COMPARATIVE STUDY OF SPACE TIME BLOCK CODES AND VARIOUS CONCATENATED TURBO CODING SCHEMES

T.H. Liew, J. Pliquett, B. L. Yeap, L-L. Yang, L. Hanzo

Dept. of Electr. and Comp. Sc., Univ. of Southampton, SO17 1BJ, UK.
Tel: +44-703-593 125, Fax: +44-703-593 045
Email: thl97r@ecs.soton.ac.uk,lh@ecs.soton.ac.uk
http://www-mobile.ecs.soton.ac.uk

ABSTRACT

Space-time block codes provide substantial diversity advantages in multiple transmit antenna assisted systems at a low decoding complexity. In this contribution, we concatenate space-time codes with three turbo coding schemes, namely Turbo BCH (TBCH) codes, Turbo Convolutional (TC) codes and Turbo Trellis Coded Modulation (TTCM) schemes for the sake of achieving significant coding gain. The issues of mapping coded bits of the TBCH and TC schemes to different protection classes of various multilevel modulation schemes is also addressed. Finally, the performance and associated complexity of the three turbo schemes is compared.

1. INTRODUCTION

The third generation (3G) mobile communication standard is expected to provide data rates up to 2 Mb/s for indoor applications [1]. In an effort to support such high rates, the capacity of band-limited wireless channels can be increased by employing multiple antennas [2]. The classic approach is to use multiple antennas at the receiver and perform Maximum Ratio Combining (MRC) of the received signals for improving the performance. However, applying receiver diversity at the mobile stations (MS) increases their complexity. Hence *receiver diversity techniques* have typically been applied at the base stations (BS). BSs provide services for many MSs and hence up-grading the BSs is economically viable. However, the drawback of this scheme is that it only provides diversity gain for the BSs.

Recently, *different transmit diversity* techniques have been introduced, in order to provide diversity gain for MSs by upgrading the BSs. In [3], Tarokh *et al.* proposed space-time trellis coding by jointly designing the channel coding, modulation, transmit diversity and the optional receiver diversity. The proposed space-time trellis codes perform extremely well at the cost of high complexity. In addressing the issue of decoding complexity, Alamouti [4] discovered a remarkable scheme for transmissions using two transmit antennas. A simple decoding algorithm was introduced, which can be generalised to an arbitrary number of receiver antennas. This scheme is significantly less complex, than space-time trellis coding using two transmitter antennas, although there is a loss in performance [5]. Despite the associated performance penalty, Alamouti's scheme is appealing in terms of simplicity and performance. This proposal motivated Tarokh *et al.* [5,6] to generalise the scheme to an arbitrary number of transmitter antennas, leading to the concept of space-time block codes. Space-time block codes were designed for achieving the maximum diversity order of $n \times m$ for n transmit and m receive antennas. However, they were not designed for achieving additional coding gain. Hence, in this contribution, we combine space-time block codes with turbo convolutional (TC) codes [7,8], turbo BCH (TBCH) codes [9] and turbo trellis coded modulation (TTCM) [10], in order to achieve additional coding gains. The performance and complexity of the different schemes will be studied comparatively.

The invention of turbo codes by Berrou [7] has resulted in achieving a performance near the Shannon limit using BPSK modulation in AWGN channels. Generally, recursive systematic convolutional codes are used as their component codes. However, block codes, such as for example binary BCH codes, can also be employed as the turbo component codes and turbo block codes can outperform their convolutional counterparts for code rates in excess of R=2/3 [9].

In order to achieve a high throughput, higher-order modulation schemes have to be used in conjunction with turbo codes. This results in a number of proposals for bandwidth-efficient turbo coding. Two approaches are particularly interesting and will be considered in this treatise. The first approach was proposed in [7,8], using suitable punctured turbo convolutional (TC) codes followed by Gray-mapping, in order to attain a high spectral efficiency. In [10], Robertson *et al.* proposed turbo trellis coded modulation (TTCM) by invoking Ungerboeck codes as the component codes. It was shown in [10] that in AWGN channels, TTCM outperforms Gray-mapping assisted TC by about 0.5 dB.

