CREST-FACTOR STUDY OF MC-CDMA AND OFDM

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ABSTRACT

The crest factor (CF) of M-ary PSK MC-CDMA systems is shown to be dependent upon the aperiodic cross-correlations as well as on the aperiodic autocorrelations of the spreading codes used. Based on this observation, the crest factor distributions of BPSK MC-CDMA using two different binary orthogonal spreading sequences - namely Walsh codes and orthogonal Gold codes - as well as two polyphase orthogonal sequences - Frank codes and Zadoff-Chu codes - are compared against that of BPSK OFDM. In a 'lightly loaded' system the polyphase sequences and the orthogonal Gold codes are shown to have better CF distributions. However, in a 'fully loaded' system, Walsh codes exhibit the best crest factor distribution amongst the investigated codes. MC-CDMA using Walsh codes was found to exhibit a peak factor reduction of 1.5 over OFDM, when the number of subcarriers is 16 and a rate 15/16 CF reduction block code is used, while maintaining the same spectral efficiency. We found that Zadoff-Chu spread MC-CDMA showed the best BER performance amongst the considered systems in Rayleigh channels, when Joint-Detection was employed, despite its heavier clipping than that of the Walsh spread scheme. This is due to its low multiple user interference.

1. INTRODUCTION

In order to capitalise on the joint benefits of direct-sequence code division multiple access (DS-CDMA) and orthogonal frequency division multiplexing (OFDM), multi-carrier CDMA (MC-CDMA) was proposed [1, 2] and has drawn significant research interests in recent years. DS-CDMA exploits frequency diversity using Rake receivers, where the number of fingers in the Rake receiver determines its maximum diversity gain. OFDM, on the other hand, is resilient to frequency selective fading and asymptotically approaches the theoretically highest 2Bd/Hz Shannonian bandwidth efficiency. One of the main drawbacks of OFDM is that the envelope power of the transmitted signal fluctuates widely, requiring a highly linear RF power amplifier. Since our advocated MC-CDMA system spreads a message symbol across the frequency domain and uses an OFDM transmitter for conveying each spread bit, its transmitted signal also exhibits a high crest factor (CF), which is defined as the ratio of peak amplitude to the root mean square (rms) value.

The fundamental problem of reducing the dynamic range of the sum of trigonometric series has been studied in many fields, such as in mathematics [3], radar engineering, measurement and in OFDM [4]. As a result, various binary and polyphase codes have been developed [5, 6, 7]. *Jones, Wilkinson* and *Barton* [4] observed that not many message codes yield high CFs and proposed a block-coding scheme, in order to eliminate those messages. Since then, various methods have been proposed for reducing the CF in OFDM.

The aim of this contribution is to analyse the envelope power behaviour of MC-CDMA and highlight that the CF problem of OFDM is mitigated by MC-CDMA, while providing an improved BER performance due to its inherent frequency diversity. In the next section, the system model is described and the envelope power of M-ary phase shift keying (MPSK) MC-CDMA signals is analysed. The relationship between the envelope power and the aperiodic autocorrelation as well as the crosscorrelation of the spreading sequences is also derived. In Section 3, various spreading codes are introduced and their CF distributions as well as signal amplitude distributions are characterised. The associated uncoded BER performance is portrayed over additive white Gaussian noise (AWGN) and Rayleigh channels for OFDM and Joint-Detection MC-CDMA in Section 4, when a clipping amplifier is used along with a block encoder, in order to reduce the CF. Several observations and conclusions are presented in the last section.

2. SYSTEM MODEL AND ENVELOPE POWER

The simplified transmitter structure of MC-CDMA is portrayed in Figure 1, where L bits of a single user are spread and transmitted in parallel using the N-chip spreading codes, C_l, $l = 0, \cdots, L-1$, mapping each chip to a different subcarrier. Alternatively, the same MC-CDMA model is applicable to an L-user scenario, where the base-station (BS) transmits the signals of L users simultaneously, spreading each bit of each user with the aid of an N-chip code to N subcarriers. For OFDM the schematic is simplified, since no spreading is applied, mapping the L bits directly to the IFFT block of Fig. 1 . The normalised complex envelope of an MPSK modulated MC-CDMA signal in a symbol period

