‘Errors-and-Erasures’ Decoding Performance of RS coded DS-CDMA Using M-ary Orthogonal Signalling

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Abstract— **In this contribution we investigate the performance of RS coded DS-CDMA using M-ary orthogonal signalling over multipath Rayleigh fading channels. ‘Errors-And-Erasures (EAE)’ RS decoding is considered, where erasures are declared based on the joint Maximum Output and Ratio Threshold Test (MOR-TT). Two classes of diversity reception techniques suitable for noncoherent M-ary detection are invoked, namely either Equal Gain Combining (EGC) or Selection Combining (SC) and their related performance is evaluated for both uncoded and coded DS-CDMA systems. The performance of the proposed MOR-TT technique is compared to that of the Ratio Threshold Test (RTT) and the Output Threshold Test (OTT), as well as that of ‘Error-Correction-Only (ECO)’ decoding. The numerical results show that when using EAE decoding, RS codes of a given code rate are capable of achieving a significantly higher coding gain than without erasure information. Furthermore, the MOR-TT technique outperforms both the RTT and the OTT techniques.**

I. Introduction

In DS-CDMA cellular systems Forward Error Correction (FEC) is often used for mitigating the effects of fading and interference. For the well-known RS codes, ‘Errors-And-Erasures (EAE)’ decoding is preferable to ‘Error-Correction-Only (ECO)’ decoding, since EAE decoding typically achieves a significantly lower codeword decoding error probability, than ECO decoding, provided a reliable erasure insertion scheme is invoked. In the context of M-ary orthogonal modulation using RS coding, a number of methods for generating reliability-based side information and their performance has been analyzed for example in [1] - [5]. In [4], [5] it has been shown that the Ratio Threshold Test (RTT) and the demodulator's Output Threshold Test (OTT) constitute a pair of promising erasure insertion schemes exhibiting low-complexity and low-delay. Specifically, in the RTT [1] an erasure is declared, whenever the ratio of the maximum to the ‘second’ maximum of the input of the Maximum-Likelihood Detector (MLD) does not exceed a pre-set threshold. By contrast, in the OTT [5] the erasure insertion is based on the observation of only the maximum of the MLD inputs, i.e. based on the demodulator's output. An erasure is activated, whenever the amplitude of the maximum of the MLD's inputs does not exceed a pre-set threshold.

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In this contribution a novel erasure insertion scheme is investigated, where an erasure is activated based on the observation of not only the maximum, but also the ratio of the maximum to the ‘second’ maximum of the inputs to the MLD associated with the M-ary demodulator. Equivalently, also the ratio of the ‘second’ maximum to the maximum can be used. We refer to this erasure insertion scheme as joint Maximum Output and Ratio Threshold Test (MOR-TT) [6]. Furthermore, we investigate the performance of a RS coded DS-CDMA system [4], [7], when M-ary orthogonal signaling is employed in conjunction with a MOR-TT-based erasure insertion scheme over dispersive Rayleigh-fading channels. Two different diversity combining schemes [4], [7] - namely Equal-Gain Combining (EGC) and Selection Combining (SC) - are considered and their performance is evaluated in the context of the proposed MOR-TT based erasure insertion scheme. The performance of the proposed MOR-TT technique is compared to that of the RTT, OTT, and to that of ECO decoding. We will show that when using EAE decoding, RS codes of a given code rate are capable of achieving a significantly higher coding gain than without erasure information, and that the MOR-TT technique outperforms both the RTT as well as the OTT techniques.

II. System and Channel Model

The DS-CDMA system using M-ary orthogonal modulation considered in this paper has been described in [5], [6], [7]. Hence, readers interested in the details of the system are referred to [7], [5], [6]. We assume that there are K active users transmitting their signals asynchronously activating their corresponding orthogonal signals and spreading sequences. The channel between the transmitter and its corresponding receiver is a multipath fading channel, and the multipath attenuations are independent Rayleigh-distributed random variables.

The receiver schematic of the studied DS-CDMA system including noncoherent M-ary demodulation, multipath diversity combining, maximum-likelihood detection (MLD), MOR-TT-based erasure insertion, deinterleaving and RS EAE decoding is shown in Fig.2 of [6]. The square-law-based noncoherent M-ary demodulator has a standard form, and hence it is the same as that used in [7]. Hence, the interested readers are referred to [7] for further details. In the receiver, multipath diversity combining invoking the EGC or the SC principle was implemented in the 'EGC or SC' block. The MLD block selects the largest from its input variables and computes the value of λ - namely the ratio of the 'second' largest to the largest of the MLD's inputs. Following the MLD stage, the next block may output an M-ary RS code symbol or insert an erasure. Finally, after symbol-based deinterleaving, the RS decoder invokes EAE decoding and then outputs the received information bits.

