORMANCE USING RS FORWARD ERROR-CORRECTION CODES

5.1. Error-Correction-Only Decoding

Upon assuming that sufficiently long randomizing symbol interleaving was invoked, the codeword decoding error probability after 'error-correction-only' decoding can be expressed as [11]:

$$P_W = \sum_{n=\lfloor(\mathcal{N}-\mathcal{K})/2\rfloor+1}^{\mathcal{N}} \binom{\mathcal{N}}{n} [P(H_0)]^n [1 - P(H_0)]^{\mathcal{N}-n}, \tag{19}$$

where $P(H_0)$ is given by Eq.(17).

5.2. Errors-and-Erasures Decoding

Let $\{U_1^i, U_2^i, \ldots, U_M^i\}$ represent the decision variables input to the MLD block of Fig.2. The ratio λ_i involved in Viterbi's RTT is computed according to the following definition [9]:

$$\lambda_i = \frac{Y_1 = \max_1\{U_1^i, U_2^i, \ldots, U_M^i\}}{Y_2 = \max_2\{U_1^i, U_2^i, \ldots, U_M^i\}}, \tag{20}$$

where $Y_1 = \max_1\{\cdot\}$ and $Y_2 = \max_2\{\cdot\}$ represent the maximum and the 'second' maximum of the decision variables of $\{U_1^i, U_2^i, \ldots, U_M^i\}$, respectively. Viterbi [9] pointed out the plausible fact that the demodulated symbols having relatively high ratio of λ_i were more reliable, than those having relatively low values of λ_i. Consequently, a pre-set threshold λ_T can be invoked, in order to erase these low-reliability symbols associated with a ratio of $\lambda_i \le \lambda_T$, which constitutes the so-called RTT.

The PDF of λ_i defined in Eq.(20) conditioned on both the correct detection hypothesis (H_1) and erroneous detection hypothesis (H_0) associated with the transmitted M-ary symbol can be expressed as:

$$f_{\lambda_i}(y|H_1) = C_{H_1} \times g_{\lambda_i}(y|H_1), \quad y \ge 1, \tag{21}$$

$$f_{\lambda_i}(y|H_0) = C_{H_0} \times g_{\lambda_i}(y|H_0), \quad y \ge 1, \tag{22}$$

where

$$g_{\lambda_i}(y|H_1) = y^{L-1}\int_0^\infty x^{2L-1}\exp\left(-x-\frac{xy}{1+\overline{\gamma}_c}\right)$$
$$\cdot[1-\Psi(xy)]^{M-1}[1-\Psi(x)]^{M-2}\Psi\left(\frac{x}{1+\overline{\gamma}_c}\right)dx, \quad (23)$$

$$g_{\lambda_i}(y|H_0) = y^{L-1}\int_0^\infty x^{2L-1}\exp(-xy)[1-\Psi(xy)]^{M-2}$$
$$\cdot\left[1-\Psi\left(\frac{xy}{1+\overline{\gamma}_c}\right)\right]\Psi(x)[1-\Psi(x)]^{M-3}$$
$$\cdot\left\{\frac{1}{(1+\overline{\gamma}_c)^L}\exp\left(-\frac{x}{1+\overline{\gamma}_c}\right)[1-\Psi(x)]\right.$$
$$\left.+(M-2)\exp(-x)\left[1-\Psi\left(\frac{x}{1+\overline{\gamma}_c}\right)\right]\right\}dx, \quad (24)$$

$$C_{H_1} = \frac{1}{\int_1^\infty g_{\lambda_i}(y|H_1)dy}, \quad (25)$$

$$C_{H_0} = \frac{1}{\int_1^\infty g_{\lambda_i}(y|H_0)dy}. \quad (26)$$

In order to erase the low-reliability RS-coded symbols, we assume that λ_T is the threshold, which activates an erasure insertion, if $\lambda_i \leq \lambda_T$. Consequently, the correct symbol probability, P_c, and the random symbol error probability, P_t, after erasure insertion can be formulated as:

$$P_c = P(H_1)\cdot\int_{\lambda_T}^\infty f_{\lambda_i}(y|H_1)dy \quad (27)$$

$$P_t = P(H_0)\cdot\int_{\lambda_T}^\infty f_{\lambda_i}(y|H_0)dy \quad (28)$$

and the symbol erasure probability P_e can be expressed as:

$$P_e = 1 - P_c - P_t. \quad (29)$$

Finally, the codeword decoding error probability P_W after 'errors-and-erasures' decoding can be expressed as:

$$P_W = \sum_{i=0}^{\mathcal{N}}\sum_{j=j_0(i)}^{\mathcal{N}-i}\binom{\mathcal{N}}{i}\binom{\mathcal{N}-i}{j}$$
$$\cdot P_t^i P_e^j(1-P_t-P_e)^{\mathcal{N}-i-j}, \quad (30)$$

where $j_0(i) = \max\{0, \mathcal{N}-\mathcal{K}+1-2i\}$, while P_t, and P_e are given by Eq.(28) and Eq.(29).

6. NUMERICAL RESULTS

Fig.3 and Fig.4 show the codeword decoding error probability of Eq.(30) over Rayleigh fading channels for the EGC scheme, when employing the RS(32,20) code over the Galois field GF(32) =GF(2^5) corresponding to 5-bit symbols using 'errors-and-erasures' decoding. In Fig.3 the codeword decoding error probability was portrayed for different values of SNR per bit and for different thresholds λ_T. By contrast, in Fig.4 the codeword decoding error probability was evaluated for different number of users and for different thresholds λ_T. From these results we conclude that for a constant SNR per bit , $\overline{\gamma}_b$ - where $\overline{\gamma}_b = L_p\overline{\gamma}_c/b$ - or for a constant number of active users, K, there exists an optimum threshold, for which 'errors-and-erasures' decoding achieves the minimum codeword decoding error probability. Hence, an inappropriate threshold leads to higher codeword decoding error probability, than the minimum seen in the figures. From Fig.3 and Fig.4 we gain an explicit insight into the characteristics of Viterbi's RTT over the dispersive Rayleigh fading channels characterised in Eq.(20), suggesting that the optimum threshold was typically around 1.5 to 2.

In Fig.5 and Fig.6 we evaluated the codeword decoding error probability of an M-ary DS-CDMA system employing either 'error-correction-only' or 'errors-and-erasures' decoding. The parameters used in these investigations were shown at the top of the figures. The optimal threshold, λ_T, was adjusted according to the average received SNR per bit in Fig.5, while in Fig.6 according to the number of active users. The results show that under dispersive multipath Rayleigh fading conditions the decoding algorithms using 'errors-and-erasures' decoding based on the RTT erasure insertion scheme outperform 'error-correction-only' decoding.

7. REFERENCES

[1] S. Verdu, *Multiuser Detection*, Cambridge University Press, Cambridge, UK, 1998.

[2] K. S. Gilhousen, *et al.*, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, Vol. 40, pp. 303-312, May 1991.

[3] EIA/TIA-95 Rev A,"Mobile Station-Base Station Compatibility Standard for Dual-Mode Wideband Spread Spectrum Cellular System" 1995.

[4] P. K. Enge, D. V. Sarwate, "Spread-spectrum multiple-access performance of orthogonal codes: Linear receivers," *IEEE Trans. on commun.*, Vol. COM-35, pp. 1309-1319, Dec. 1987.

[5] L-L. Yang and C-S. Li, "DS-CDMA performance of random orthogonal codes over Nakagami multipath fading channels," in *Proc. of IEEE ISSSTA'96*, pp. 1282-1286, Mainz, Germany, 22-25 Sep 1996.

[6] L. M. A. Jalloul and J. M. Holtzman, " Performance analysis of DS/CDMA with non-coherent M-ary orthogonal modulation in multipath fading channels," *IEEE J. Select. Areas Commum.*, Vol.12, No. 5, pp.862-870, Sept. 1994.

[7] V. Aalo, O. Ugweje and R. Sudhakar, "Performance analysis of a DS/CDMA system with noncoherent M-ary orthogonal modulation in Nakagami fading," *IEEE Trans. on Veh. Technol.*, Vol. 47, No. 1, pp. 20-29, Feb. 1998.

[8] R. E. Blahut, *Theory and Practice of Error Control Codes*, Addison-Wesley Pub. Co., 1983.

[9] A. J. Viterbi, "A robust ratio-threshold technique to mitigate tone and partial band jamming in coded MFSK systems," in *IEEE Military Commun. Conf. Rec.*, pp.22.4.1-22.4.5, Oct. 1982.