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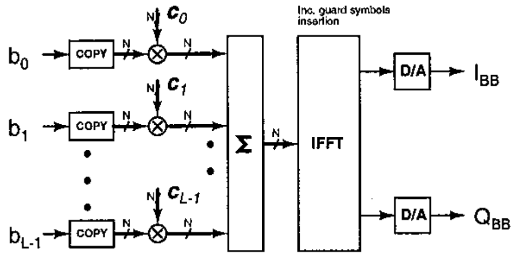


Figure 1: MC-CDMA Transmitter Model: $\mathbf{b_i}$ and $\mathbf{C_i}$ are the ith message symbol and spreading sequence, respectively.

may be represented as :

$$s(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} b_l \, c_{l,n} \, e^{j2\pi n \frac{t}{T}} , \qquad (1)$$

where L is the number of simultaneously transmitted bits or transmitting users, N represents the number of subcarriers and processing gain, $\{b_l\}$, $l = 0, \cdots, L-1$, is the lth user's message symbol such that $|b_l| = 1$, $C_l = \{c_{l,n}\}$, $l = 0, \cdots, L-1, n = 0, \cdots, N-1$, is the l-th user's complex spreading sequence, T is the message symbol period and t is in $[0, T)$. In the rest of this paper, we will cast our discussions in the context of L transmitting users, noting again that our considerations are also valid for the parallel transmission of L bits. The spreading sequence is orthogonal and satisfies $\sum_{n=0}^{N-1} c_{i,n} \, c_{j,n}^* = N\delta_{i,j}$, where c^* is the complex conjugate of c and $\delta_{i,j}$ is the Kronecker delta.

Let d_n be $\sum_{l=0}^{L-1} b_l \, c_{l,n}$. Then, following the approach in [8, 9], the envelope power, $|s(t)|^2$, may be expressed as :

$$\begin{aligned} |s(t)|^2 &= s(t)s^*(t) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} |d_n|^2 + \frac{2}{N} \mathrm{Re} \left\{ \sum_{n=1}^{N-1} D_n e^{j2\pi n \frac{t}{T}} \right\} , \end{aligned} \qquad (2)$$

where D_n is :

$$D_n = \sum_{i=0}^{N-n-1} d_i d_{i+n}^* . \qquad (3)$$

The first term in Eq. (2) is the average power and it can be shown that :

$$\begin{aligned} \frac{1}{N} \sum_{n=0}^{N-1} |d_n|^2 &= \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{l=0}^{L-1} b_l \, c_{l,n} \right) \left(\sum_{l=0}^{L-1} b_l^* \, c_{l,n}^* \right) \\ &= L . \end{aligned} \qquad (4)$$

Thus, the CF of $s(t)$ is entirely dependent upon the second term of Eq. (2), which can be expressed as :

$$D_n = \sum_{l=0}^{L-1} A_n^l + \sum_{l=0}^{L-1} \sum_{l'=0, l' \neq l}^{L-1} b_l b_{l'}^* \, X_n^{l,l'} , \qquad (5)$$

where A_n^l represents the **aperiodic autocorrelations of the l-th user's spreading code, defined by :**

$$A_n^l \triangleq \sum_{i=0}^{N-n-1} c_{l,i} \, c_{l,i+n}^* \qquad (6)$$

and $X_n^{l,l'}$ represents the **aperiodic cross-correlations between the spreading codes of user l and l', defined by :**

$$X_n^{l,l'} \triangleq \sum_{i=0}^{N-n-1} c_{l,i} \, c_{l',i+n}^* . \qquad (7)$$

Upon substituting Eq. (4) and (5) in Eq. (2), $|s(t)|^2$ becomes :

$$\begin{aligned} &|s(t)|^2 = \\ &L + \frac{2}{N} \mathrm{Re} \sum_{n=1}^{N-1} \left(\sum_{l=0}^{L-1} A_n^l + \sum_{l=0}^{L-1} \sum_{l' \neq l}^{L-1} b_l b_{l'}^* X_n^{l,l'} \right) e^{j2\pi n \frac{t}{T}} \\ &= \sum_{l=0}^{L-1} |s_l(t)|^2 + \mathrm{Re} \sum_{l=0}^{L-1} \sum_{l'=0, l' \neq l}^{L-1} b_l b_{l'}^* \frac{2}{N} \sum_{n=1}^{N-1} X_n^{l,l'} e^{j2\pi n \frac{t}{T}} , \end{aligned} \qquad (8)$$

where $|s_l(t)|^2$ is the envelope power of a single user, which is given by :

$$|s_l(t)|^2 = 1 + \frac{2}{N} \mathrm{Re} \sum_{n=1}^{N-1} A_n^l e^{j2\pi n \frac{t}{T}} . \qquad (9)$$

