

# HYBRID GENETIC ALGORITHM BASED DETECTION SCHEMES FOR SYNCHRONOUS CDMA SYSTEMS

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## ABSTRACT

In this paper, we applied a hybrid genetic algorithm (GA) scheme as a suboptimal multiuser detection technique in bit-synchronous code division multiple access (CDMA) systems over a Gaussian channel as well as over a single-path Rayleigh fading channel. The proposed hybrid GA scheme attempts to search for the users' transmitted bit sequence that optimizes the correlation metric employed. Simulation results showed that the BER performance of our proposed multiuser detector approaches the single-user performance bound, while imposing a lower complexity as compared to that of the optimum multiuser detection, especially for a high number of users. Furthermore, the parameters of the hybrid GA scheme can be adjusted, in order to attain a tradeoff between the computational complexity, BER performance and the detection time.

## 1. INTRODUCTION

Code division multiple access (CDMA) [1] constitutes an attractive multiuser scheme that allows users to transmit at the same carrier frequency in an uncoordinated manner. However, this creates multiuser interference (MUI), which – if not controlled – can seriously deteriorate the quality of reception. Numerous methods have been proposed for reducing the amount of MUI present in the received signal such as power control, the optimization of signature sequences and sectorised antennas. Nevertheless, these techniques have their limitations in combating the effects of MUI, particularly in conjunction with the conventional single user detector since the MUI is treated as noise.

On the other hand, the so-called multiuser detector [2] treats the MUI as a part of the information, rather than noise. Hence, by processing this additional information significant performance improvements can be achieved. In a bit-synchronous CDMA system supporting  $K$  users the optimal multiuser detector proposed by Verdú [3] searches exhaustively for a point in the discrete solution space that

maximises the associated correlation metric given by [4]:

$$\Lambda(\mathbf{r}_K, \mathbf{b}_K) = 2\Re\{\mathbf{r}_K^T \mathbf{C} \mathbf{b}_K\} - \mathbf{b}_K^T \mathbf{C} \mathbf{R}_s \mathbf{C}^* \mathbf{b}_K, \quad (1)$$

where  $\mathbf{r}_K = [r_1 r_2 \dots r_K]^T$  denotes the cross-correlation of the received signal with each of the  $K$  users' signature sequences,  $\mathbf{b}_K = [\sqrt{\xi_1} b_1 \dots \sqrt{\xi_K} b_K]^T$  is a vector of the  $K$  users' transmitted bit during a bit interval scaled by the corresponding transmit energy,  $\mathbf{C} = \text{diag}[c_1, c_2, \dots, c_K]$  is a diagonal matrix of the users' complex CIR coefficients and  $\mathbf{R}_s$  is the correlation matrix of the signature sequences, with elements  $\rho_{jk}(0)$  given by:

$$\rho_{jk}(0) = \int_0^{T_b} g_j(t) g_k(t) dt, \quad (2)$$

where  $g_j(t)$  is the signature sequence of the  $j$ th user and  $T_b$  is the bit duration. The points in the discrete solution space represent all the possible combinations of the users' transmitted bit sequences. Hence for a  $K$  user system, the total number of points in the solution space is  $2^K$ . Clearly, the size of the solution space grows exponentially with the number of users. Hence, the optimal detector becomes impractical, when there is a high number of users. This leads to numerous so-called suboptimal multiuser detector proposals [5], which sacrifice performance for the sake of a reduced complexity.

Genetic algorithms (GAs) [6–8] have been employed for solving many optimization problems in numerous fields. While GAs are not perfect, i.e. they do not always find the optimal point, they are very efficient in attaining a near-optimal solution in a much shorter time as compared to the conventional point-by-point exhaustive search, especially in large solution spaces. GA-based multiuser detection has been proposed for example in [9, 10]. In these proposals good initial 'guesses' concerning the possible user bit sequences were required, in order to attain an improved BER performance within a reasonable amount of time, when compared to suboptimal detectors, such as the decorrelator and the MMSE detector [5]. Hence, initially a tentative decision concerning the transmitted bit sequence must be made using a detector, such as the conventional single-user detector or a decorrelator, in order for GA-based multiuser detection to be feasible.