[10] S. W. Kim and W. Stark, "Performance limits of Reed-Solomon coded CDMA with orthogonal signaling in a

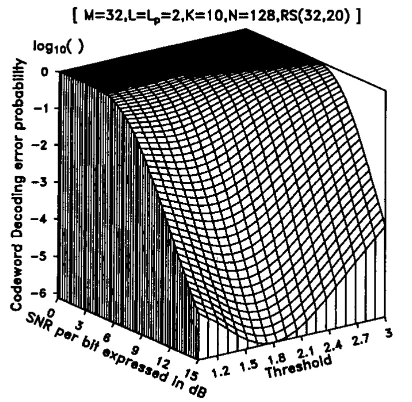

Figure 3: Codeword decoding error probability versus the average SNR per bit, $\overline{\gamma}_b$, and versus the threshold, λ_T, for the RS(32,20) FEC code using 'errors-and-erasures' decoding over the Rayleigh fading channel characterised by Eq.(4), evaluated from Eq.(21)-(30).

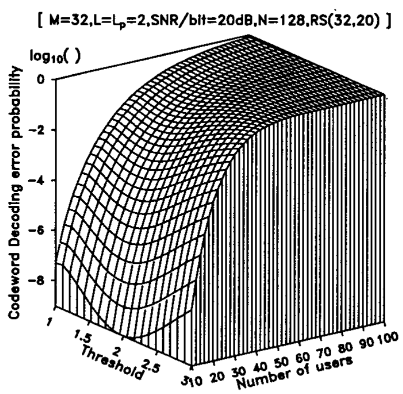

Figure 4: Codeword decoding error probability versus the number of active users, K, and versus the threshold, λ_T, for the RS(32,20) FEC code using 'errors-and-erasures' decoding over the Rayleigh fading channel characterised by Eq.(4). The results were evaluated from Eq.(21)-(30).

Rayleigh-fading channel," *IEEE Trans. on Commun.*, Vol.46, No. 9, pp.1125-1134, Sept. 1998.

[11] J. G. Proakis, *Digital Communications*, (3rd Ed.) New York: McGraw-Hill, 1995.

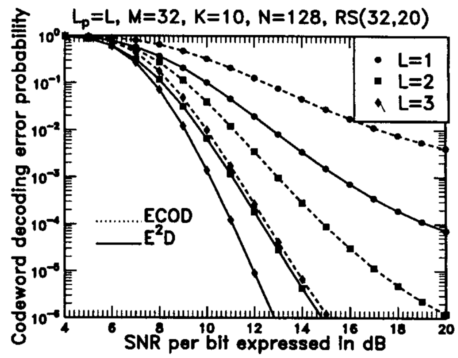

Figure 5: Codeword decoding error probability versus the average SNR per bit, $\overline{\gamma}_b$, for the RS(32,20) FEC code using 'error-correction-only' decoding (ECOD) and 'errors-and-erasures' decoding (E^2D) with parameters of $M = \mathcal{N} = 32$, $\mathcal{K} = 20$, $K = 10$, $N = 128$, $L_p = L = 1,2,3$ over the dispersive Rayleigh fading channel characterised by Eq.(4).

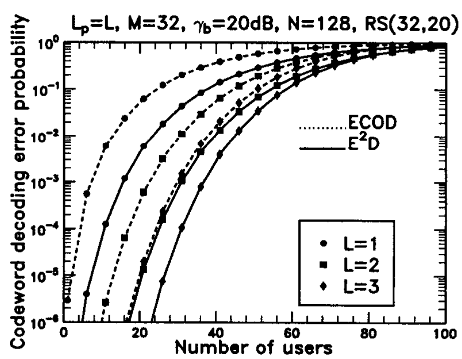

Figure 6: Codeword decoding error probability versus the number of active users, K, for the RS(32,20) FEC code using 'error-correction-only' decoding (ECOD) and 'errors-and-erasures' decoding (E^2D) with parameters of $M = \mathcal{N} = 32$, $\mathcal{K} = 20$, $\overline{\gamma}_b = 20dB$, $N = 128$, $L = L_p = 1,2,3$ over the dispersive Rayleigh fading channel characterised by Eq.(4).