2. SYSTEM OVERVIEW

The schematic of the proposed concatenated space-time block codes and the different turbo coding schemes is shown in Figure 1. The information source at the transmitter generates random data bits. The information bits will be encoded by three different turbo coding schemes, as shown in Figure 1. For the first turbo coding scheme, namely TBCH(32,26), we have used extended BCH(32,26) codes as the component codes. The information bits and parity bits of the BCH(32,26) component encoders are not punctured, which results in a code rate of R=0.68. In the second turbo coding scheme, three different recursive systematic convolutional (RSC) codes are employed. Specifically,

0-7803-6465-5/00 $10.00

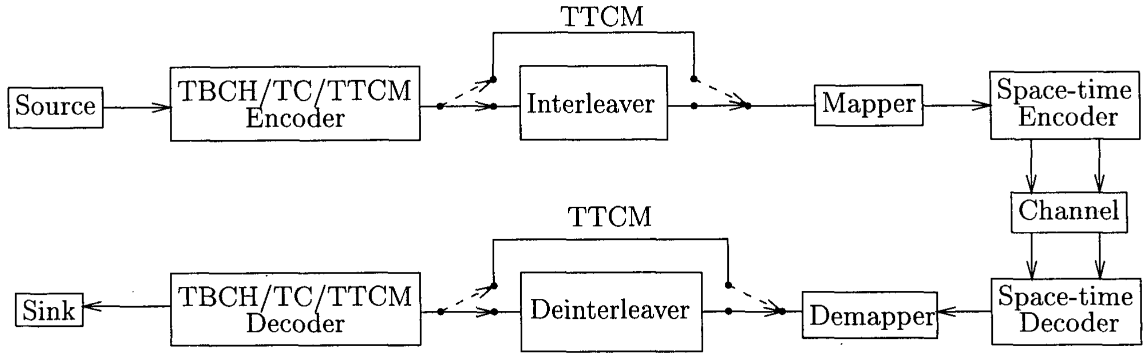

Figure 1: System overview of space-time block codes and different turbo schemes.

these codes have a constraint length of 3 in conjunction with octal generator polynomials of $(7_8, 5_8)$, a constraint length of 4 in conjunction with octal generator polynomials of $(13_8, 15_8)$, as well as a constraint length of 5 along with octal generator polynomials of $(23_8, 35_8)$. This results in three TC codes, namely the TC(2,1,3), TC(2,1,4) and the TC(2,1,5) schemes. Note that TC(2,1,4) has been proposed for employment in the third generation wireless systems [1]. The parity bits are punctured alternately, while all systematic information bits are transmitted. The resulting code rate is hence R=0.50. The third turbo scheme considered is the TTCM arrangement proposed in [10]. The constraint length of Ungerboeck's code is 3 and the octal generator polynomials are $(11_8, 2_8, 4_8)$. Again, the code rate is R=0.67.

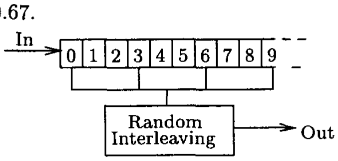

Figure 2: Random separation interleaving.

Only the output bits of the TBCH and TC schemes are interleavered, as seen in Figure 1. We apply random separation based or pseudo random interleaving for dispersing the bursty errors within a transmitted symbol. Explicitly, Figure 2 shows an example of the random separation based interleaving employed. If 8-PSK modulation is used, then 3 bits per constellation point are transmitted. Hence, for every 3-bit spaced position, the bits will be randomly interleaved. For an example, in Figure 2 we randomly interleaved the bit positions $0, 3, 6, 9, ...$ Similarly, bit positions $1, 4, 7, ...$ and $2, 5, 8, ...$ were also randomly interleaved. The objective of random separation based interleaving is to randomly interleave the bits within the same protection class of the 8-PSK symbol. The output bits of the TTCM scheme are passed directly to the mapper in Figure 1, where we employed two different mapping techniques. Gray-mapping [8] was used for the TBCH and TC schemes, whereas set-partitioning [10] was utilised for the TTCM scheme.