Eq. (8) show that the envelope power depends on the aperiodic autocorrelations A_n^l of Eq. (6) and the crosscorrelations $X_n^{l,l'}$ of Eq. (7) of the spreading sequences. The CF is defined as :

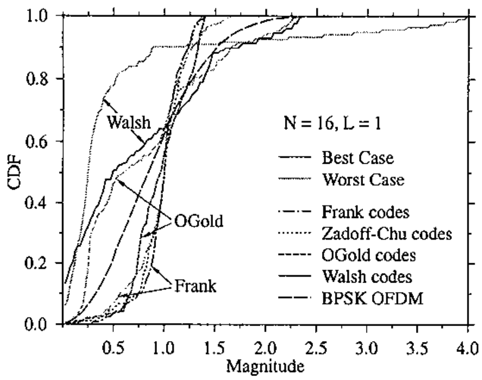
$$\mathrm{CF} \triangleq \max_{0 \le t \le T} \frac{|s(t)|}{\sqrt{L}} , \qquad (10)$$

and the peak factor (PF) is as CF^2 .

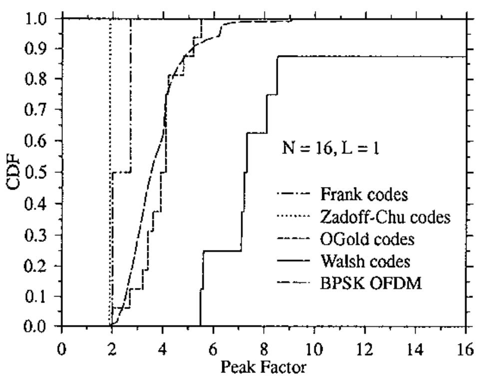
It can be observed in Eq. (9) that for the single user case the crosscorrelation terms do not contribute to the envelope power. However, as L increases, the crosscorrelation terms of Eq. (7) play an increasingly dominant role.

3. SPREADING SEQUENCES AND PEAK FACTORS

In our forthcoming study two binary sequences - Walsh codes and orthogonal Gold codes - along with two so-called root-unity polyphase sequences - namely the so-called Frank codes [5] and Zadoff-Chu codes[6, 7] - were considered as the orthogonal spreading codes for our BPSK MC-CDMA scheme. We assumed that the number of subcarriers, N, is 16. However, the results shown in this section apply also to MC-CDMA schemes with $16 \times m$ subcarriers, where the adjacent spread signals are m-subcarriers apart, in order to achieve independent fading on each subcarrier, which improves the diversity gain. For the single user case ($L = 1$), the MC-CDMA envelope power is governed by Eq.(9), where only the aperiodic autocorrelations A_n^l of Eq.(6) play a role. The associated magnitude distributions and the peak factor distributions are depicted in Figure 2(a) and 2(b), respectively. In Figure 2(a) the best and the worst cases for Frank, orthogonal Gold and Walsh codes are depicted in order to show their magnitude cumulative distribution function (CDF), depending on the particular choice of a specific spreading code. The two polyphase codes - namely

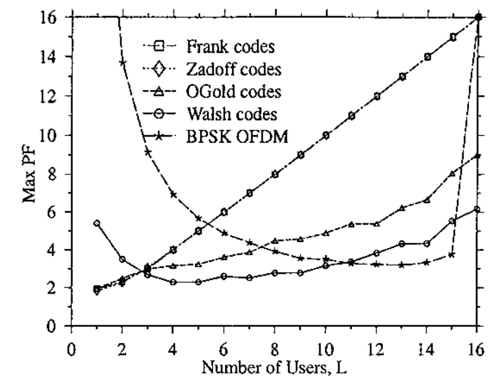


(a) CDF of $|s(t)|$

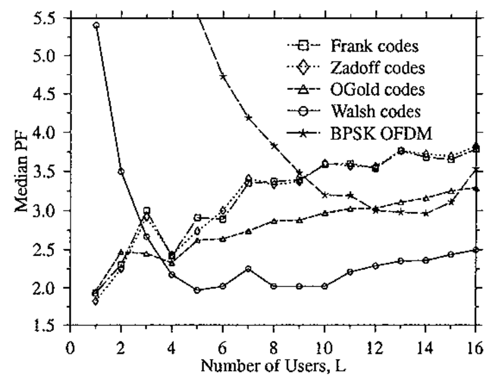


(b) CDF of PF (CF^2)

