# UPPER-BOUND PERFORMANCE OF A WIDEBAND BURST-BY-BURST ADAPTIVE MODEM

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Abstract - In this contribution, adaptive modulation is applied in conjunction with a Decision Feedback Equalizer (DFE) in order to mitigate the effects of the slowly varying wideband multi-path Rayleigh fading channel. The upper-bound mean BER and Bits Per Symbol (BPS) performance of this scheme is determined by utilizing the pseudo-SNR at the output of the DFE, in order to switch the modulation schemes on a burst-by-burst basis. The performances of each individual modulation schemes and their amalgamated adaptive scheme were compared in terms of information throughput for a low- and a high- BER arrangement. The results indicate both SNR gains of approximately $1-3$dB and $7-9$dB, for the high- and low- BER arrangement, respectively using the proposed burst-by-burst adaptive wideband scheme in a noise limited environment.

## 1. INTRODUCTION

Adaptive Quadrature Amplitude Modulation (AQAM) employs a higher-order modulation scheme when the channel quality is favourable, in order to increase the throughput and conversely, a more robust lower order modulation scheme when the channel exhibits a deep fade in order to improve the mean BER performance. Recent developments in AQAM over narrow-band channels have been pioneered by Webb and Steele [1]. The concept of variable rate AQAM was then further developed by Sampei *et al* [2], Goldsmith and Chua [3] as well as Torrance *et al.*[4]. In most of these contributions, the quality of the channel was determined on the basis of the short term Signal to Noise Ratio (SNR) of the received burst.

However, in a wideband environment, due to the presence of intersymbol interference (ISI), the short term SNR is inadequate for measuring the quality of the channel. In addition, the wideband channel-induced degradation is combated not only by the employment of AQAM but also by equalization. Hence we propose to utilize the signal to noise plus residual ISI ratio at the output of the equalizer, termed as the pseudo-SNR in order to switch the modulation modes. This criterion will be defined formally in Section 3. Previous contributions on invoking AQAM over wideband channels are due, for example, to Morinaga *et al* [5] and Lim *et al* [6].

## 2. SYSTEM OVERVIEW

The system schematic of the joint AQAM and equalization scheme is depicted in Figure 1. At the receiver, the Channel Impulse Response (CIR) is estimated, which is then used to calculate the DFE coefficients in the DFE Coefficient Computation block by solving Equations 2 and 3, which will be described at a later stage. Subsequently, the coefficients are used to equalize the ISI-corrupted received signal. In addition, both the channel estimate and the DFE coefficients are utilized to compute the pseudo-SNR at the output of the DFE. The calculated pseudo-SNR is then compared against a set of optimized switching threshold levels, $f_n$, stored in a look-up table, and subsequently the appropriate modulation scheme is selected for the next transmission burst. The switching methodology, using the pseudo-SNR can be summarized as follows:

$$\text{Modulation Mode} = \begin{cases} NOTX & \text{if } \gamma_{dfe} < f_1 \\ BPSK & \text{if } f_1 \leq \gamma_{dfe} < f_2 \\ 4QAM & \text{if } f_2 \leq \gamma_{dfe} < f_3 \\ 16QAM & \text{if } f_3 \leq \gamma_{dfe} < f_4 \\ 64QAM & \text{if } \gamma_{dfe} \geq f_4 \end{cases}, \quad (1)$$

where $f_n, n = 1...4$ are the pseudo-SNR thresholds levels, which are set according to Section 5.

The channel impulse response, $h_i$, used in our investigation is derived from the COST 207 [7] channel model for a typical urban area (TU), which is shown in Figure 2. Each path in this discretised channel impulse response is faded independently according to a Rayleigh distribution, where the multi-path components are non-uniformly spaced on a grid of symbol-spaced legitimate potential positions. The Rayleigh fading statistics assumed a normalised Doppler frequency of $3.3615 \times 10^{-5}$. Variations due to path loss and shadowing are assumed to be eliminated by power control. The minimum mean square error (MMSE) DFE is characterized by the forward coefficients, $C_m$ and the backward coefficients, $b_q$, which can be written as [8]:

$$\sum_{m=0}^{N_f-1} C_m \left[ \sum_{v=0}^{l} h_v^* h_{v+m-l} \sigma_S^2 + N_o \delta_{m-l} \right] = h_l^* \sigma_S^2 \qquad l = 0.....N_f - 1, \quad (2)$$