Following the mapper, the modulated symbols are passed to the space-time block encoder, as shown in Figure 1. A space-time block code is defined by a $p \times n$ transmission matrix $\mathbf{G}$, where the entries of the matrix are linear combinations of the input symbols $x_1, x_2, ..., x_k$ and their conjugates. The number of transmitter antennas is n. The $p \times n$ matrix $\mathbf{G}$ – which defines the space-time block code – is based on a complex generalised orthogonal design, as defined in [4,5]. In our system, we used the simplest space-time block code defined in [4,5] as:

$$\mathbf{G}_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} . \qquad (1)$$

The code rate of the space-time code is given by k/p, and hence in this example the code rate is unity. All symbols in the same row of the matrix $\mathbf{G}$ are transmitted simultaneously from n different transmit antennas, while all entries in the same column are transmitted from the same antenna in p successive transmission instants.

The number of receiver antennas constitutes a design parameter, but it was fixed to one in our system. Since at high bit rates the channel does not change significantly for p consecutive symbols, the orthogonality of the matrix $\mathbf{G}$ enables us to separate the signals x_1 and x_2 transmitted from the different antennas. This then allows us to invoke the Log-MAP algorithm [11] independently for the decoding of the signals received from the different antennas. The soft outputs associated with the received bits or symbols are passed to the deinterleaver or TTCM decoder, respectively, as seen in Figure 1. The deinterleaved soft outputs of the received bits will then be passed to TBCH or TC decoders. All turbo decoder schemes apply the Log-MAP decoding algorithm [7,9,10]. The decoded bits are finally passed to the sink for calculation of the Bit Error Rate (BER), as shown in Figure 1.

3. SIMULATION RESULTS

All simulation results were obtained over uncorrelated or – synonymously – perfectly interleaved narrow-band Rayleigh fading channels. This assumption does not contradict to requiring a near-constant channel magnitude and phase over p consecutive symbols, since upon applying a sufficiently high interleaving depth the channels' fading envelope can be indeed uncorrelated. We assumed that the narrow-band fading amplitudes received from each transmitter antenna were mutually uncorrelated Rayleigh distributed processes. It was also assumed that the fading amplitudes were constant across p (number of rows in the matrix $\mathbf{G}_2$) consecutive symbols, where we had $p = 2$. The average signal power received from each transmitter antenna was the same. Furthermore, we assumed that the receiver had a perfect estimate of the channels' fading amplitudes. The channels' fading amplitude can be estimated with the aid of pilot symbols [12].

3.1. Performance of 16-QAM based schemes

Before we compare the performance of the three different turbo schemes studied, we will investigate, how the mapping of the TC coded bits to 16-QAM symbols affects the performance of the codes. In the Gray-mapping assisted 16-QAM constellation, there are two protection classes [12], class I and II, depending on the bit position. There are four bits per symbol in the 16-QAM constellation, and two of the bit positions are more protected, than the remaining two bits.

