ADAPTIVE RATE RRNS CODED OFDM TRANSMISSION FOR MOBILE COMMUNICATION CHANNELS

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ABSTRACT

The novel class of non-binary maximum minimum distance redundant residue number system (RRNS) codes is invoked in the context of adaptively RRNS coded, symbol-by-symbol Adaptive Orthogonal Frequency Division Multiplexing (AOFDM), in order to combat the effects of frequency-selective fading inflicted by dispersive wideband channels. The system's performance can be adjusted, in order to maintain a given target bit error rate (BER) and bit per symbol (BPS) performance. The proposed adaptive RRNS scheme outperforms the convolutional constituent code based turbo coded benchmarker system for channel Signal-to-Noise Ratios (SNR) in excess of about 15 dB at a target BER of 10^{-4}.

1. INTRODUCTION

Symbol-by-symbol Adaptive Orthogonal Frequency Division Multiplexing (AOFDM) has been proposed [1]-[4], in order to improve the system's Bit Error Rate (BER) and throughput performance, by invoking an appropriate AOFDM mode, which is determined on the basis of the near-instantaneous perceived channel quality. Since the channel quality is both time- and frequency-dependent, adaptation in both time- and frequency-domain was shown to be beneficial. In this contribution the potential benefits of combining AOFDM with adaptive or variable rate channel coding schemes are explored. We will demonstrate that the family of Redundant Residual Number System (RRNS) [5, 6, 7, 8] based codes constitute an attractive alternative.

Similarly to the AOFDM schemes discussed in [1]-[4] adaptive coding relies on the fundamental principles of channel quality estimation, AOFDM / coding mode adaptation and signalling of the AOFDM / coding modes employed. Adaptation of the AOFDM / coding modes relies on a duplex link, where both receivers can estimate the prevalent near-instantaneous channel conditions on the basis of the received OFDM symbols. This knowledge can then be used to adapt the reverse link's transmitter directly, which is referred to as open-loop adaptation, or by instructing the remote transmitter to employ a required set of AOFDM / coding mode parameters for transmission. This closed-loop adaptation does not rely on the channel's reciprocity and can therefore operate also in the presence of co-channel interference. In this treatise we will concentrate on the upper-bound performance study of the proposed adaptive OFDM / coding mode regime and assume perfect channel estimation as well as error-free AOFDM / coding mode signalling. We assume a duplex adaptive Wireless Asynchronous Transfer Mode (WATM) system communicating over the three-path Rayleigh fading channel of reference [3], having a normalised Doppler frequency of $1.23 \cdot 10^{-5}$. Consecutive timeslots are used for the up- and down-link in a Time-Division Duplex (TDD) frame.

In order to efficiently react to the time- and frequency dependent channel transfer function fluctuations, the adaptive error correction codec has to be able to vary its code rate rapidly - ie without latency - according to the time-variant channel conditions. Hence high-latency turbo codes cannot be readily employed in this context. Ideally, the error correction capability of the code would be adjustable for each data bit's expected BER independently, although this is clearly irrealistic. For our experiments, short block length codes of less than 72 bits per code word were employed, in order to allow flexible adaptation of the channel code parameters, while delivering reasonable error protection for the data bits. Specifically, a range of adaptive Redundant Residual Number System (RRNS) [5, 6, 7, 8] based codes (ARRNS) are proposed and investigated in this contribution due to their advantages highlighted below.

2. ADAPTIVE REDUNDANT RESIDUAL NUMBER SYSTEM CODES

The RRNS codes employed in our investigations are systematic, implying that k of the n code residues contain the original data bits and the additional $(n-k)$ redundant residues can be employed for error correction at the decoder. The error correction capability of the code is $t = \lfloor \frac{n-k}{2} \rfloor$ residues [10]. An advantageous property of RRNS codes is that a high number of residues can be generated and transmitted, if necessary, but in the absence of transmission errors the original meassage can be recovered without any redundant moduli [5, 10, 11]. This property lends itself to the proposed adaptive coding strategy, which will be outlined during our forthcoming discourse.