$$b_q = \sum_{m=0}^{N_f-1} C_m h_{m+q} \qquad q = 1.....N_b, \quad (3)$$

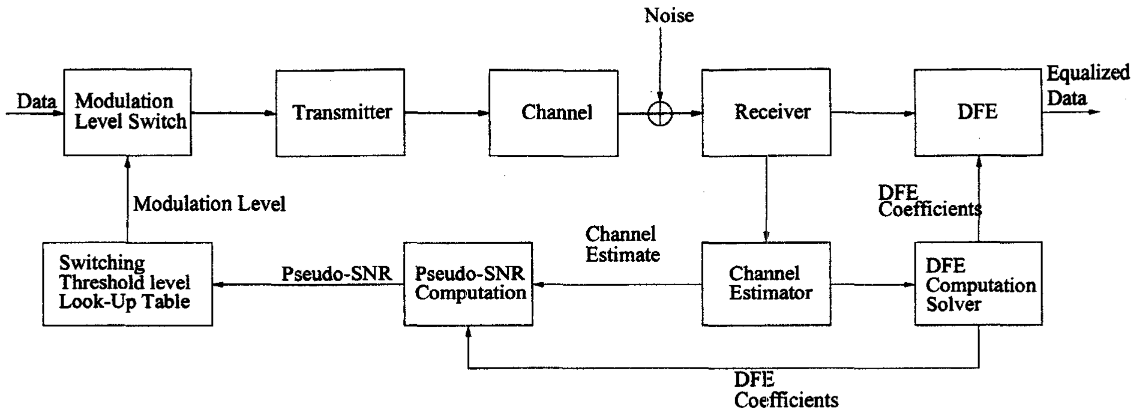


Figure 1: Schematic of the joint AQAM and equalization scheme.

where $N_f$ and $N_b$ represent the number of taps in the forward and backward filters, which were set to 35 and 7, respectively; $\sigma_S^2$ denotes the variance of the transmitted signal and the superscript $*$ represents complex conjugation. Furthermore, $N_o$ is the single sided power spectral density of white noise, while $h_i$ and $\delta$ denote the $i$th path of the CIR and the Dirac delta function, respectively.

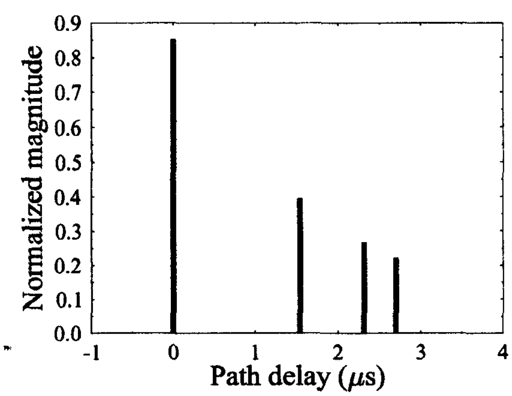


Figure 2: The impulse response of a COST 207 Typical Urban (TU) channel.

### 2.1. Assumptions

In deriving the upper bound performance of this joint AQAM and equalization scheme, the following assumptions are made:

1. The CIR is constant across the transmission burst, but varies from burst to burst by assuming that the channel is slowly varying.
2. The pseudo-SNR at the output of the equalizer in the receiver is estimated perfectly prior to transmission. This can only be assumed in a TDD scenario[8], where the channel is reciprocal in the uplink and downlink transmission or when there is a reliable, low-delay feedback path between the transmitter and the channel quality estimator at the receiver, which was assumed by Goldsmith and Chua, for example [3].
3. At the receiver, perfect channel compensation is applied in order to derive the upper bound performance. In reality, imperfect channel compensation will incur a performance degradation at the receiver which is difficult to quantify analytically or numerically. Consequently, the degradation due to imperfect channel compensation is neglected here in order to achieve a numerical solution for the upper-bound performance.
4. The receiver assumed perfect knowledge of the modulation scheme used in its received transmission burst. In reality, some form of signalling must be used to convey the modulation scheme used to the receiver.
5. The equalizer employed in this scheme is the DFE, where error propagation is neglected. This will simplify the calculation of the numerical upper bound performance. The residual ISI at the output of the DFE is assumed to be Gaussian distributed for the purpose of mean BER calculations. This assumption will be justified by an experiment at a later stage.