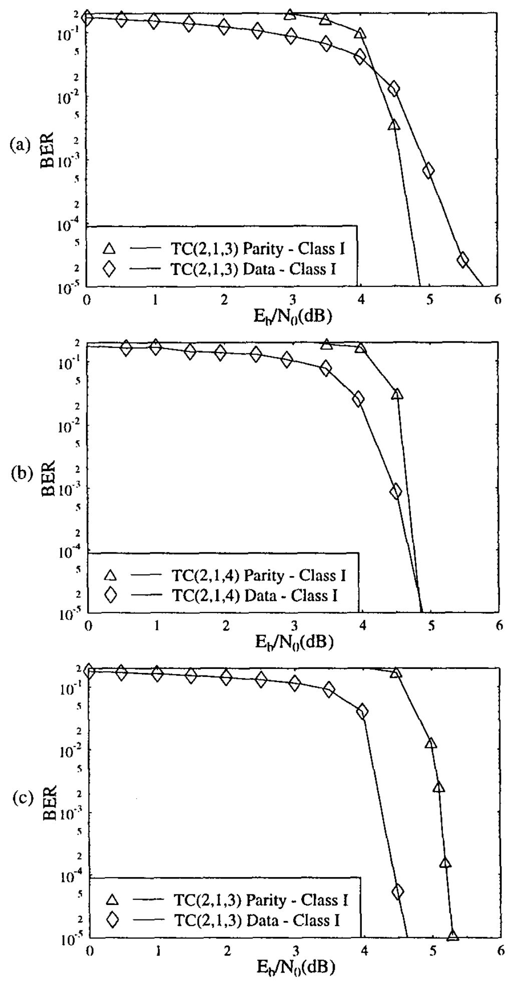

Figure 3: Performance comparison of various data and parity bit allocation schemes for the (a) TC(2,1,3), (b) TC(2,1,4) and (c) TC(2,1,5) codes. All simulation results were obtained upon employing the space-time code $\mathbf{G}_2$ using one receiver and 16-QAM over uncorrelated Rayleigh fading channels at an effective throughput of 2 BPS.

In Figure 3, we compare the performance of various parity and data bit mapping schemes for the (a) TC(2,1,3), (b) TC(2,1,4) and (c) TC(2,1,5) codes. The curve marked by triangles represents the performance of TC codes, when allocating the parity bits to protection class I and the data bits to protection class II. On the other hand, the performance curve marked by diamonds indicates the allocation of data bits to protection class I, while the parity bits are assigned to protection class II.

In Figure 3(a), we can see that at low E_b/N_0 values the performance of the TC(2,1,3) code, when allocating the parity bits to protection class I is worse, than upon allocating the data bits to protection class I. However, for E_b/N_0 values in excess of about 4 dB, the situation is reversed. At a BER of 10^{-5}, there is a performance gain of about 1 dB when using the TC(2,1,3) arrangement with the parity bits allocated to protection class I. We surmize that by protecting the parity bits better, we render the TC(2,1,3) code more powerful. It is common that stronger channel codes perform worse, than weaker codes at low E_b/N_0 values, but outperform their less powerful counterparts for a higher E_b/N_0 values.

In Figure 3(b) we show the performance of the TC(2,1,4) code using the same data and parity bit allocation, as in Figure 3(a). The figure clearly shows that the TC(2,1,4) scheme exhibits a better performance for E_b/N_0 values below about 5 dB, if the data bits are more strongly protected than the parity bits. It is also seen from the figure that the situation is reversed for E_b/N_0 values above this point.

Let us now consider the same performance curves in the context of the significantly stronger TC(2,1,5) code in Figure 3(c). The figure clearly shows that better performance is yielded in the observed range, when the data bits are more strongly protected. Unlike in Figure 3(a) and 3(b), there is no visible crossing point in Figure 3(c). However, if we were to extrapolate the curves in Figure 3(c), they might cross at BER$\approx 10^{-10}$. The issue of data and parity bit mapping to multilevel modulation schemes was also addressed by Goff *et. al.* [8]. However, the authors only investigated the performance of the TC(2,1,5) code and stated that better performance is achieved by protecting more strongly the data bits. Additionally, we note here that the situation was reversed for the TC(2,1,3) code, where better performance was achieved by protecting the parity bits better.

Hence, from the three subfigures of Figure 3, we can draw the following conclusion for the mapping of the data and parity bits in conjunction with TC codes. For weaker turbo codes, such as the TC(2,1,3) arrangement, it is better to protect the parity bits more strongly. On the other hand, for stronger turbo codes, such as the TC(2,1,4) and TC(2,1,5) schemes, better performance is achieved by protecting more strongly the data bits. Based on these facts, we continue our investigations by exploring the effect of interleavers, in an effort to achieve an improved performance.