Figure 2: The magnitude and crest factor Cumulative Distribution Function (CDF) of BPSK MC-CDMA and BPSK OFDM for a spreading factor of $N = 16$ and $L = 1$ user



(a) Maximum Peak Factor



(b) Median Peak Factor

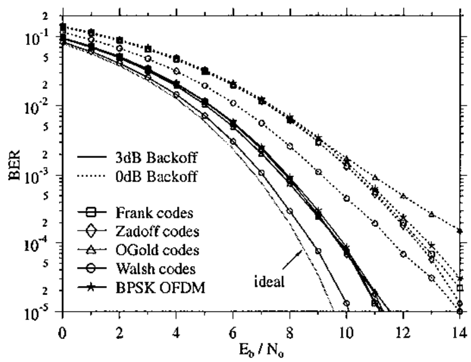
Figure 3: The maximum and median PF (CF^2) for various spreading codes for BPSK MC-CDMA. For BPSK OFDM various-rate peak-factor reduction codes (N, L) were used, which were normalised by N/L for fair comparison. The spreading factor was $N = 16$.

the Frank and the Zadoff-Chu codes, show desirable, step-function-like CDF distributions.

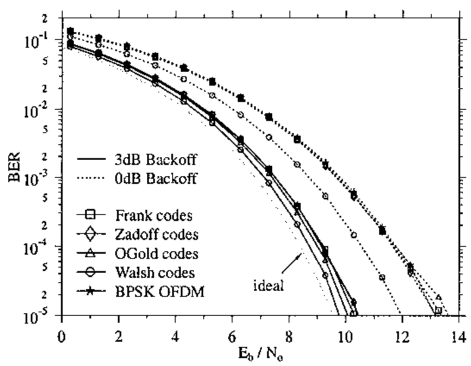
All the Zadoff-Chu codes [6, 7] showed the same magnitude distribution and hence the same peak factor. In fact, we can show that for Zadoff-Chu codes :

$$|s_l(t)|^2 = \left| s_0 \left(t - \frac{Tl}{N} \right) \right|^2 , \qquad (11)$$

which states that the lth user's power envelope is a time-shifted version of the first user's power envelope, retaining all the magnitude statistics. By contrast, Frank codes form two equivalent classes in terms of their magnitude statistics. Since Frank codes and Zadoff-Chu codes have zero off-peak periodic autocorrelations, their aperiodic autocorrelations are also low, which is typically quantified in terms of the so-called merit factor (MF), defined as the ratio of peak to off-peak aperiodic autocorrelations given in Eq. (6), $\text{MF} \triangleq A_0^2/(2\sum_{n=1}^{N-1}|A_n)|^2)$. All of the Zadoff-Chu codes exhibit MF=6.7, while the Frank codes have MF=8 or 4. The merit factors of orthogonal Gold codes lie between 0.6 and 1.8, except for one of them, which is 4.0, and those for Walsh codes are below 0.52 . With reference to Eq. (8) and (10), this explains the preponderance of high peak factors in the distributions. Please note that the $2n$-th and $(2n+1)$-th Walsh code pairs have the same peak factor due to an equivalence transform [10], resulting in $N/2$ different peak factors, as it can be observed in Figure 2(b) . As the number of users, L, is increased, the aperiodic crosscorrelations begin to play a role, as suggested by Eq. (8). When there are L number of bits to be transmitted simultaneously, we have $\binom{N}{L}$ choices of selecting spreading codes for each class. Figure 3(a) depicts the maximum peak factors over all 2^L messages for the 'best case' spreading code combinations over all possible $\binom{N}{L}$ choices. For our BPSK OFDM modem, (N,L) non-linear PF reduction codes [4] were used in order to arrive at the maximum peak factors plotted, which were multiplied by N/L to normalise them for the same average power as MC-CDMA. Walsh-spread MC-CDMA shows the lowest maximum peak factors for $L \geq 3$ amongst the four MC-CDMA systems. Figure 3(a) also shows that the specific peak factor reduction block codes used for BPSK OFDM, for which the code rates are higher than 3/4, are more effective in terms of reducing the maximum peak factor, than Walsh-spread MC-CDMA with the same bandwidth efficiency and without block coding. If we apply (L,L-1) block codes in order to reduce the peak factors in MC-CDMA, the peak factor reduction is more effective for Walsh-spread MC-CDMA, than for BPSK OFDM, as shown