Following from this line of argument, the code rate — and accordingly the error correction capability of the code — can be readily varied by transmitting only a fraction of the generated redundant residues. If the channel conditions are favourable, then only the systematic information-bearing residues are transmitted, resulting in a unity-rate

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code with no added redundancy and no error correction capability. Upon transmitting two redundant residues with the data bits, the resulting code can correct one residue error for a code rate of $\frac{n}{n+2}$. More of the redundant residues can be transmitted, lowering the code rate and improving the code's error resilience at the cost of a lower effective information throughput, when the channel quality degrades.

In our investigations RRNS codes employing 8 bits per residue have been chosen. Three systematic information-bearing residues — corresponding to 24 useful data bits per code word — and up to six redundant residues have been employed. The code parameters for these codes are shown in Table 1. As it can be seen from the table, the code rates vary from 0.33 to 1 and the codes $(9,3)$, $(7,3)$, $(5,3)$ and $(3,3)$ are used, correcting $t = 3, 2, 1,$ and 0 residues per code word, respectively.

Mode c_w	0	1	2	3
Code	(3,3)	(5,3)	(7,3)	(9,3)
n_c	3	5	7	9
k_c	3	3	3	3
t_c	0	1	2	3
R	1	0.6	0.43	0.33

Table 1: RRNS coding modes used for the code rate adaptation employing 8-bit residues

We note that residue-based interleaving has a better performance than bit-interleaving, since bit-interleaving would increase the probability of residue errors due to spreading bursts of erroneous bits across residues. Since the RRNS decoding algorithm is symbol based, the increased residue error rate due to bit interleaving would degrade the system's performance. The code rate adaptation reacts to the time- and frequency-varying channel conditions experienced in a duplex link. Each receiver exploits the channel quality information extracted from the last received OFDM symbol for determining the coding parameters of the next transmitted frame.

The choice of the coding mode for each code word in the OFDM symbol is determined on the basis of the estimated channel transfer function. The predicted bit error probabilities p_e are calculated for all bits to be transmitted in an OFDM symbol, based on the estimated subcarrier SNR - with the aid of frequency-domain pilots - and the AOFDM mode to be employed. If adaptive modulation is to be employed in conjunction with adaptive coding, then the number of bits per OFDM symbol and the mapping of bits to subcarriers can change from one OFDM symbol to the next. Hence the coding scheme adaptation algorithm operates on the basis of the estimated BER, rather than relying on the estimated channel transfer function. Once the vector of estimated bit error probabilities $p_e(n)$ for the number of bits per AOFDM symbol N_b is known, the total number of bits to be transmitted is split into blocks of K bits, where K is the number of bits per RRNS residue. The error correction capability of the code in each RRNS code word is a given number of residues or non-binary symbols, not bits. Hence — as argued before — interleaving of bits would increase the residue error rate at the decoder's input and lower the system's performance.

From the values of $p_e(n)$, the estimated residue error rate $p_r(r)$ for the $N_r = \lfloor N_b/K \rfloor$ residues in the OFDM symbol can be calculated as:

$$p_r(r) = 1 - \prod_{n=0}^{K-1} \left(1 - p_e(r \cdot K + n)\right). \tag{1}$$

The remaining $N_b - K \cdot N_r$ data bits of the OFDM symbol that are not allocated to any residue are filled with padding bits and hence contain no useful data. The mapping of the residues with index r to the RRNS code words is based on the estimated residue error probabilities $p_r(r)$. A square-shaped block residue interleaver, $I(r)$, is used to map the stream of residues to the residue positions in the transmitted OFDM symbol. The interleaver function used for the experiments is defined below.