In Figure 4, we show the performance of the TC(2,1,4) code using different mapping methods. Specifically, we have repeated the performance curves of Figure 3(b) in Figure 4, as marked by the triangles and diamonds. The curve marked by hearts shows the performance of the TC(2,1,4) arrangement using a random interleaver. Figure 4 clearly shows that the random-interleaved performance is between the performance of the TC(2,1,4) code protecting the data bits and parity bits more strongly, respectively, when E_b/N_0

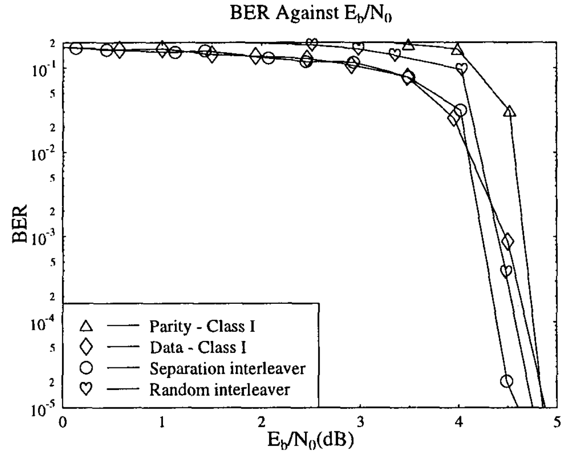

Figure 4: Performance comparison between different mapping methods for the TC(2,1,4) code in conjunction with the space-time code $\mathbf{G}_2$ using one receiver and 16-QAM over uncorrelated Rayleigh fading channels at an effective throughput of 2 BPS.

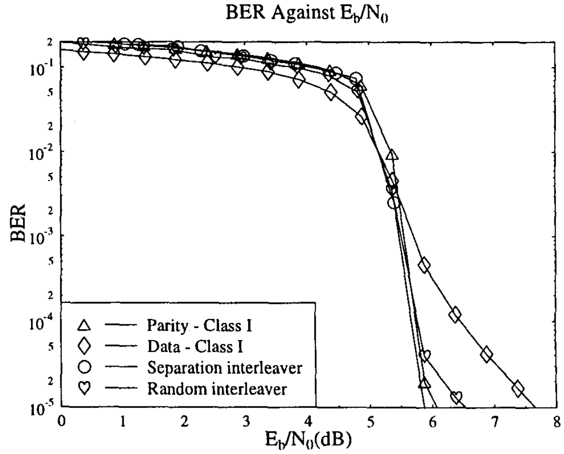

Figure 5: Performance comparison between different bit mapping methods for the TBCH(32,26) code in conjunction with the space-time code $\mathbf{G}_2$ using 8-PSK over uncorrelated Rayleigh fading channels at an effective throughput of 2 BPS.

is low. However, its performance improves and it outperforms the above two schemes for E_b/N_0 values in excess of 4.5 dB. This is because as the BER reduces, the performance gap between the curves marked by the triangles and diamonds is also reduced. Hence, randomly mapping the coded bits to the two 16-QAM protection classes has no significant effect, since the associated performances are similar. However, the random interleaver disperses the bursty channel errors associated with a transmitted symbol, hence improving the performance of the TC(2,1,4) scheme. The TC(2,1,4) code protecting the data bits more strongly and applying a random separation based interleaver outperforms the TC(2,1,4) code using a random interleaver by approximately 0.2 dB at a BER of 10^{-5}. The random separation based interleaver was then also applied to the other TC codes, namely to the TC(2,1,3) and TC(2,1,5) schemes. Our simulation results demonstrate that at a BER of 10^{-5} the random separation based interleaver attains the best performance, albeit only by a small margin.