III. Erasure Insertion Schemes

In this section the three erasure insertion schemes considered in this paper are defined. Let $\{U_1^i, U_2^i, \ldots, U_M^i\}$ represent the decision variables after combining the multipath components using EGC or SC, which form the inputs of the MLD block in Fig.2 of [4]. We express the maximum and the 'second' maximum of $\{U_1^i, U_2^i, \ldots, U_M^i\}$ as

$$Y_1 = \max_1 \{U_1^i, U_2^i, \ldots, U_M^i\}, \quad (1)$$

$$Y_2 = \max_2 \{U_1^i, U_2^i, \ldots, U_M^i\}. \quad (2)$$

Furthermore, let H_1 and H_0 represent the hypotheses of correct decision and erroneous decision, respectively, concerning an M-ary symbol using the MLD algorithm. Hence, we have $P(H_0) = 1 - P(H_1)$.

In the context of the OTT, the decision variable subjected to an erasure insertion is Y_1, i.e., the actual demodulator output is observed. Let Y_T be a threshold associated with making an erasure decision based on the OTT. Then, if $Y_1 \leq Y_T$, the associated demodulated symbol is replaced by an erasure. Otherwise, if $Y_1 > Y_T$, the demodulator outputs a RS code symbol.

The ratio involved in Viterbi's RTT is defined as the ratio of the maximum to the 'second' maximum [1], or equivalently, it can be defined as the ratio of the 'second' maximum to the maximum, which can be expressed as

$$\lambda = \frac{Y_2}{Y_1}, \; 0 \leq \lambda \leq 1. \quad (3)$$

It was shown in [1], [4] that the demodulated symbols having a relatively low ratio of λ are more reliable, than those having relatively high values of λ. Consequently, a preset threshold λ_T can be invoked, in order to erase these low-reliability symbols associated with a ratio of $\lambda \geq \lambda_T$.

In the context of the joint MOR-TT [6], the erasure insertion is based on the observation of both the maximum Y_1 of Eq.(1) and the ratio λ of Eq.(3). Specifically, let $f_{Y_1,\lambda}(y, r|H_1)$ and $f_{Y_1,\lambda}(y, r|H_0)$ be the joint two-dimensional (2D) PDFs of the maximum Y_1 and the ratio λ associated with the MOR-TT, conditioned on both the correct detection and erroneous detection of M-ary symbols. It can be shown that if a demodulated symbol has a maximum output value of Y_1 and a ratio of λ, that had fallen in the main range of $f_{Y_1,\lambda}(y, r|H_0)$, the symbol concerned must be a low-reliability symbol and hence it must be replaced by an erasure. By contrast, if a demodulated symbol has values of Y_1 and λ that had fallen outside the main range of $f_{Y_1,\lambda}(y, r|H_0)$, then this symbol is likely to be correct and hence the corresponding RS code symbol can be forwarded to the RS decoder. Consequently, in order to erase the low-reliability RS coded symbols in the context of MOR-TT, we assume that Y_T and λ_T are two thresholds, which activate an erasure insertion, whenever $Y_1 \leq Y_T$ and $\lambda \geq \lambda_T$.

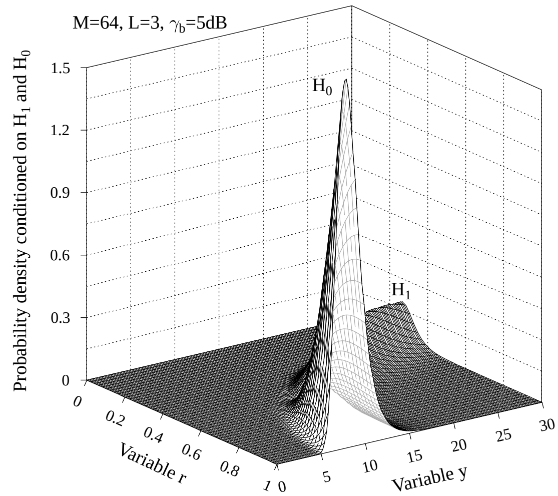

Fig. 1. The 2D joint PDFs of $f_{Y_1,\lambda}(y, r|H_1)$ and $f_{Y_1,\lambda}(y, r|H_0)$ for the EGC scheme using the parameters of $M = 64$, $L = 3$, γ_b =5dB and $K = 1$ over dispersive Rayleigh-fading channels.