A received code word of the codec mode c_w is irrecoverable, if more than t_c of the received residues are in error. The RRNS code word error probability p_w for word w can be calculated as:

$$p_w(w) = p(R_r(w) > t_{c_w}) = 1 - P(R_{r_w} \leq t_{c_w}), \tag{2}$$

where $R_r(w)$ is the number of residue errors in code word w, and $p_w(w)$ can be calculated from the residue error probabilities $p_r(r)$ as:

$$p_w(w) = 1 - p\left[R_r(w) = 0\right] - \ldots - p\left[R_r(w) = t_{c_w}\right]. \tag{3}$$

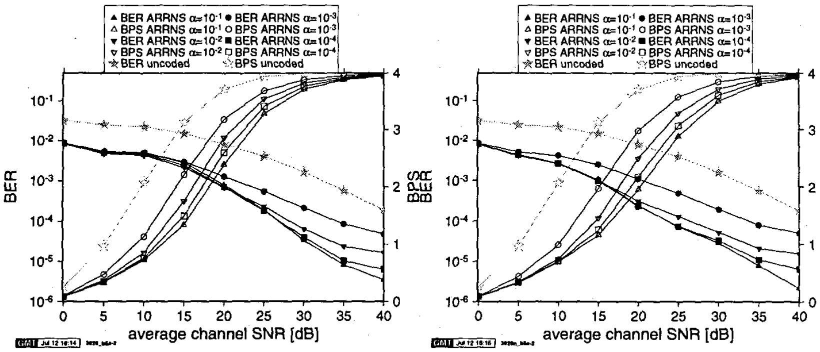
Upon elaborating further:

$$p\left[R_r(w) = 0\right] = \prod_{r=0}^{n_{c_w}} \left(1 - p_r(I(r_{0,w} + r))\right) \tag{4}$$

$$\begin{aligned} p\left[R_r(w) = 1\right] &= \sum_{r=0}^{n_{c_w}-1} p_r(I(r_{0,w} + r) \\ &\quad \cdot \prod_{s=0, s\neq r}^{n_{c_w}-1} \left(1 - p_r(I(r_{0,w} + s))\right) \\ &= p\left[R_r(w) = 0\right] \cdot \sum_{r=0}^{n_{c_w}-1} \frac{p_r(I(r_{0,w} + r))}{1 - p_r(I(r_{0,w} + r))} \end{aligned} \tag{5}$$

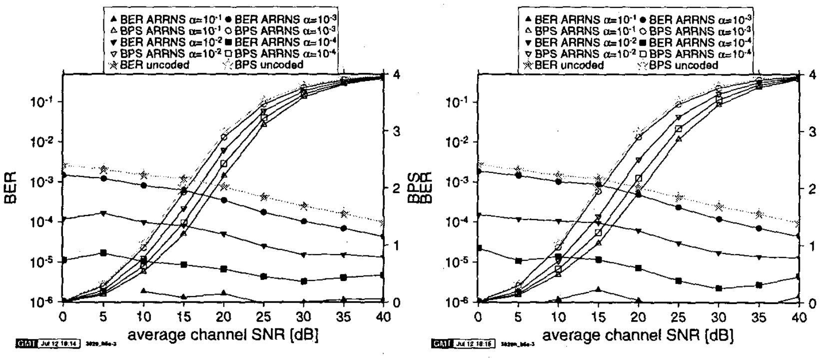
$$\begin{aligned} p\left[R_r(w) = 2\right] &= \frac{1}{2} \cdot p\left[R_r(w) = 0\right] \\ &\quad \cdot \sum_{r=0}^{n_{c_w}-1} \left[\frac{p_r(I(r_{0,w} + r))}{1 - p_r(I(r_{0,w} + r))} \right. \\ &\quad \left. \cdot \sum_{s=0, s\neq r}^{n_{c_w}-1} \frac{p_r(I(r_{0,w} + s))}{1 - p_r(I(r_{0,w} + s))}\right] \end{aligned} \tag{6}$$