3.2. Performance of 8-PSK based schemes

In the Gray-mapping assisted 8-PSK constellation, there are also two protection classes, depending on the bit position. From the three bits of the 8-PSK constellation two of the bit positions are more protected, than the remaining bit. In Figure 5, we portray the performance of the TBCH(32,26) scheme for four different bit mapping methods. Firstly, one data bit and one parity bit was mapped to the two better protected 8-PSK bit positions. The corresponding BER curve was marked by the triangles in Figure 5. According to the second method, the data bits were mapped to the two better protected bit positions of the 8-PSK symbol. This scenario was marked by diamonds in Figure 5. As we can see from the figure, the first mapping method yields a substantial coding gain of 1.5 dB at a BER of 10^{-5} over the second method. By applying the random separation based interleaver of Figure 2, while still better protecting one of the data bits and the parity bit than the remaining bit, we disperse the bursty errors associated with a transmitted symbol over several BCH codewords of the turbo BCH code. As shown in Figure 5, the performance curve marked by the circles shows a slight improvement compared to the above mentioned first method, although the difference is marginal. Finally, we show the performance of applying random interleaving, which randomly distributes the data and parity bits between the two 8-PSK protection classes. It can be seen that the associated performance is worse than that of the first mapping method. We note here, however that the preference orders found above for uncorrelated Rayleigh fading channels may change over correlated or dispersive fading channels.

3.3. Performance Comparison of the turbo schemes

Having investigated the above different mapping methods, in the rest of this contribution we apply the best mapping, yielding the highest coding gain for the investigated TBCH and TC schemes. The performance of the TBCH(32,26) code as well as that of the TC(2,1,3), TC(2,1,4), TC(2,1,5) and TTCM schemes in conjunction with the concatenated space-time code $\mathbf{G}_2$ over uncorrelated Rayleigh fading channels is shown in Figure 6. The different modulation schemes were chosen such that the effective throughput of the system was 2 bits per symbol (BPS). The total latency of each scheme was fixed to approximately 10,000 information bits. The Log-MAP decoder [7, 9, 10] was used for each scheme and the number of iterations was fixed to eight. The performance of the unity-rate space-time code $\mathbf{G}_2$ using QPSK over uncorrelated Rayleigh fading channels is also shown in Figure 6 for comparison. The coding gain of each scheme at a BER of 10^{-5} over the unity-rate $\mathbf{G}_2$-coded QPSK modulation is shown in Table 1. In the table, we also tabulated the octal generator polynomials, the associated number of states, the coding rate, the estimated complexity and memory requirements for each turbo scheme. The complexity

	TBCH	TC(2,1,3)	TC(2,1,4)	TC(2,1,5)	TTCM
Generator	45	7,5	13,15	23,35	11,4,2
No of States	64	4	8	16	16
Code Rate	0.68	0.50	0.50	0.50	0.67
Complexity	118	8	16	32	32
Memory	1536	40,000	80,000	160,000	40,000
Gain(dB)	17.5	19.5	19.7	19.9	17.2

Table 1: Coding gains of the TBCH, TC(2,1,3), TC(2,1,4), TC(2,1,5) and TTCM schemes over the unity-rate $\mathbf{G}_2$-coded QPSK scheme in uncorrelated Rayleigh fading channels.

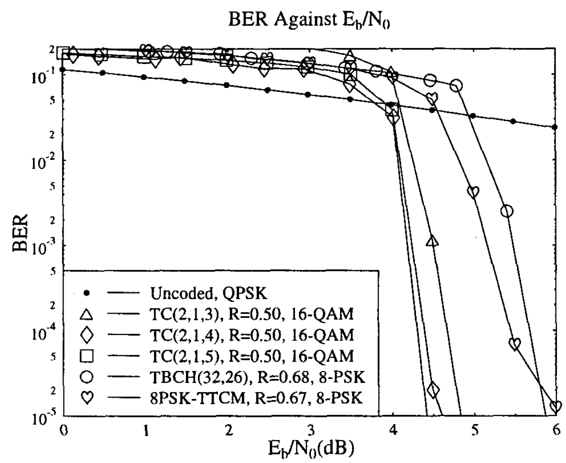

Figure 6: Performance comparison between the investigated TBCH, TC and the TTCM scheme along with the concatenated space-time code $\mathbf{G}_2$ at an effective throughput of 2 BPS over uncorrelated Rayleigh fading channels.