IV. Statistics Properties of MOT-TT

The properties of the proposed MOR-TT based erasure insertion schemes can be studied with the aid of the 2D PDFs of $f_{Y_1,\lambda}(y, r|H_1)$ and $f_{Y_1,\lambda}(y, r|H_0)$ shown in Fig.1 and Fig.2 associated with EGC and SC, respectively. The range of parameters used is shown in the captions. In the figures, M is the dynamic range of M-ary symbols, L represents the number of diversity components combined, K is the number of users transmitting simultaneously and finally, $\overline{\gamma}_b$ is the average Signal to Noise Ratio (SNR) per bit, which was obtained by computing $\overline{\gamma}_b = L\overline{\gamma}_c / \log_2 M$, where $\log_2 M$ is the number of bits per symbol. From the 2D surfaces of Fig.1 and Fig.2 we observe that for the given parameters the peak of the distribution of $f_{Y_1,\lambda}(y, r|H_1)$ is located at a relatively high value of y or a relatively low value of r, while the peak of the distribution of $f_{Y_1,\lambda}(y, r|H_0)$

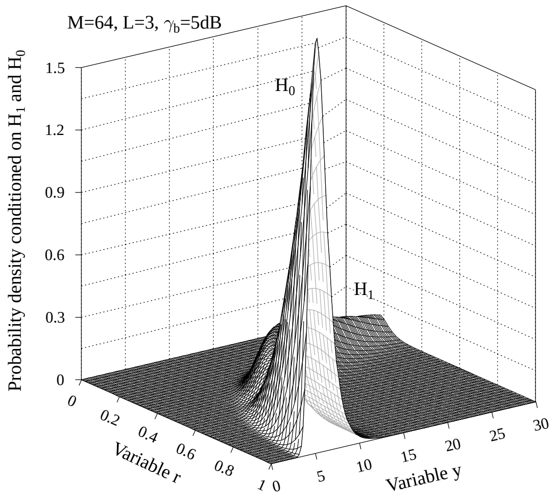

Fig. 2. The 2D joint PDFs of $f_{Y_1,\lambda}(y,r|H_1)$ and $f_{Y_1,\lambda}(y,r|H_0)$ for the SC scheme using the parameters of $M=64$, $L=3$, γ_b =5dB and $K=1$ over dispersive Rayleigh-fading channels.

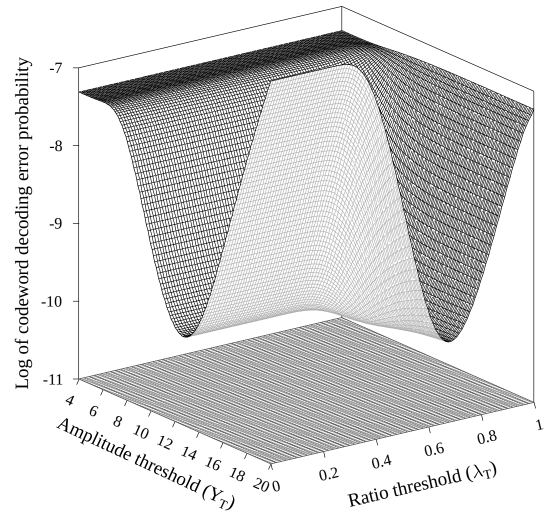

Fig. 3. **EGC**: Codeword decoding error probability versus the amplitude threshold, Y_T and the ratio threshold, λ_T for the RS(32,20) FEC code using 'errors-and-erasures (EAE)' decoding based on the MOR-TT erasure insertion scheme over dispersive Rayleigh fading channels.