$$\begin{aligned} p\left[R_r(w) = 3\right] &= \frac{1}{3!} \cdot p\left[R_r(w) = 0\right] \\ &\quad \cdot \sum_{r=0}^{n_{c_w}-1} \left[\frac{p_r(I(r_{0,w} + r))}{1 - p_r(I(r_{0,w} + r))} \right. \\ &\quad \cdot \sum_{s=0, s\neq r}^{n_{c_w}-1} \left[\frac{p_r(I(r_{0,w} + s))}{1 - p_r(I(r_{0,w} + s))} \right. \\ &\quad \left.\left. \cdot \sum_{t=0, t\neq q, r}^{n_{c_w}-1} \frac{p_r(I(r_{0,w} + t))}{1 - p_r(I(r_{0,w} + t))}\right]\right], \end{aligned}$$



(a) target BER 5% non–interleaved

(b) target BER 5% interleaved



(c) target BER 0.5% non–interleaved

(d) target BER 0.5% interleaved

Figure 1: BER and BPS throughput versus average channel SNR for **hard–decision** ARRNS–coded 512–subcarrier OFDM transmission employing adaptive modulation over the Rayleigh fading time–dispersive WATM channel of reference [3]; (a)(b) – uncoded adaptive modulation target BER 5% (c)(d) – uncoded target BER 0.5% (a)(c) non–interleaved (b)(d) – interleaved. The stipulated WER values were $\alpha = 10^{-1}, 10^{-2}, 10^{-3}$ and 10^{-4}. The light grey curves show the uncoded BER and BPS throughput.

where $r_{0,w}$ is the index of the first residue in code word w.

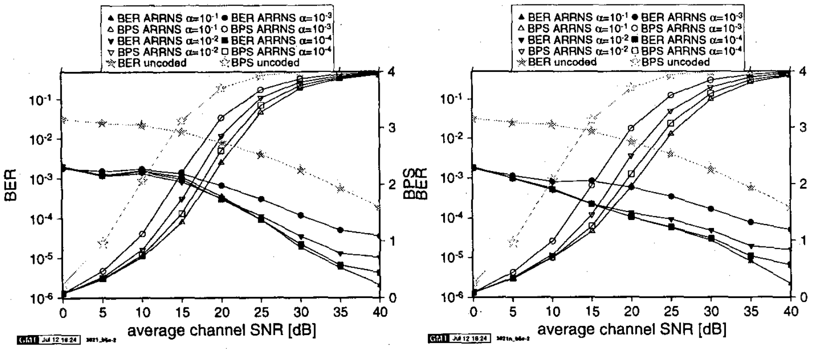
The code rate adaptation algorithm calculates the word error probability $p_w(w)$ for the RRNS code word index w for the lowest–power codec mode of Table 1, $c = 0$. If the code Word Error Rate (WER) is higher than its target value α - ie $p_w(w) > \alpha$ for $c = 0$ - then the next stronger ARRNS coding mode, namely $c = 1$ is selected, and the word error probability is evaluated again. If the new RRNS code word error probability exceeds the threshold α, then the next stronger codec mode is evaluated, until the estimated RRNS code word error probability falls below the threshold α, or until the highest–power codec mode is selected. The target WER α is supplied to the algorithm and it can be used to control the adaptation process.

3. ARRNS/AOFDM TRANSCEIVERS

In this section we will demonstrate that upon combining AOFDM [1]-[4] with the above ARRNS coding regime, an attractive system accrues, which exhibits an excellent performance - in particular at low SNRs - due to amalgamating transmission blocking for the low-quality subcarriers with ARRNS coding. We have advocated here the target–BER adaptive modulation algorithm of [2] due to its high performance and convenient adjustability to different target BERs and invoked three different modulation modes, as well as "no transmission", on a subband-by-subband basis. Specifically, 0, 1, 2 and 4 Bits-Per-Symbol (BPS) Quadrature Amplitude Modulation (QAM) schemes [1] - corresponding to Binary Phase Shift Keying (BPSK), Quarternary Phase Shift Keying (QPSK), and 16-QAM were used.

