Reduced Complexity Turbo Equalization for Space-time Trellis Coded Systems

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Abstract— **In this paper, a reduced complexity turbo equalizer referred to as TEQ-IQ, is proposed for space-time trellis coded (STTC) systems using the in-phase/quadrature-phase cancellation technique developed for single-transmitter and single-receiver systems. The TEQ-IQ scheme is capable of approximating the performance of the conventional turbo equalizer (TEQ-CT), while achieving a complexity reduction factor of 1.4 and 468.2 for a 4-PSK 32-state STTC system communicating over two-path and five-path symbol-spaced and equal tap-weight Rayleigh fading channels, respectively.**

I. Introduction

The 3rd Generation (3G) mobile radio standards [1] aim for offering a variety of mobile multimedia services ranging from low to high data rate services, supporting bit rates of at least 144 kb/s in vehicular scenarios, 384 kb/s in scenarios providing in-door coverage from out-door and 2 Mb/s in indoor picocellular environments. With the availability of high speed transmission capabilities demanding Internet applications and user expectations have emerged. In order to cope with increasingly more demanding high data rate applications, such as high-quality Internet video-streaming, research has been focused on developing spectrum efficient techniques.

A transmit-diversity technique, known as space-time trellis coding (STTC) [2, 3] has been developed for overcoming the limited capacity offered by the hostile fading wireless channels. Transmit diversity techniques can also be utilized for mitigating the effects of ISI and have been advocated for providing additional diversity gains for the Mobile Stations (MSs) by upgrading the Base Stations (BSs). In space-time trellis coding symbols are encoded across the antennas and are simultaneously transmitted. This is an effective scheme, since it combines the benefits of forward error correction (FEC) coding and transmit diversity, in order to obtain performance gains. The trellis structure employed in STTC decoders allows Soft-in/Soft-out (SISO) algorithms, such as the Log-Maximum *A Posteriori* (MAP) [4] and the Max-Log-MAP [5,6] algorithms, to be readily implemented, hence paving the way towards the implementation of turbo equalization schemes, such as the scheme proposed by Bauch *et al.* [7]. However, this is achieved naturally at the cost of higher computational complexity.

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Motivated by these trends, we propose a reduced complexity STTC turbo equalizer referred to here as TEQ-IQ, which adopts the philosophy of separate In-phase/Quadrature-phase (I/Q) operations developed for single-transmitter and single-receiver systems in [8]. The performance of the TEQ-IQ scheme is compared to that of the conventional turbo equalizer (TEQ-CT). We commence our discourse on the TEQ-IQ scheme by presenting an overview of the investigated system in Section II. This is followed by a discussion of the TEQ-IQ principles in Section III. Sections IV and V analyses the complexity incurred and present our simulation results. Finally, we conclude in Section VI.

II. System Overview

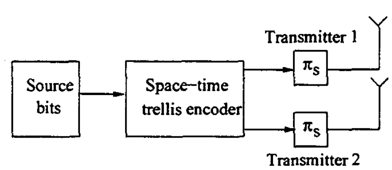

Fig. 1. Transmitter of the space-time trellis coded system, where π_s represents the space-time trellis code's interleaver.

Figure 1 shows the transmitter of the STTC system, consisting of a STTC encoder using $T_x = 2$ transmit antennas. The source bits are passed to the STTC encoder, represented using the notation STTC(M,n), where M represents the modulation mode used and n the number of states of the STTC encoder. These STTC codes can be characterized using trellis diagrams. For example, the STTC(4,32) code associated with 4-PSK modulation can be characterized by the trellis diagram illustrated in Figure 2. Upon receiving an input symbol, the STTC produces an output symbol in each transmitter arm of Figure 1. These output symbols are displayed at the right-

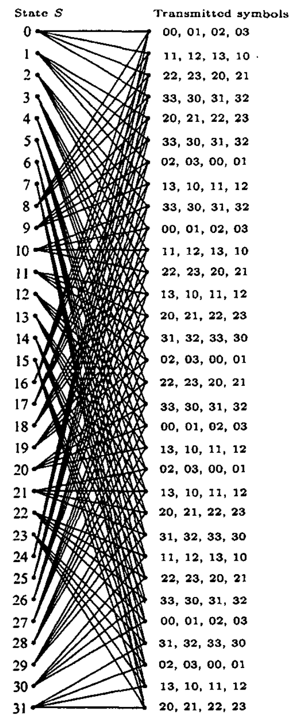

Fig. 2. Trellis diagram of the systematic 32-state, 4-PSK STTC [2].

hand side of each trellis state in Figure 2, which can be treated as the STTC codewords. At the output of the STTC encoder, the encoded symbols are interleaved by a random STTC interleaver represented as π_s in Figure 1. Note that the same interleaving rule is used for all transmit antennas.