In our system, a GA based scheme is employed in conjunction with a local search in order to improve the initial

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guesses using a population of randomly generated possible solutions, while maintaining a high diversity of solutions. In this way, the GA is capable of converging to the best solution in a short time. Such a GA/local search combination is known as a *hybrid GA* [7, 11]. In our forthcoming discussions, we will investigate the BER performance gain of the hybrid GA based multiuser detector over a purely GA based detector in a bit-synchronous CDMA system over a Gaussian channel. Furthermore, we will evaluate the effects of varying the GA parameters on the bit error rate (BER) performance, in order to strike a balance between detection complexity and performance. We then further investigate the BER performance of the GA-based multiuser detector over a single-path Rayleigh fading channel, assuming perfect knowledge of the users' channel impulse response (CIR) coefficients. Our simulation results showed that a GA based multiuser detector can achieve a near-optimum BER performance over a single-path Rayleigh fading channel without the aid of local search. Following this introduction, we will describe the proposed hybrid GA scheme in the context of suboptimal multiuser detection. Our simulation results and conclusions are presented in Section 3 and Section 4, respectively.

## 2. GENETIC ALGORITHM HYBRID-BASED MULTIUSER DETECTOR

In hybrid GA based multiuser detection schemes GAs combined with an element of local search are used to determine the specific combination of the users' transmitted bits, i.e. the transmitted bit vector  $\mathbf{b}_K$  that maximizes the correlation metric (objective function or fitness function) given in Eqn. (1). The set of possible bit combinations of the bit sequence constitute the search space. Hence for a  $K$ -user bit-synchronous system, there are a total of  $2^K$  possible bit sequences in each bit-interval.

GAs maintain a population of  $n$  so-called strings or individuals [7], which correspond to the points in the search space. Hence, each *individual* contains a sequence of  $K$  binary bits, referred to as the *string length*. GAs commence their search for the optimum solution with an initial population of *individuals*, which is created randomly. This initial population, denoted here as  $\mathbf{g}(0) = [\hat{\mathbf{b}}_K^1 \dots \hat{\mathbf{b}}_K^n]$  occurs at the so-called 0th generation, where  $\hat{\mathbf{b}}_K^i, i = 1, \dots, n$  is the  $i$ th *individual* corresponding to the estimated transmitted data vector of the  $K$  users and  $n$  is the population size. Subsequently, a new population of  $n$  *individuals*, i.e.  $\mathbf{g}(x)$  of the  $x$ th generation, is created – or evolved – from the previous population, i.e. from  $\mathbf{g}(x-1)$  of the  $(x-1)$ th generation, by means of some simple probabilistic genetic operations [7]. Three basic genetic operators are used here, namely the so-called *reproduction* (or selection), *crossover* and *mutation* [7].

As suggested by the terminology, the so-called *reproduction* process selects two individuals from the old population in order to produce two new individuals, or so-called *offspring*, for the next generation population. Hence, these individuals are also known as *parents*. The individuals are selected as parents based on their figure of merit calculated from the objective function given in Eqn. (1) through a selection mechanism known as *truncation selection* [8]. In truncation selection, only a limited number of  $M$  individu-

als, where  $M \leq n$ , corresponding to the  $M$  highest figure of merit in the population during a generation can be selected and they all have the same selection probability, i.e.  $1/M$ . Note that this selection mechanism is different from the so-called *proportionate selection* scheme employed in [10]<sup>1</sup>, since GAs using the proportionate selection scheme can lead to a so-called *premature convergence* [8], which prevents the GA from embarking on any further explorations.

After reproduction, parents in the so-called mating pool are randomly selected to undergo the so-called *crossover* with a probability of  $p_c$ , in order to produce offspring for the next generation population. There are several forms of crossover, such as the *one-point*, *two-point* and *uniform crossover* [8]. Here, uniform crossover [12] is used, which invokes a so-called *crossover mask* defined as a sequence of randomly generated 1s and 0s of equal probability of string-length  $K$ . Bits are exchanged between the selected pair of parents at locations corresponding to a 1 in the crossover mask. An illustration of the uniform crossover process is shown in Figure 1.

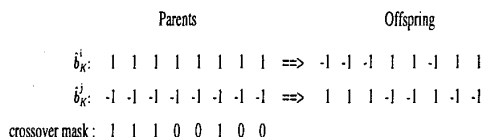


Figure 1: An example of uniform crossover

Finally, *mutation* refers to the alteration of the value of a bit in the offspring with a probability of  $p_m$ . Mutation is necessary, in order to prevent premature convergence to suboptimal solutions. Parents do not always produce fitter offspring. There is also a finite chance that individuals having a high figure of merit in the sense of Eqn. (1) are not selected as parents. Hence, in order to prevent the loss of high-merit individuals, we identify the best individual in a population and replace the worst offspring by it in the new population. Hence the highest-merit individual of a particular generation is propagated to the next generation. This is known as *elitism*.