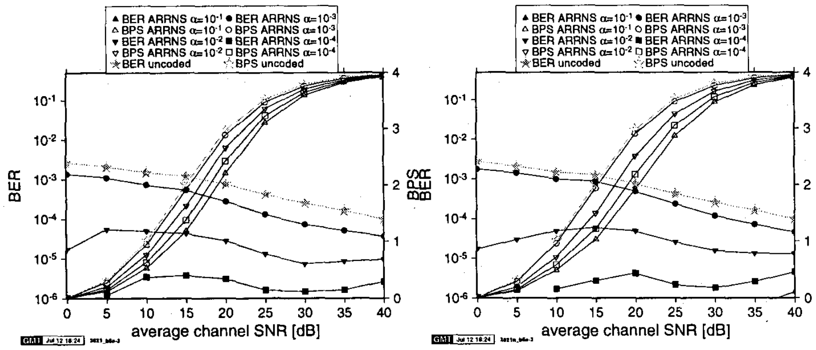
The transmission parameter adaptation is performed in two steps. First the AOFDM modulation modes are allocated to the subcarriers according to the algorithm outlined in [2]. In this contribution we assumed perfect knowledge of the channel transfer function and perfect modulation mode detection, concentrating on the system's upper–bound performance. Blind AOFDM modem mode detection was the topic of [2].

Following this step, the number of bits N_b to be trans-



(a) target BER 5% non–interleaved

(b) target BER 5% interleaved



(c) target BER 0.5% non–interleaved

(d) target BER 0.5% interleaved

Figure 2: BER and BPS throughput versus average channel SNR for **soft–decision** assisted ARRNS–coded 512–subcarrier AOFDM employing adaptive modulation over the Rayleigh fading time–dispersive WATM channel of reference [3]; (a)(b) – uncoded adaptive modulation target BER 5% (c)(d) – uncoded target BER 0.5% (a)(c) non–interleaved (b)(d) – interleaved. The stipulated WER values were $\alpha = 10^{-1}, 10^{-2}, 10^{-3}$ and 10^{-4}. The light grey curves show the uncoded BER and BPS throughput. The corresponding hard–decision based results were plotted in Figure 1.

mitted in the next OFDM symbol and their estimated bit error probabilities $p_e(n)$, are known. On the basis of this, the ARRNS code rate adaptation algorithm calculates the residue error rates $p_r(r)$ from Equation 1, constructs the interleaver $I(r)$ for the correct number of residues and invokes the appropriate codec modes for the ARRNS code words, as outlined above. Let us now consider the performance of the proposed hard- as well as soft-decision assisted AARNS/AOFDM system over the WATM channel of reference [3].

Figure 1 gives an overview of the hard-decision assisted ARRNS/AOFDM system's BER and throughput performance over the fading time–dispersive WATM channel of [3]. Two target BER values have been stipulated, both with and without interleaving of the transmitted residues. Figures 1(b) and 1(a) portray the system's performance, if an uncoded target BER of 5% is assumed for the ARRNS/AOFDM scheme with and without interleaving, respectively. It can be seen that the coded BER is then below 1% for all simulated ARRNS/AOFDM modem configurations and our results demonstrated - not shown here due to lack of space - that the SNR gain is significantly higher for the ARRNS/AOFDM system, than for example for fixed-mode QPSK ARRNS transmission. The BER performance is limited, however, by the limited error correction capability of the RRNS (9,3) mode when the SNR is very low.