The transmission burst employed in our investigations consists of 144 data symbols. Additionally, two-path and five-path Rayleigh fading channel impulse responses (CIR)s having equal symbol-spaced tap weights were used. The Rayleigh fading statistics obeyed a normalised Doppler frequency of 3.3615×10^{-5}, where the fading magnitude and phase was kept constant for the duration of a transmission burst. Furthermore, in order to characterize the best possible performance of these systems, we have assumed that the CIR was perfectly estimated at the receiver. At the receiver, $R_x = 2$ receive antennas were employed. The received signal at the ith receiver can be written as:

$$\begin{aligned} r^i(t) &= \sum_{k=1}^{T_x=2} s^k(t) * h^{ki}(t) + n^i(t) \qquad \text{i=1} \ldots R_x = 2, \\ &= r_I^i(t) + j r_Q^i(t), \end{aligned} \tag{1}$$

where

$$\begin{aligned} r_I^i(t) &= \sum_{k=1}^{T_x=2} \left(s_I^k(t) * h_I^{ki}(t) - s_Q^k(t) * h_Q^{ki}(t) \right) + n_I^i(t) \\ j r_Q^i(t) &= \sum_{k=1}^{T_x=2} j \left(s_Q^k(t) * h_I^{ki}(t) + s_I^k(t) * h_Q^{ki}(t) \right) + j n_I^i(t), \end{aligned} \tag{2}$$

and $s^k(t)$, $r^i(t)$ and $n^i(t)$ denote the symbol transmitted from the kth transmitter, the received signal on the ith receive antenna and the AWGN imposed on the signal received by the ith receive antenna, respectively. The notation $h^{ki}(t)$ refers to the CIR corresponding to the kth transmit and ith receive link, whereas $*$ represents convolution.

III. Principles of Reduced Complexity Turbo Equalization

As shown in Equation 2, the received I/Q signals, namely $r_I^i(t)$ and $r_Q^i(t)$, become dependent on $s_I^k(t)$ and $s_Q^k(t)$ after transmission over a complex channel. We refer to the inter-dependency between $s_I^k(t)$ and $s_Q^k(t)$ in the received quadrature signals $r_I^i(t)$ and $r_Q^i(t)$ as **cross-coupling**. This cross-coupling of the transmitted signal's quadrature components requires the receiver to consider an increased number of signal combinations, hence necessitating a high number of equaliser trellis states. However, we can reduce the number of trellis states to be considered significantly, when the cross-coupling is removed such that the quadrature components of the decoupled channel output $r^{i'}(t)$ are only dependent on $s_I^k(t)$ or $s_Q^k(t)$. Before commencing the decoupling operation, the quadrature components of the signal estimates, namely $s_I^k(t)$ and $s_Q^k(t)$ of all transmitters, must be generated. These quadrature signal estimates are obtained by converting the LLR values generated by the STTC decoder using:

$$\begin{aligned} E\{s_I^k(t)\} = \hat{s}_I^k(t) &= \sum_{u=0}^{\sqrt{\mathrm{M}}-1} s_{I;u}^k \cdot P[s_{I;u}^k | \hat{s}^k(t)] \\ E\{s_Q^k(t)\} = \hat{s}_Q^k(t) &= \sum_{u=0}^{\sqrt{\mathrm{M}}-1} s_{Q;u}^k \cdot P[s_{Q;u}^k | \hat{s}^k(t)], \end{aligned} \tag{3}$$

where $E\{\cdot\}$ denotes the expectation or averaging operation, $\sqrt{\mathrm{M}}$ is the number of constellation points in each quadrature arm of a particular modulation mode and $\hat{s}^k(t)$ is the Multi-variable Decision Feedback Equalizer (MV-DFE) [9] symbol estimate of transmitter k.