The GA terminates at the  $(y-1)$ th generation, where  $y > 0$ . The *individual corresponding to the highest fitness value is the detected users' bit sequence*.

In our system, we additionally invoked a local search in the form of *hill-climbing* [7], in order to provide good initial guesses concerning the 0th generation population. The initial population is first randomly generated using an unbiased coin. This ensures that the GA has a highly diversified search range at the beginning. The fitness values of the individuals of the initial population are recorded. The local search is performed for each individual independently. Initially, the first bit of an individual is complemented and the corresponding fitness value is compared to the recorded original fitness value. If the new fitness value – associated with the inverted first bit of the individual concerned – is higher than the original recorded fitness value, then the complemented bit is confirmed to be more likely and the new fitness value is recorded. Otherwise, the original bit and the recorded fitness value are retained. This process

<sup>1</sup>There was no mention on the type of selection mechanism employed in [9]

is performed serially, until the last bit of the individual is determined. Hence for a population of  $n$  individuals, the total number of fitness evaluations required is equivalent to  $(K + 1) \times n$ . If the GA terminates after the  $y$ th generation, then the total number of fitness evaluations equals to  $(K \times n) + (y \times n)$ . Note that if no local search is invoked, then the number of objective function evaluations is lower, namely  $(y \times n)$ . Let us now consider the performance of the proposed system.

### 3. SIMULATION RESULTS

#### 3.1. AWGN channel

In this section, computer simulations are presented in order to illustrate the Bit Error Rate (BER) performance of the hybrid GA-based multiuser detector highlighted in the previous section. All the results in this section were based on evaluating the BER performance of a bit-synchronous 20-user CDMA system over a Gaussian channel. The signature sequences were randomly generated 31-chip per bit sequences and the transmit bit energy  $\xi_k$  was assumed to be equal for all users.

Fig. 2 shows the BER performance against  $\xi_k/N_0$  for the hybrid GA based multiuser detector with various population sizes  $n$ . Also shown in the same figure is the BER performance of the GA-based detector without the aid of local search for a population size of  $n = 40$ . The GA parameters used in this case are:  $p_c = 1, p_m = 0.1, M = 5$  and  $y = 20$ . As the figure shows, a BER degradation is observed, when no local search is incorporated in the optimization process. Furthermore, upon increasing the population size, the BER performance of the hybrid GA-based detector approaches that of the single-user case. The error floor observed for  $n = 40$  was caused by the limitation of the given set of GA parameters. We will demonstrate at a later stage that it is possible to improve the BER performance by adjusting the various GA parameters at the expense of additional computational complexity as well as upon tolerating a longer convergence time. The parameter in round brackets in the legend denotes the number of times the objective function of Eqn. (1) was evaluated upon detecting each of the users' transmitted bits during a bit interval. Hence, while achieving near single-user performance, the associated additional computational complexity has to be tolerated.

Fig. 3 shows the effects of evolution in terms of the number of generations on the BER performance. When using a high population size – such as  $n = 120$  – the hybrid GA multiuser detector is capable of reaching the optimal value within about 20 generations. By contrast, a population size of  $n = 40$  requires more generations, before it reaches its residual BER, which is higher than that of the population size of 120. Similar observations are valid for  $n = 80$ .

The choice of the crossover and mutation probabilities  $p_c$  and  $p_m$  also has an effect on the convergence rate and the residual BER. This is illustrated in Fig. 4 using various values of  $p_c$  and  $p_m$  for  $n = 40$  and  $M = 5$ . As it can be seen in the figure,  $p_c = 1, p_m = 0.01$  can cause the optimization to converge initially faster but fail to find a satisfactory solution, while the system using  $p_c = 1, p_m = 0.2$  is slow to converge. We also notice that the combination of  $p_c = 0.6$  and  $p_m = 0.03$  gives the best BER performance as compared to the other simulated values shown in Fig. 4.