is located at a relatively low value of y and a relatively high value of r. The above observations in turn imply that $f_{Y_1,\lambda}(y,r|H_1)$ is mainly distributed over the area having relatively high values of y or relatively low values of r, while $f_{Y_1,\lambda}(y,r|H_0)$ is mainly distributed over the area having relatively low values of y and relatively high values of r. Therefore, if a demodulated symbol has a maximum output value of Y_1 and a ratio of λ that had fallen in the main area of $f_{Y_1,\lambda}(y,r|H_0)$, the symbol concerned must be a low-reliability symbol and hence must be replaced by an erasure. By contrast, if a demodulated symbol has values of Y_1 and λ that had fallen outside the main area of $f_{Y_1,\lambda}(y,r|H_0)$, then this symbol is likely to be correct and hence the corresponding RS code symbol can be forwarded to the RS decoder. Consequently, in order to erase the low-reliability RS coded symbols, we can assume that Y_T and λ_T are two thresholds, which activate an erasure insertion, whenever $Y_1 \leq Y_T$ and $\lambda \geq \lambda_T$. Based on the above erasure decision scheme, the symbol erasure probability, P_e, and the random symbol error probability, P_t, after erasure insertion can be formulated as

$$\begin{aligned} P_e &= P(H_1)\cdot\int_0^{Y_T}\int_{\lambda_T}^{1} f_{Y_1,\lambda}(y,r|H_1)dydr \\ &+ P(H_0)\cdot\int_0^{Y_T}\int_{\lambda_T}^{1} f_{Y_1,\lambda}(y,r|H_0)dydr, \quad (4) \\ P_t &= P(H_0)\cdot\left[1-\int_0^{Y_T}\int_{\lambda_T}^{1} f_{Y_1,\lambda}(y,r|H_0)dydr\right], (5) \end{aligned}$$

where $f_{Y_1,\lambda}(y,r|H_1)$ and $f_{Y_1,\lambda}(y,r|H_0)$ are the joint PDFs of Y_1 and λ corresponding to the EGC or SC diversity combining scheme, respectively.

If we assume that the symbol errors and symbol erasures within a codeword are independent due to using sufficient channel interleaving, then the codeword decoding error probability after EAE RS decoding can be expressed in the form of [2]:

$$\begin{aligned} P_W &= \sum_{i=0}^{\mathcal{N}}\sum_{j=j_0(i)}^{\mathcal{N}-i}\binom{\mathcal{N}}{i}\binom{\mathcal{N}-i}{j} \\ &\cdot P_t^i P_e^j (1-P_t-P_e)^{\mathcal{N}-i-j}, \quad (6) \end{aligned}$$

where $j_0(i) = \max\{0, \mathcal{N}-\mathcal{K}+1-2i\}$, while P_e and P_t represent the symbol erasure probability and random symbol error probability before decoding, respectively, which are given by Eq.(4) and Eq.(5).

V. Examples of Decoding Performance

In this section the performance of RS coded M-ary CDMA using EAE decoding associated with the proposed MOR-TT erasure insertion scheme is estimated with the aid of the equations from [6] and compared to that of using 'error-correction-only' decoding, and that of using 'erasures-and-erasures' decoding associated with either the RTT or OTT erasure insertion schemes [1], [5], [4].

As an example, Fig.3 and 4 show the codeword decoding error probability of (6) over dispersive Rayleigh fading channels in the context of EGC (Fig.3) and SC (Fig.4) using the proposed joint MOR-TT. The RS(32,20) code defined over the Galois Field GF(32)=GF(2^5) corresponding to 5-bit symbols was used and EAE decoding based on the proposed MOR-TT insertion scheme was employed. In addition to the parameters we defined previously, in Fig.3 and 4 the variable N represents the number of chips of the spreading sequences within a bit interval, and N is related to the spreading factor, N_s, by $N_s = N \times \log_2 M$. From

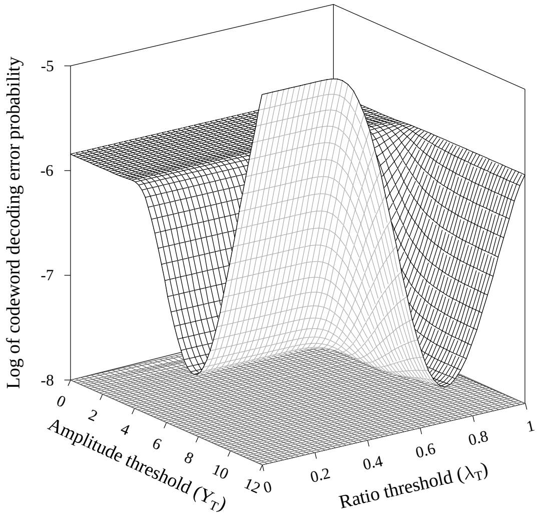

Fig. 4. **SC**: Codeword decoding error probability versus the amplitude threshold, Y_T and the ratio threshold, λ_T for the RS(32,20) FEC code using EAE decoding based on the MOR-TT erasure insertion scheme over dispersive Rayleigh fading channels.