Subsequently, these estimated symbols are convolved with the in-phase and quadrature-phase CIR estimates of the kth transmit and ith receive link, in order to generate $\hat{s}_I^k(t) * \hat{h}_I^{ki}(t)$, $j\hat{s}_I^k(t) * \hat{h}_Q^{ki}(t)$, $\hat{s}_Q^k(t) * \hat{h}_Q^{ki}(t)$ and $j\hat{s}_Q^k(t) * \hat{h}_I^{ki}(t)$, where $k = 1 \ldots T_x$ and $i = 1 \ldots R_x$ as seen in Figure 3. Let us now show, how the received signals can be decoupled with the aid of linear processing, such that we obtain the received signals $r_I^{i'}(t)$ and $r_Q^{i'}(t)$, which are only dependent on a particular quadrature component, namely on $s_I^k(t)$ or $s_Q^k(t)$, rather than on both. Commencing with $r_I^{i'}(t)$, we can generate this signal by removing $\hat{s}_Q^k(t) * \hat{h}_Q^{ki}(t)$ and $j\hat{s}_Q^k(t) * \hat{h}_I^{ki}(t)$ from the in-phase component of the received signal $r^i(t)$, yielding:

$$\begin{aligned} r_I^{i'}(t) &= r^i(t) + \sum_{k=1}^{T_x=2} \left(\hat{s}_Q^k(t) * \hat{h}_Q^{ki}(t) - j\hat{s}_Q^k(t) * \hat{h}_I^{ki}(t)\right) \\ &= \sum_{k=1}^{T_x=2} (s_I^k(t) * h_I^{ki}(t) + js_I^k(t) * h_Q^{ki}(t) + \\ &\quad e(\hat{s}_Q^k(t), \hat{h}_Q^{ki}(t)) + je(\hat{s}_Q^k(t), \hat{h}_I^{ki}(t))) + n^i(t), \end{aligned} \tag{4}$$

where $e(\hat{s}_Q(t), \hat{h}_Q(t))$ and $e(\hat{s}_Q(t), \hat{h}_I(t))$ are the error terms expressed as:

$$\begin{aligned} e(\hat{s}_Q^k(t), \hat{h}_Q^{ki}(t)) &= r_I^i(t) + \hat{s}_Q^k(t) * \hat{h}_Q^{ki}(t) \\ je(\hat{s}_Q^k(t), \hat{h}_I^{ki}(t)) &= jr_Q^i(t) - j\hat{s}_Q^k(t) * \hat{h}_I^{ki}(t), \end{aligned} \tag{5}$$

which arise, when inaccurate CIR estimates and low-confidence M-QAM symbol estimates are generated. Similarly, $r_Q^{i'}(t)$ is obtained by subtracting $\hat{s}_I^k(t) * \hat{h}_I^{ki}(t)$ and $j\hat{s}_I^k(t) * \hat{h}_Q^{ki}(t)$, where $k = 1 \ldots T_x$ and $i = 1 \ldots R_x$ from $r^i(t)$. Although errors are introduced in the decoupling operation, it is seen in the simulation results of Section V, that the imperfect decoupling effects are compensated through successive turbo equalization iterations and the performance approaches that of the turbo equalizer utilising the conventional full-complexity trellis-based equalizer.

The structure of the turbo equalizer, which incorporates the I/Q decoupling principle is illustrated in Figure 3.

After the decoupling operation has been completed, the resultant signals corresponding to the in-phase signal components of all transmitters are directed to a SISO I/Q equalizer. We have utilized the Max-Log-MAP algorithm [5, 6] in our equalizer, since it constitutes a good compromise in terms of the achievable performance and the computational complexity imposed. Upon receiving signals corresponding to the in-phase signal components, the SISO equalizer determines the transition metric associated with the transition from state S to state $\grave{S}$, which can be expressed as:

$$\Gamma(S, \grave{S}) = \sum_{i=1}^{R_x=2} \left(r_I^{i'}(t) - \hat{r}_I^{i'}\right)^2 / 2\sigma^2, \tag{6}$$

where $\hat{r}_I^{i'}$ is the estimate of the in-phase component of the decoupled received signal $r_I^{i'}$ in Equation 4. Subsequently, the forward and backward recursion values are computed, in order to generate the LLR values of the transmitted symbols. Similarly, the quadrature-phase signal component is processed by the second I/Q equalizer. These LLR values are deinterleaved by the STTC deinterleaver of Figure 3 and passed to the STTC decoder. The STTC decoder computes the *a posteriori* LLRs and subsequently the extrinsic LLRs, which are directed back to the I/Q equalizer of Figure 3 for processing in the next iteration. This iterative process is repeated, until the termination criterion stipulated is satisfied. In our investigations the iterations were curtailed, when no significant further performance gains could be obtained through additional iterations.

Having described the I/Q turbo equalization operation, we now proceed further to analyse the complexity incurred by the proposed scheme.