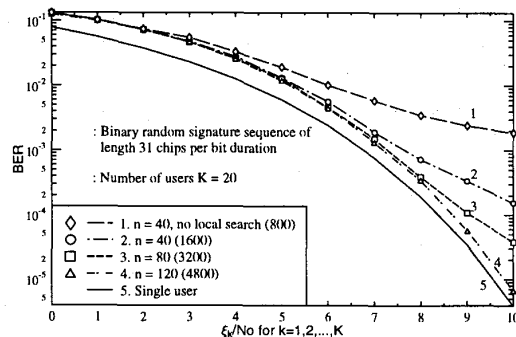


Figure 2: The BER performance of the hybrid GA multiuser detector as a function of  $\xi_k/N_0$  with population sizes of  $n = 40, 80, 120$  using binary random signature sequences of length 31. The GA parameters used are the probability of crossover given by  $p_c = 1$ , the probability of mutation given by  $p_m = 0.1$ , the truncation selection criterion of  $M = 5$  and the number of generations namely  $y = 20$ . The parameter in round brackets in the legend denotes the number of times the objective function of Eqn. (1) was evaluated upon detecting each of the users' transmitted bits during a bit interval

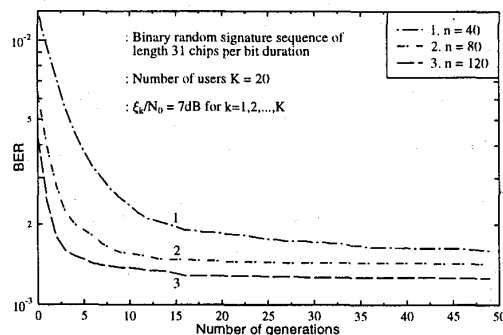


Figure 3: The BER performance of the hybrid GA multiuser detectors as a function of the number of generations with population sizes of  $n = 40, 80, 120$  using binary random signature sequences of length 31 at  $\xi_k/N_0 = 7\text{dB}$  for  $k = 1, 2, \dots, K$ . The GA parameters used are the probability of crossover given by  $p_c = 1$ , the probability of mutation specified by  $p_m = 0.1$  and the truncation selection criterion of  $M = 5$

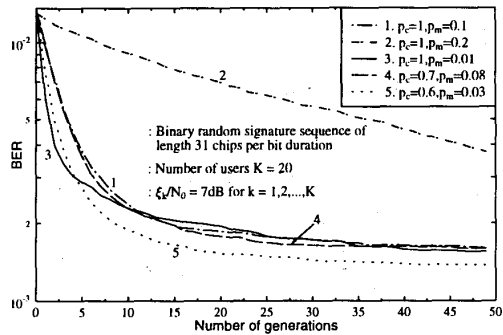


Figure 4: The BER performance of the hybrid GA multiuser detectors as a function of the number of generations with various probability values of crossover  $p_c$  and mutation  $p_m$  for a population size of  $n = 40$  using binary random signature sequences of length 31 at  $\xi_k/N_0 = 7\text{dB}$  for  $k = 1, 2, \dots, K$ . The truncation selection criterion was  $M = 5$

Hence, the BER performance shown in Fig. 2 for the hybrid GA-based detector is certainly not optimal.

### 3.2. Single-path Rayleigh fading channel

In this section, computer simulations are presented in order to illustrate the BER performance of the GA-based multiuser detector **without any local search**. All the results were based on evaluating the BER performance of a bit-synchronous 10-user CDMA system over a **single-path Rayleigh fading channel**. The signature sequences were randomly generated 31-chip per bit sequences and the users' CIR coefficients were assumed to be known at the receiver.

Fig. 5 shows the BER performance of the GA based detector without any local search for different number of generations  $y$  and for  $n$  strings. The single-user bound was also plotted for comparison. As it can be seen from the figure, the combination of  $n = 40$  strings and  $y = 10$  generations – which constitutes  $40 \times 10 = 400$  objective function evaluations according to Eqn. (1) – was capable of achieving a near-optimum single-user-like BER performance without the aid of a local search. For  $\xi_k/N_0$  values beyond 40 dB, the system exhibited an error floor due to the performance limitations of the GA in conjunction with the given  $y$  and  $n$  values studied. At lower values of  $y$  and  $n$ , the error floor occurred at a lower  $\xi_k/N_0$  value. For instance, at  $y = 10$  and  $n = 20$ , which constitutes 200 objective function evaluations according to Eqn. (1), the error floor occurred at an  $\xi_k/N_0$  value of about 32 dB, while for  $\xi_k/N_0$  values up to 24 dB, the detector exhibited near-optimum BER performance. Hence, the proposed GA-based detector was capable of offering a trade-off between computational complexity and an optimum BER performance. We also note that GAs using a higher number of generations per signalling interval gave a better BER performance, than GAs having a larger population size  $n$  and a lower number of generations  $y$  at

the same computational complexity. For example, in Fig. 5 the BER performance of the  $n = 20, y = 10$  scenario was better, than that of the  $n = 40, y = 5$  arrangement, both of which requires 200 objective function evaluations according to Eqn. (1).