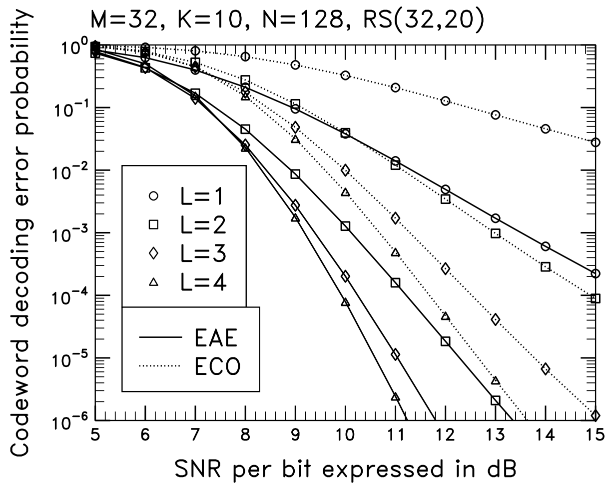

Fig. 5. **EGC**: Codeword decoding error probability versus the average SNR per bit, $\overline{\gamma}_b$ for the RS(32,20) FEC code using 'error-correction only (ECO)' decoding and EAE decoding with parameters of $M = \mathcal{N} = 32$, $\mathcal{K} = 20$, $K = 10$, $N = 128$, and a diversity order of $L = 1, 2, 3, 4$ over dispersive Rayleigh fading channels.

the results of Fig.3 and 4 we observe that there exists an optimum threshold value of λ_T or Y_T, for which the EAE decoding achieves the minimum codeword decoding error probability. This observation in turn implies that for given values of M, for a diversity order of L, for K number of users, for a SNR per bit of $\overline{\gamma}_b$ as well as for a given RS code, there exist optimum thresholds Y_T and λ_T, for which the EAE decoding using the joint MOR-TT erasure insertion scheme achieves the minimum codeword decoding error probability. This minimum codeword decoding error probability is lower, than that associated with using the RTT alone or OTT alone. Note that for both the EGC scheme of Fig.3 and for the SC arrangement of Fig.4, the point corresponding to $Y_T = 0$ and $\lambda_T = 1$ represents the codeword decoding error probability using ECO decoding. Therefore, we can observe that for both EGC and SC, the EAE decoding outperforms the ECO decoding, if the appropriate thresholds are invoked. However, if the threshold Y_T is too high and simultaneously the threshold λ_T is too low, too many erasures will be activated, potentially erasing correct demodulated symbols. Consequently, the codeword decoding error probability using EAE decoding might be significantly higher, than that using ECO decoding.

In Fig.5 and 6 we evaluated the codeword decoding error probability of a RS coded M-ary DS-CDMA system employing either ECO or EAE decoding for the EGC (Fig.5) and SC (Fig.6) schemes using diversity orders of $L = 1, 2, 3$ and 4. In the computations we assumed that the optimal thresholds, Y_T and λ_T, were employed for any given value of the average received SNR per bit, $\overline{\gamma}_b$. The remaining parameters were also specified in the figures. The results show that under dispersive Rayleigh fading conditions, while assuming a constant average SNR per bit and a constant number of multipath diversity components combined - i.e., a constant complexity - the decoding algorithms using EAE decoding based on the joint MOR-TT erasure insertion scheme outperform ECO decoding. As shown in Fig.5 for EGC using diversity orders of $L = 3$ and 4 and at the codeword decoding error probability of 10^{-6}, the EAE decoding scheme can achieve a gain of about 3.3dB and 2.3dB, respectively, over the ECO decoding scheme. Similarly, in the context of the SC scheme of Fig.6 using a given diversity order, the EAE decoding scheme can also achieve a significant gain over the ECO decoding scheme. Furthermore, the results show that for a given number of combined paths, L and for a given SNR per bit, $\overline{\gamma}_b$ the EGC scheme has a lower codeword decoding error probability, than the SC scheme. This is because EGC is the optimal diversity combining scheme for a noncoherent demodulation technique.