Upon reducing the uncoded AOFDM scheme's target BER to 0.5%, the BER of the ARRNS/AOFDM system can be influenced over the whole SNR range by varying the target WER α. Figures 1(d) and 1(c) depict the corresponding BER and BPS throughput, where for a WER of $\alpha = 10^{-1}$, the achieved ARRNS/AOFDM BER is better than $2 \cdot 10^{-3}$, while for $\alpha = 10^{-2}$ a BER of $2 \cdot 10^{-4}$, and for $\alpha = 10^{-3}$ a BER of $2 \cdot 10^{-5}$ are never exceeded.

Comparing the interleaved performance with the non-interleaved results, it can be seen that the BER performance of the corresponding modems is fairly similar. The throughput is slightly higher however for the non-interleaved systems, demonstrating the efficiency of the hard-decision based ARRNS/AOFDM schemes in terms of combatting

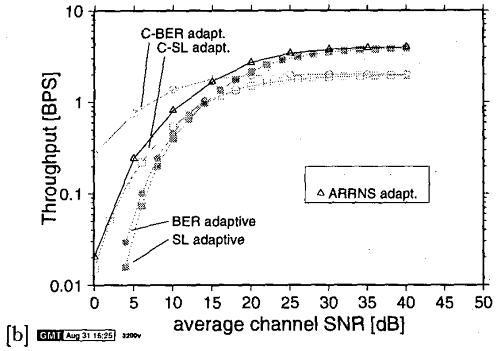


Figure 3: BPS Throughput versus average channel SNR for AOFDM over the dispersive WATM channel of reference [3] for a maximum BER of 10^{-4}. The lightly shaded curves present the performance of the variable throughput AOFDM schemes from [2] with and without convolutional turbo coding. The variable throughput systems include: convolutionally turbo coded (C-) and uncoded switching level adaptive (SL) and target-BER adaptive (BER) systems, as well as the joint adaptive RRNS/AOFDM system.

the bursty errors of frequency-selective fading. More explicitly, this is the consequence of the dispersion of residue errors by the interlever, which reduces the efficiency of the AOFDM/ARRNS regime.

In the previous literature only hard-decision based RRNS decoding has been proposed [10, 11]. However, upon invoking the approach of [8], our RRNS decoder becomes capable of exploiting soft outputs provided by the demodulator at the receiver. Specifically, soft decoding of the ARRNS codes can be implemented by combining the classic Chase algorithm [12] with the hard decision based ARRNS decoder, which is the topic of our forthcoming discussions.

Figure 2 portrays the **soft-decision** decoded performance of the proposed AOFDM/ARRNS system. As an example, observe in the figure that a BER of less than 10^{-4} was registered for the 0.5% target BER system for a WER of $\alpha = 10^{-2}$ and that the interleaved system exhibits a lower throughput and worse BER performance, than that of the non-interleaved system.

4. COMPARISON AND CONCLUSIONS

In conclusion, Figure 3 shows the bit per symbol (BPS) throughput of the various ARRNS/AOFDM transmission systems studied for a target BER of 10^{-4}. The lightly shaded curves represent the variable throughput systems' performance graphs from reference [2]. The ARRNS/AOFDM system employed no interleaving, had an uncoded AOFDM target BER of 1%, and used a target WER of $\alpha = 10^{-2}$. It should be noted that the above BPS performance figures do not take into account the signalling overhead required for conveying the ARRNS/AOFDM modes and hence constitute the upper-bound performance of the system. These performance figures suggest that the proposed ARRNS/AOFDM scheme outperforms the benchmarkers of [2] - which were also defined in the caption of Figure 3 - in BPS terms for channel SNRs in excess of about 15dB over the WATM channel of reference [3], while below 15dB the turbo-convolutional code of [2] exhibits a higher BPS throughput. Our future work will be focused on incorporating turbo BCH codes in this system, in an effort to further assess its performance potential in conjunction with AOFDM. Our further related research invoking turbo trellis coding in conjunction with adaptive beam-steering and interference cancellation was summarised in [13, 14].

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