## 3. THE PSEUDO SIGNAL TO NOISE RATIO OF THE DECISION FEEDBACK EQUALIZER

In Section 2 we have introduced the concept of using the pseudo-SNR at the output of the DFE as a criterion to switch the modulation schemes and hence we have to investigate the validity of this criterion. Let us begin by providing the Equations used to calculate the pseudo-SNR at the output of the DFE following Cheung's approach [9]:

$$\gamma_{dfe} = \frac{E\left[|S_k \sum_{m=0}^{N_f-1} C_m h_m|^2\right]}{\sum_{q=-(N_f-1)}^{-1} E\left[|f_q S_{k-q}|^2\right] + N_o \sum_{m=0}^{N_f-1} |C_m|^2}, \quad (4)$$

where $f_q = \sum_{m=0}^{N_f-1} C_m h_{m+q}$ and $S_k$ is the transmitted signal at time $k$.

The pseudo-SNR is composed of the wanted signal, the effective noise at the output of the DFE and the residual Gaussian-like ISI, hence, the term pseudo-SNR. Hence, in obtaining the mean BER performance, we approximated the residual interference as additional Gaussian noise at the output of the DFE. However, to ensure that this assumption is valid when applied to the calculation of the mean BER, several experiments were conducted using fixed modulation schemes in a fading multi-path environment.

The mean BER performance for each of the fixed modulation schemes in the fading multi-path channels was numerically calculated as follows :

$$\begin{aligned}
P^b_{BPSK_{MR}}(\gamma) &= \int_0^\infty P^b_{BPSK}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\
P^b_{4QAM_{MR}}(\gamma) &= \int_0^\infty P^b_{4QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\
P^b_{16QAM_{MR}}(\gamma) &= \int_0^\infty P^b_{16QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\
P^b_{64QAM_{MR}}(\gamma) &= \int_0^\infty P^b_{64QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe},
\end{aligned} \quad (5)$$

where $\gamma$ is the average channel SNR and $p(\gamma_{dfe})$ is the PDF of the instantaneous pseudo-SNR, $\gamma_{dfe}$, at the output of the DFE for a given $\gamma$. In order to obtain $p(\gamma_{dfe})$, the instantaneous values of $\gamma_{dfe}$ were calculated using Equation 4 throughout our experiments. Subsequently, the discretised PDF was constructed and inserted into Equation 5 to calculate the numerical mean BER performance of each fixed modulation scheme. The integration was approximated by the trapezoidal rule [10] with the limits set to the lowest and highest values of the instantaneous pseudo-SNR. The notations $P^b_{BPSK}$, $P^b_{4QAM}$, $P^b_{16QAM}$ and $P^b_{64QAM}$ are the individual theoretical BER performances of each modulation scheme in a Gaussian channel, which were derived in Reference [8] and the subscript $MR$ in Equation 5 denotes their performances in a Rayleigh fading multipath channel.

The results shown in Figure 3 displayed close correspondence between the numerical and simulated performance for each individual modulation scheme. Consequently, we can justify the utilization of the output pseudo-SNR of the DFE, $\gamma_{dfe}$, as a criterion to switch the modulation modes and in calculating the mean BER upper-bound performance.

## 4. NUMERICAL AVERAGE UPPER BOUND PERFORMANCE

We can now use the psuedo-SNR as the criterion to switch the modulation modes and apply certain concepts and principles of narrow-band AQAM [1] - [4] in order to formulate the following numerical upper bound BER performance for this joint scheme:

$$P_a(\gamma) = B^{-1} \cdot \begin{bmatrix} 1 \cdot \int_{f_1}^{f_2} P^b_{BPSK}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\ + 2 \cdot \int_{f_2}^{f_3} P^b_{4QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\ + 4 \cdot \int_{f_3}^{f_4} P^b_{av16QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \\ + 6 \cdot \int_{f_4}^{\infty} P^b_{av64QAM}(\gamma_{dfe})\; p(\gamma_{dfe}) d\gamma_{dfe} \end{bmatrix}, \quad (6)$$

where $B$ is the mean number of Bits Per Symbol (BPS), which can be written as:

$$B = \begin{bmatrix} 1 \cdot \int_{f_1}^{f_2} p(\gamma_{dfe}) d\gamma_{dfe} \\ + 2 \cdot \int_{f_2}^{f_3} p(\gamma_{dfe}) d\gamma_{dfe} \\ + 4 \cdot \int_{f_3}^{f_4} p(\gamma_{dfe}) d\gamma_{dfe} \\ + 6 \cdot \int_{f_4}^{\infty} p(\gamma_{dfe}) d\gamma_{dfe} \end{bmatrix}. \quad (7)$$

The numerical performance was calculated for two different adaptive modulation schemes. In the first scheme, using the COST 207 Typical Urban(TU) Rayleigh fading channel of Figure 2, the pseudo-SNR switching thresholds were set for