¿From the previous results we conclude that both EGC and SC exhibit similar characteristics both in terms of their PDFs and in terms of the associated codeword decoding error probabilities, although the EGC outperforms the SC. Therefore, in Fig.7 we used EGC as an example and compared the EAE RS decoding performance using the MOR-TT, RTT and OTT. The results for the proposed MOR-TT were evaluated from Equations in [6], while for the RTT and OTT from the equations derived in [4] and [5]. From the results we observe that the proposed MOR-TT erasure scheme outperforms both the RTT and OTT erasure insertion schemes.

VI. Conclusions

In this contribution the performance of RS coded DS-CDMA systems using the proposed MOR-TT based EAE

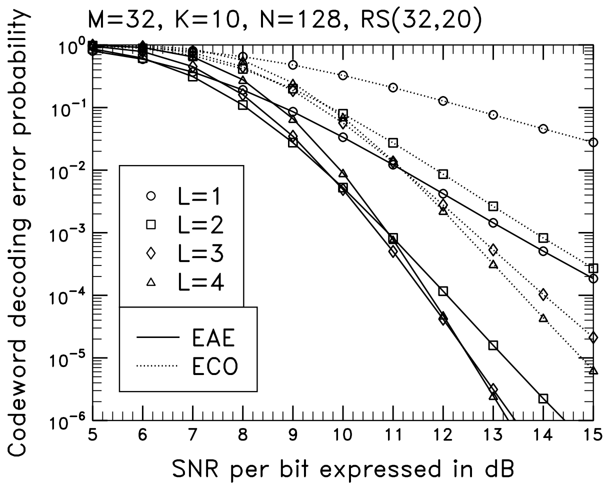

Fig. 6. **SC**: Codeword decoding error probability versus the average SNR per bit, $\overline{\gamma}_b$ for the RS(32,20) FEC code using ECO decoding and EAE decoding with parameters of $M = \mathcal{N} = 32$, $\mathcal{K} = 20$, $K = 10$, $N = 128$, and a diversity order of $L = 1, 2, 3, 4$ over dispersive Rayleigh fading channels.

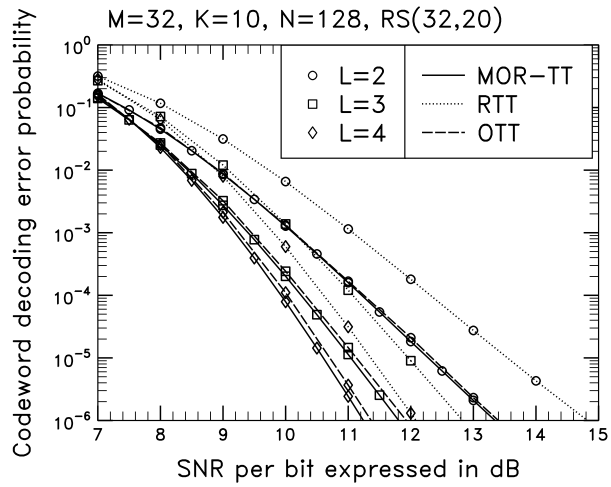

Fig. 7. **EGC**: Comparison of codeword decoding error probability for RS(32,20) EAE decoding using RTT, OTT and joint MOR-TT erasure insertion schemes.

RS decoding has been investigated and compared to that using ECO decoding as well as to that using EAE RS decoding based on the low-complexity, low-delay RTT and OTT erasure insertion schemes, when noncoherent M-ary orthogonal modulation and diversity reception using the EGC or SC schemes were considered. We assumed that the transmitted signals were subjected to dispersive Rayleigh fading. The distributions of the joint PDFs associated with the MOR-TT have been studied and their properties have been analyzed. The proposed MOR-TT technique was compared to the RTT and OTT erasure insertion techniques. Specifically, the performance of EAE RS decoding employing the MOR-TT was compared to that of ECO RS decoding without using side-information. The numerical results suggested that when using EAE decoding, RS codes of a given code rate can achieve a significantly higher coding gain than without erasure information, provided that appropriate thresholds were invoked. Therefore, DS-CDMA systems using the proposed EAE decoding could support more simultaneous users at a given error performance, than without erasure information. Furthermore, the numerical results demonstrated that the MOR-TT technique outperforms both the RTT and the OTT techniques.

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