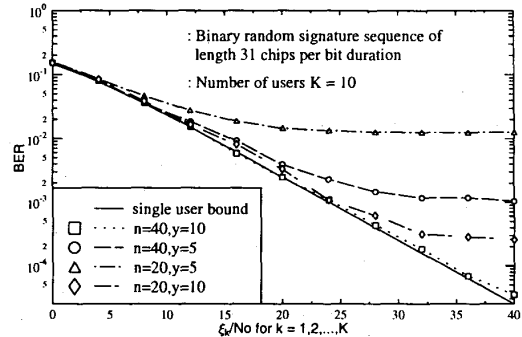


Figure 5: BER performance of the GA-based detector without local search with various population size  $n$  and for  $y$  generations in conjunction with **known CIR coefficients** over narrowband Rayleigh-fading channels for all users using binary random signature sequences of length 31. The GA parameters used are the probability of crossover given by  $p_c = 1$ , the probability of mutation specified by  $p_m = 0.1$  and the truncation selection criterion of  $M = 5$

Fig. 6 shows the near-far resistance of the proposed GA-based detector as compared to that of the conventional matched filtering or correlation detector, both assuming perfect knowledge of the CIR estimates, which for our narrowband Rayleigh channel implies the knowledge of the channel attenuation and phase rotation. The average received bit energy for the desired user remained unchanged, while the energies of all other users were varied in the range of 0-20 dB higher, than that of the desired user. We can see that the GA-based detector was near-far resistant, similarly to the optimum maximum likelihood (ML) detector [3] (not shown in the figure), since both detectors are based on the ML correlation metric.

## 4. CONCLUSION

In conclusion, the optimum multiuser detector [3] has a computational complexity that is exponentially proportional to the number of users. Thus its implementation becomes impractical, when there is a high number of users. In order to circumvent the complexity problem, GAs have been used to solve complex optimization problems in many fields. However, the convergence rate of GAs is typically too slow for real-time data detection. For mitigating this impediment, we proposed a hybrid GA approach in order to tackle the slow convergence problem. The hybrid GA scheme uses an element of local search in order to provide good initial guesses concerning the possible solutions. We have shown that with a high population size – ensuring a sufficiently

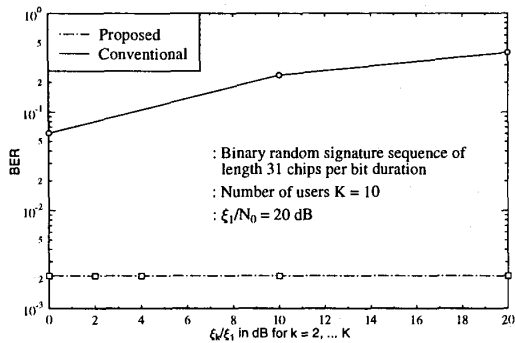


Figure 6: Relative near-far resistance of the GA-based detector without local search and the conventional detector in terms of the BER with **perfect channel estimation** over narrowband Rayleigh-fading channels for all users using binary random signature sequences of length 31. The GA parameters used are the probability of crossover given by  $p_c = 1$ , the probability of mutation specified by  $p_m = 0.1$ , the truncation selection criterion of  $M = 10$ , the population size of  $n = 40$  and the number of generations given by  $y = 10$

diversified search [7] – the BER performance of a hybrid GA based multiuser detector approaches that of a single-user scenario with a much lower computational complexity, when compared to that of an optimum multiuser detector. We have also shown that the GA-based detector is capable of achieving a near-optimum single-user-like BER performance without the aid of a local search over a single-path Rayleigh fading channel with perfect channel magnitude and phase estimation. The GA-based detector is also near-far resistant. Our future work will attempt to extend these advances to residue number system based multiuser systems [13].

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