(a) **Without** amplitude limiting coding



(b) **With** (16,15) amplitude limiting coding

Figure 4: The BER of BPSK MC-CDMA and BPSK OFDM over AWGN channels for a SF of $N = 16$ and for $L = 16$ users (or $L = 16$ parallel bits).

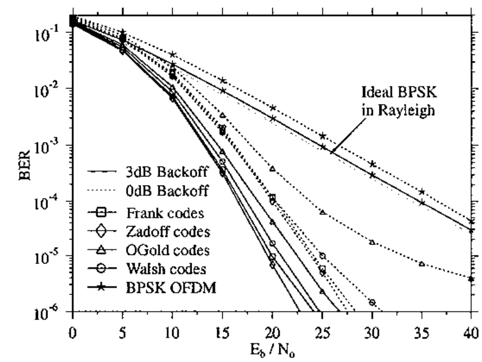
in Figure 3(b) . In fact, for $L > 3$, Walsh-spread MC-CDMA turned out to be the most effective in terms of reducing the peak factors amongst the compared schemes. The high peak factors of the polyphase codes are due to their high aperiodic crosscorrelations of Eq. (7). For example, the relation $X_n^{l,l+n} = N - n$, holds for both polyphase codes. Let us define a relative measure of the aperiodic crosscorrelations, XMF, as XMF $\triangleq \sum_n \sum_l \sum_{l' \neq l}^{N-1} |X_n^{l,l'}|^2$. Then, the corresponding values are 30336, 30414.5, 28736 and 22400 for Frank, Zadoff-Chu, orthogonal Gold and Walsh codes. The ordering of the codes by XMF broadly follows their ordering based on their peak factors. Let us now consider the effects of clipping amplifier on the BER.

4. CLIPPING AMPLIFIER AND BER COMPARISON

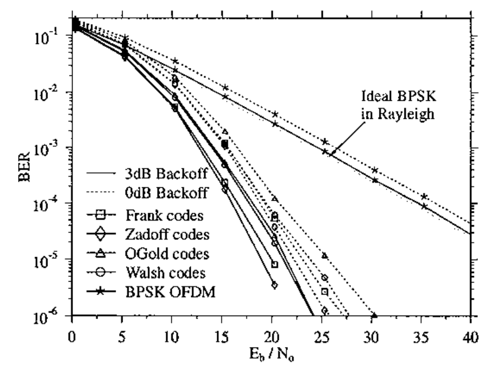
A simple unit-gain clipping amplifier can be modeled as :

$$A(x) = \begin{cases} x & \text{if } x < x_{max} \\ x_{max} & \text{otherwise .} \end{cases} \quad (12)$$

Let us assume that the MC-CDMA and OFDM signals are normalised, such that their average power becomes unity. Clipping the signal at $x_{max} = \sqrt{p}$ is equivalent to an input backoff of $10\log_{10}(p)$ dB.



(a) **Without** amplitude limiting coding



(b) **With** (16,15) amplitude limiting coding

Figure 5: The BER of BPSK JD MC-CDMA and BPSK OFDM in Rayleigh channel for a SF of $N = 16$ and for $L = 16$ users. MMSE Joint Detection was used for MC-CDMA. Independent fading of each subcarrier was assumed.

Simulations were carried out, in order to investigate the BER performance (Figure 4 and 5) of BPSK MC-CDMA and BPSK OFDM at a range of different clipping levels. Figure 4 shows the associated BER results over AWGN channels. Observe that when no amplitude limiting code is applied, 3dB backoff in the amplifier is sufficient for Walsh-spread MC-CDMA to achieve a BER of 10^{-5} with 0.5dB higher E_b/N_o, than the ideal system having no dynamic range limitation. When a (16,15) block code is applied in conjunction with a 3dB backoff in order to reduce the peak factor, Walsh-spread MC-CDMA failed to improve the BER performance because of the energy-per-bit (E_b) loss due to the introduction of one redundant bit. In fact, in this case all MC-CDMA systems - except for the Walsh-spread system - and the OFDM system showed little difference in their BER performance over AWGN channels.