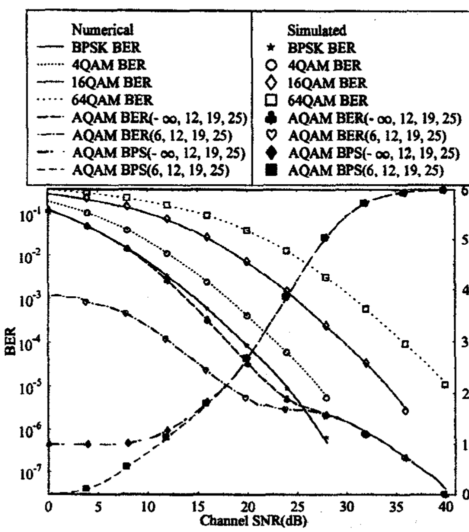


Figure 3: Numerical and simulated performance of the joint AQAM and DFE, employing BPSK, 4QAM, 16QAM and 64QAM over the COST 207 **TU Rayleigh fading channel** of Figure 2 using the assumptions in Section 2.1. The numerical solutions were calculated using Equation 5 for the fixed modulation schemes and Equations 6 and 7 were used to calculate the numerical solution for the joint AQAM and DFE.

each of the modulation modes, such that a BER of approximately 1% was maintained according to the fixed modulation performance of Figure 3. Consequently, the levels were set as follows : $f_1 = -\infty$dB, $f_2 = 12$dB, $f_3 = 19$dB and $f_4 = 25$dB, where the transmitter constantly transmitted data and in the second scheme, the thresholds were set at: $f_1 = 6$dB, $f_2 = 12$dB, $f_3 = 19$dB and $f_4 = 25$dB, resulting in no transmission, whenever the instantaneous received pseudo-SNR dipped below the threshold $f_1$.

The performance of the joint AQAM and equalization scheme was evaluated over the TU channel and the results are displayed in Figure 3 in terms of the mean BER and BPS. It was observed that a good correspondence was achieved between the simulated and numerical performance, further justifying our numerical model based on Equations 6 and 7. The simulated and numerical fixed modulation results are also shown in these Figures as a comparison to the AQAM schemes used.

Let us now analyse in detail the results shown in Figure 3 for the COST 207 TU channel of Figure 2. In the AQAM scheme, which did not incorporate transmission blocking (indicated by the $-\infty$ lowest threshold in the captions), the adaptive performance is better or equal to the performance using BPSK in terms of the mean BER and BPS for the channel SNR range between 0dB and 25dB. The adaptive mean BER performance is even better than the BPSK performance in the channel SNR range of 8dB to 24dB. This is because, due to the increased instantaneous pseudo-SNR at the output of the DFE, a better mean BPS performance is achieved than that of BPSK and hence a given number of bit errors is averaged over a higher number of transmitted bits. At higher SNRs, the mean BER and BPS performance begins to converge to the 64QAM performance, since the probability of encountering a low instantaneous pseudo-SNR is small due to the high average channel SNR. In the joint scheme utilizing transmission blocking in Figure 3 (indicated by the lowest threshold of 6 in the captions), at low channel SNRs the mean BER performance was below approximately 0.1%. This low mean BER was a result of the employment of "transmission blocking", when the pseudo-SNR was below 6dB. Conversely, the mean BPS performance degraded, when compared to the "non-blocking" adaptive scheme. However, at higher SNRs the performance of the two different schemes converged, since the probability of "transmission blocking" was reduced.

## 5. SWITCHING LEVEL OPTIMIZATION

In Section 4 we observed that transmission blocking can be used in the joint AQAM and DFE scheme in order to target a certain required performance. Consequently, we can set the switching levels appropriately in order to support a transmission quality of 1% and 4.5 in terms of mean BER and BPS, respectively, which will be termed as the **High-BER** system. The other targeted performance, where a mean BER of 0.01% and a mean BPS value of 3 is expected will be termed as the **Low-BER** system.

The threshold optimization was conducted in the context of the COST 207 TU Rayleigh fading channel of Figure 2 for both transmission systems following Torrance's approach [11]. The result of the optimization process is listed in Table 1. The results are shown in Figure 4 for the COST 207 TU

| | $f_1$(dB) | $f_2$(dB) | $f_3$(dB) | $f_4$(dB) |
|---|---|---|---|---|
| **High-BER** | 3.6363 | 6.226 | 11.645 | 17.685 |
| **Low-BER** | 8.246 | 10.468 | 16.798 | 23.759 |

Table 1: The optimized switching levels $f_n$ of the joint AQAM and DFE for the **High-BER** and **Low-BER** systems over the TU Rayleigh fading channel.