IV. Complexity Analysis

For the sake of simplicity, the complexity of the STTC turbo equalizers can be quantified in terms of the number of associated trellis transitions per information bit. Therefore, the complexity of the equalizer, which is dependent on the number of trellis transitions per coded bit, must be normalized by the overall throughput T_r, which is the number of bits per symbol, yielding the number of transitions per information bit.

For the conventional full-complexity trellis-based equalizer CT-EQ, it can be shown that the complexity denoted by Λ[CT-EQ] and associated with equalizing M-QAM signals transmitted over a channel having complex weights and a delay spread of τ_d symbols can be expressed as:

$$\begin{aligned} \Lambda[\text{CT-EQ}] &= (\text{Num. of states} \cdot \text{Num. of transitions})\,/T_r \\ &= \left(\mathrm{M}^{\mathrm{T_x}\tau_d} \cdot \mathrm{M}^{\mathrm{T_x}}\right)/T_r = \left(\mathrm{M}^{\mathrm{T_x}(\tau_d+1)}\right)/T_r, \end{aligned} \tag{7}$$

where again M, T_x and T_r denote the number of constellation points, the number of transmitters and the overall throughput, respectively. By contrast, the complexity of

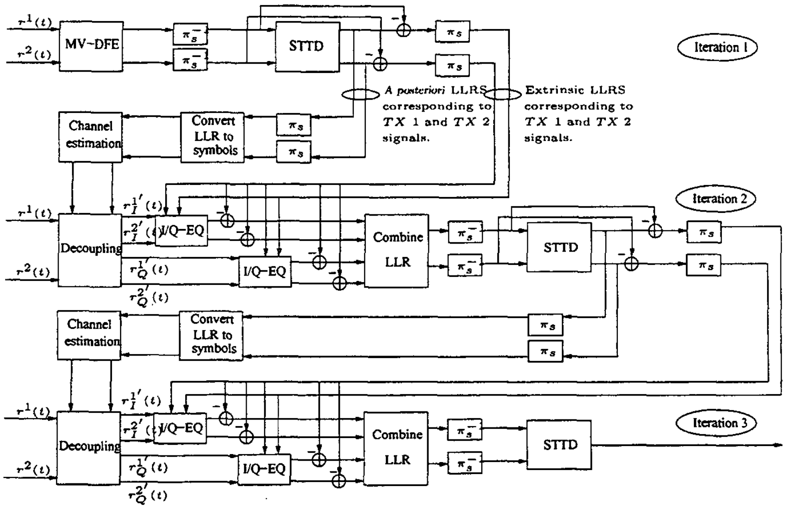

Fig. 3. Schematic of the reduced complexity turbo equalizer in conjunction with a two-transmitter, two-receiver based STTC scheme performing three turbo equalization iterations.

a single I/Q equalizer (I/Q-EQ) trellis stage is given by:

$$\Lambda[\text{I/Q-EQ}] = (\text{Num. of states} \cdot \text{Num. of transitions})/\mathrm{T_r} = \left(\sqrt{\mathrm{M}}^{\,T_x(\tau_d+1)}\right)/T_r. \tag{8}$$

For the STTC(M,n) decoder, where M and n denote the number of constellation points and the number of states in the encoder, the complexity $\Lambda[\mathrm{STTC(M,n)}]$ incurred is:

$$\Lambda[\mathrm{STTC(M,n)}] = (\mathrm{n} \cdot \mathrm{M})/\mathrm{T_r}. \tag{9}$$

Having determined the complexity of the equalizer and that of the STTC decoder as a function of the number of trellis transitions per information bit, the complexity of the TEQ-CT can be estimated as:

$$\Lambda[\text{TEQ-CT}] = \frac{\left(\left(\mathrm{M}^{\mathrm{T_x}(\tau_d+1)}\right) + n \cdot \mathrm{M}\right)}{T_r} \cdot \text{Itr[TEQ-CT]}, \tag{10}$$

while the complexity of TEQ-IQ as:

$$\Lambda[\text{TEQ-IQ}] = \frac{n \cdot \mathrm{M}}{T_r} + \frac{\left(2 \cdot \left(\sqrt{\mathrm{M}}^{\,T_x(\tau_d+1)}\right) + n \cdot \mathrm{M}\right)}{T_r} \times (\text{Itr[TEQ-IQ]} - 1), \tag{11}$$

where Itr[ ] denotes the number of turbo equalization iterations. A factor of two was introduced in Equation 11, since two I/Q-EQs are required for performing the equalization. In Equation 11, the first term on the right hand side represents the complexity incurred in the first TEQ-IQ iteration, where a MV-DFE and a STTC decoder was employed. The remaining terms correspond to the complexity of the subsequent TEQ-IQ iterations. For the sake of simplicity we have assumed that the complexity of the MV-DFE is negligible, when compared to the complexity of the I/Q-EQ and CT-EQ. Therefore, the complexity of the TEQ-IQ in the first iteration is only dependent on the complexity of the STTC decoder.