was calculated on the basis of the number of trellis branches per information bit [10]. As we can see from the table, the TBCH arrangement is the most complicated turbo scheme in the set investigated, and yet, the performance gains of the less complex schemes are better. The associated memory requirement is proportional to the number of states in a trellis. Since the TBCH(32,26) code used can be broken into smaller codewords, which exhibit a lower number of states in the trellis, its memory requirement is the lowest. The TC(2,1,3) code constitutes a good compromise, since it exhibits a low complexity and low memory requirements, while providing an attractive coding gain.

4. CONCLUSION

In this contribution we investigated the concatenation of three different turbo schemes in conjunction with space-time block codes. Initially, different mapping methods were investigated in the context of the TBCH and TC codes using a random separation channel interleaver. The coding gain, complexity and memory requirement of the schemes studied was compared. We concluded that the TC(2,1,3) scheme constitutes a good compromise in terms of coding gain, complexity and memory requirement. Our future work will be focused on correlated and wideband channels in conjunction with burst-by-burst adaptive modulation [12].

5. ACKNOWLEDGEMENTS

The financial support of the European Union under the auspices of the Pan-European TRUST project and that of the EPSRC, Swindon UK is gratefully acknowledged.

6. REFERENCES

[1] P. Chaudhury, "The 3GPP Proposal for IMT-2000", *IEEE Communications Magazine*, Vol. 37, no. 12, pp. 72-81, December 1999.

[2] G. J. Foschini, Jr. and M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas", *Wireless Personal Communications*, Vol. 6, no. 3, pp. 311-335, March 1998.

[3] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction", *IEEE Transactions on Information Theory*, Vol. 44, no. 2 pp. 744-765, March 1998.

[4] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications", *IEEE Journal on Selected Areas in Communications* , pp. 1451-1458, October 1998.

[5] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-Time Block Codes from Orthogonal Designs", *IEEE Transactions on Information Theory*, pp. 1456-1467, July 1999.

[6] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-Time Block Coding for Wireless Communications: Performance Results", *IEEE Journal on Selected Areas in Communications*, pp. 451-460, March 1999.

[7] C. Berrou and A. Glavieux, "Near Optimum Error Correcting Coding and Decoding: Turbo-Codes", *IEEE Transactions on Communications*, Vol. 44, no. 10, pp. 1261-1271, October 1996.

[8] S. L. Goff, A. Glavieux and C. Berrou, "Turbo-codes and high spectral efficiency modulation", *Proc IEEE International Conference on Communications*, New Orleans, USA pp. 645-649, May 1994.

[9] J. Hagenauer, E. Offer and L. Papke, "Iterative Decoding of Binary Block and Convolutional Codes", *IEEE Transactions on Information Theory*, Vol. 42, no. 2, pp. 429-445, March 1996.

[10] P. Robertson and T. Worz, "Bandwidth-Efficient Turbo Trellis-Coded Modulation Using Punctured Component Codes", *IEEE Journal on Selected Areas in Communications*, pp. 206-218, February 1998.

[11] G. Bauch, "Concatenation of Space-Time Block Codes and Turbo-TCM", *Proc IEEE International Conference on Communications*, Vancouver, Canada pp .1202-1206, June 1999.

[12] L. Hanzo, W. Webb and T. Keller, "Single- and Multicarrier Quadrature Amplitude Modulation", *John Wiley & Sons, Ltd*, Wessex, England, 2000, ISBN 0471492396.