Rayleigh fading channel of Figure 2. The targeted mean BERs of 1% and 0.01% for both systems were achieved for all channel SNRs. Furthermore, the targeted mean BPS values of 4.5 and 3 were achieved at approximately 18dB and 19dB channel SNRs, respectively.

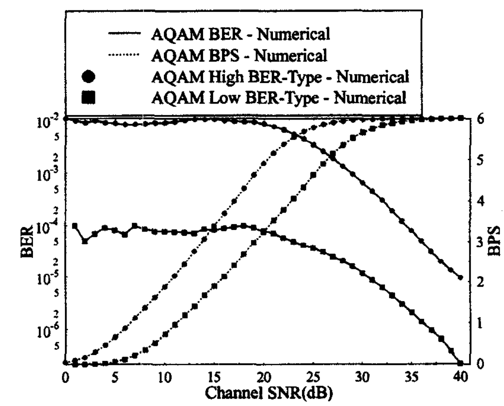


Figure 4: Numerical mean BER and BPS performance of the wideband equalized adaptive modem for the **High-BER** and **Low-BER** transmission systems using the assumptions in Section 2.1 and Equations 6 and 7. The switching levels used in these experiments are listed in Table 1.

## 6. THE BPS THROUGHPUT PERFORMANCE

In this Section, the system throughput in terms of BPS will be used to compare the performances of conventional fixed modulation schemes and the joint adaptive scheme. The BPS throughput was evaluated for the individual fixed modulation schemes at the targeted mean BER of 1% and 0.01 by utilizing the results shown in Figure 3. Similarly, the BPS throughput of the joint scheme was evaluated by using the results displayed in Figure 4. The BPS throughput achieved for the **High-BER** and **Low-BER** scheme over the COST 207 TU channels is shown in Figure 5. The system throughput of the **High-BER** scheme was higher than that of the **Low-BER** scheme for the same energy due to the more relaxed BER requirement of **High-BER** scheme.

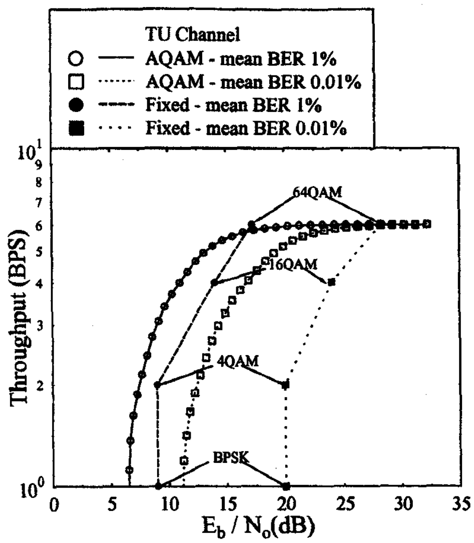


Figure 5: System throughput of the joint AQAM scheme and fixed modulation schemes over the COST 207 **TU Rayleigh Fading channel** of Figure 2 for the **High-BER** and **Low-BER** transmission systems using the assumptions in Section 2.1. The switching levels used in these experiments are listed in Table 1.

The achieved BPS throughput of the joint scheme is higher than that of the BPSK, 4QAM and 16QAM schemes for the same energy. However, at higher $\frac{E_b}{N_o}$, the channel capacity performance of both schemes converges, since 64QAM becomes the dominant modulation mode for the joint scheme. Improvements of approximately 1 – 3dB and 7 – 9dB in terms of $\frac{E_b}{N_o}$ were recorded for **High-BER** and **Low-BER** transmissions, respectively, when compared to the fixed modulation schemes.

## 7. CONCLUSIONS

In this paper we have extended the application of AQAM to a wideband channel environment by jointly exploiting the benefits of AQAM and equalization. A new criterion, the pseudo-SNR at the output of the DFE was introduced to switch the modulation modes and the numerical upper-bound performance characterized by Equations 6 and 7 was presented. The performance was then optimized for a **High-BER** and a **Low-BER** transmission arrangement in Section 5. Lastly, the performance of each individual fixed modulation scheme was compared to the joint scheme in terms of their system throughput where gains of approximately 1 – 3dB and 7 – 9dB were recorded for the **High-BER** and a **Low-BER** transmission arrangement, respectively.

This upper-bound performance will be degraded by imperfect channel estimation and compensation as well as by the latency incurred in estimating the channel prior to transmission. However, this numerical upper-bound performance highlights the flexibility and performance advantages of the joint AQAM and DFE scheme, when compared to the individual fixed modulation schemes.

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