V. Results and Discussion

Figure 4 shows the performance comparison of the TEQ-CT and TEQ-IQ schemes in conjunction with the STTC(4,32) system using a 2304-symbol STTC interleaver. It was observed for the TEQ-CT scheme that no significant performance improvements were attained after two turbo equalization iterations. By comparison, the TEQ-IQ required three critical turbo equalization iterations. Note that the term *critical number of iteration* refers to the number of iterations that have to be performed, such that further iterations beyond this value do not yield significant gains in BER performance terms. After performing their respective number of critical turbo equalization iterations, it was demonstrated in Figure 4 that the TEQ-IQ scheme was capable of approximating the performance of the TEQ-CT arrangement. By using Equations 10 and 11, it was noted that this was achieved at a complexity reduction of a factor 1.7. As we will show in our next scenario studied, the complexity reduction fac-

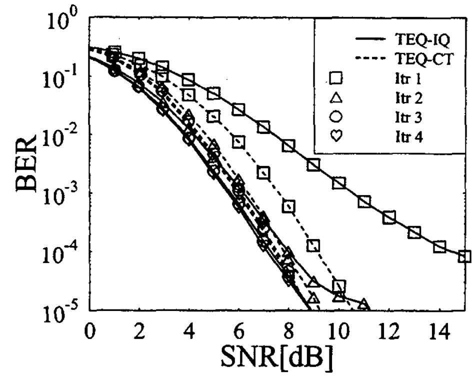

Fig. 4. Performance comparison of the TEQ-CT and TEQ-IQ in conjunction with the STTC(4,32) scheme using 2304-symbol STTC interleaver for transmission over a two-path, equal-weight, and symbol-spaced Rayleigh fading CIR having a normalised Doppler frequency of 3.3615×10^{-5}.

tor becomes more significant for channels having long delay spreads. Here, the performance of the STTC (4,32) system communicating over channels having a CIR associated with long delay spreads *i.e.* over symbol-spaced five-path, equal tap-weight Rayleigh fading channel, is investigated. For this CIR, the TEQ-CT scheme cannot be implemented, since the complexity of the trellis-based equalizer alone is already associated with 10^6 transitions per trellis interval.

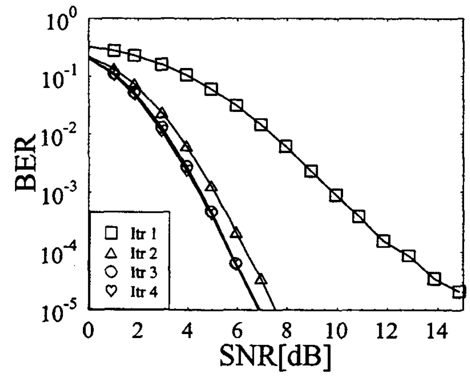

Fig. 5. Performance of the TEQ-IQ in conjunction with the STTC(4,32) scheme using 2304-symbol STTC interleaver for transmission over a five-path, equal-weight, and symbol-spaced Rayleigh fading CIR having a normalised Doppler frequency of 3.3615×10^{-5}.

Despite experiencing long delay spreads, it was demonstrated that the TEQ-IQ was capable of detecting signals transmitted over such channels, while attaining good BER performances. Assuming that the TEQ-CT could have been simulated and two critical turbo equalization iterations were required, the TEQ-IQ would have achieved a complexity reduction of a factor of 468.2.

VI. Conclusions

In conclusion, it was observed that the TEQ-IQ is capable of reducing the complexity of the STTC system, without significantly sacrificing the achievable performance. For channels having short delay spreads, such as the two-path, symbol-spaced and equal tap-weight CIR, the TEQ-IQ scheme approximated the performance of the TEQ-CT arrangement, while achieving a complexity reduction factor of 1.4. For the five-path, symbol-spaced and equal-weight Rayleigh fading channel, the TEQ-CT scheme could not be implemented, since the complexity of the trellis-based equalizer alone already corresponds to 10^6 transitions per trellis interval. However, it was observed that the TEQ-IQ was capable of detecting signals transmitted over such channels, while attaining good BER performances. Assuming that the TEQ-CT could have been simulated and two critical turbo equalization iterations were required, the TEQ-IQ would have achieved a complexity reduction by a factor of 468.2.

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