The BER performance of MMSE Joint-Detection (JD) MC-CDMA [11] and OFDM is shown in Figure 5 over Rayleigh channels, where independent subcarrier fading and perfect channel estimation are assumed. The independent subcarrier fading can be achieved in a system, where many subcarriers are employed in conjunction with frequency domain interleaving, provided that the number of resolvable multipaths in the channel is higher than or equal to the spreading gain. Figure 5(a) shows the advantage of MC-CDMA systems over OFDM. MC-CDMA exhibited a significantly better BER performance than OFDM, providing the same spectral efficiency. This is only true, however, when there is sufficient diversity gain in order to compensate for the multiple user interference (MUI) or multi-bit interference. It was observed in other simulations that a diversity order of two or three is sufficient for MMSE JD-MC-CDMA to overcome the MUI. We can observe that JD MC-CDMA is more sensitive to amplitude clipping than OFDM. However, Walsh-spread MC-CDMA was not the best performing scheme this time, *ie* over Rayleigh channels. Two polyphase codes were more effective, than the binary spreading codes in this environment. Considering that the polyphase-spread signals suffer from higher distortion due to clipping, this result is surprising. It implies that the MUI of polyphase-spread systems is lower than that of the binary spreading systems and the MUI difference plays a greater role in determining their BER performance. The BER performance of MMSE JD MC-CDMA and OFDM systems with amplitude limiting coding is shown in Figure 5(b). The BER improvement was around 1dB for MC-CDMA systems at a 10^{-5} BER and at 3dB backoff, in comparison to the uncoded system. This BER improvement was more noticeable at 0dB backoff, where amplitude clipping occurs heavily.

When MMSE single user detection was used for our MC-CDMA systems, we found that the amplitude limiting coding degraded the BER performance at 3dB backoff for the Walsh-spread system. This means that the amplitude limiting coding is not optimal in reducing the overall BER, because it may select the message codes which incur large MUI, resulting in a degraded BER performance.

5. CONCLUSIONS

In addition to the more widely recognised benefits of MC-CDMA - such as its substantial frequency diversity gain - this paper showed that MC-CDMA is also capable of reducing the CF problem. We also investigated the relationship between the envelope power and the properties of various spreading sequences. It was shown that the envelope power is determined by the sums of the aperiodic autocorrelations in Eq. (6) and the aperiodic crosscorrelations of the spreading sequences in Eq. (7). We showed furthermore that the former property is more important for low number of users, L, while the latter plays a more dominant role for highly loaded conditions, *ie* for a high L.

When we consider that the weighted sums of the aperiodic autocorrelations and the aperiodic crosscorrelations are bounded by a constant [12], it is apparent that there is no 'magic' sequences, which exhibit "good" CF distributions for all values of L. For example, the orthogonal Gold code is better for small L values, but Walsh codes are preferred for large L values. The two polyphase codes considered were also found attractive for $L = 1$, while their CF distributions for $L = N$ were the least attractive amongst the codes studied.

When we set $L{=}N$, in order to maintain the same spectral efficiency as OFDM, Walsh-spread MC-CDMA showed a significant peak factor reduction over OFDM with the same rate block coder in a 'fully loaded' condition. When 3dB backoff was applied to the transmitter amplifier, the Walsh-spread MC-CDMA system showed more than 1dB SNR gain at a BER of 10^{-5} in Figure 4 over OFDM in an AWGN environment. For independent Rayleigh channels over each subcarrier, MMSE JD MC-CDMA employing Zadoff-Chu spreading codes was found to be the best in Figure 5, owing to its small MUI, despite more clipping, than the other MC-CDMA systems. We found that a 3dB amplifier backoff without amplitude limiting coding is a reasonable choice for MC-CDMA systems, which outperformed OFDM in all the investigated environments. Thus, MC-CDMA is expected to find many practical applications, where OFDM is employed now, as DSP technology develops.

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