BLOCK TURBO CODED BURST-BY-BURST ADAPTIVE RADIAL BASIS FUNCTION DECISION FEEDBACK EQUALISER ASSISTED MODEMS

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ABSTRACT

A novel adaptive modem scheme is to be presented for transmissions over wideband mobile channels, which employs a Radial Basis Function (RBF) based decision feedback equaliser, in order to mitigate the effects of the dispersive wideband channel. Turbo codes are invoked for improving the bit error rate (BER) and bits per symbol (BPS) performance of the scheme, which is shown to give a significant improvement in terms of mean BPS performance compared to that of the uncoded RBF equaliser assisted adaptive modem.

1. INTRODUCTION

The principles of adaptive quadrature amplitude modulation (AQAM) schemes were presented for example by Webb et al. [1] and Wong [2] et al. Chen, McLaughlin, Mulgrew and Grant [3] proposed a range of so-called RBF network based channel equalisers, which are capable of detecting the received signalling symbols even in a scenario, where the phasors become linearly non-separable due to the intersymbol interference inflicted by the channel. In this situation conventional equalisers would be unable to remove the effects of intersymbol interference (ISI) and hence would exhibit a residual BER. Additionally, Chen *et. al.* [4] introduced decision feedback in their RBF-based equaliser, in order to reduce its computational complexity. The RBF decision feedback equaliser (RBF DFE) was then extended to higher-order QAM schemes, which were investigated in [5]. Turbo coding was proposed by Berrou, Glavieux and Thitimajshima and its performance was shown to approximate the Shannonian limit [6]. The reader is referred to the above references for background reading.

The upper bound performance of the joint adaptive QAM and RBF DFE for multipath Rayleigh fading channels has been investigated in [7]. In this contribution we invoke turbo codes in the AQAM/RBF DFE system in order to improve the BER and BPS performance of the current adaptive scheme. An overview of the turbo coded AQAM/RBF DFE is given in the next section.

2. SYSTEM OVERVIEW

In order to maximise the BPS throughput of the system, high code rates in excess of 2/3 are desirable. Consequently, block codes were chosen as the component codes in preference to Recursive Systematic Convolutional (RCS) codes, since turbo block coding has generally shown better results for coding rates above 2/3 [8]. At the transmitter, binary BCH(n, k) codes are used as the component codes in the turbo encoder, where n and k denote the number of encoded bits and the number of information bits, respectively. At the receiver, the RBF DFE based on the optimal Bayesian decision function [4] is capable of estimating the *a-posteriori* probability of the transmitted symbols. Therefore from the symbol probability estimates we can obtain the so-called log likelihood ratio (LLR) of the bits representing the symbols, where the LLR of a data bit u_k is defined as:

$$\mathcal{L}(u_k) = ln(\frac{P(u_k = +1)}{P(u_k = -1)}). \quad (1)$$

The above bit LLRs provide the soft decision values required by the turbo decoders.

The probability of error for the detected bits can be estimated from the soft output of the turbo decoder. Referring to Equation 1 and assuming $P(u_k = +1) + P(u_k = -1) = 1$, the probability of error of the detected bit is given by

$$P_{error}(u_k) = \begin{cases} P(u_k = -1), & \text{if } \mathcal{L}(u_k) \geq 0 \\ P(u_k = +1), & \text{if } \mathcal{L}(u_k) < 0 \end{cases} . \quad (2)$$

The probability of the bit having the value of +1 or -1 can be rewritten in terms of the LLR of the bit, $\mathcal{L}(u_k)$, as follows:

$$P_{error}(u_k) = \frac{1}{1 + e^{|\mathcal{L}(u_k)|}}, \quad (3)$$

where $|\mathcal{L}(u_k)|$ is the magnitude of $\mathcal{L}(u_k)$. Therefore, since the magnitude of the bit LLR is related to the probability of bit error according to Equation 3, the average value of the LLR magnitude of all the bits in a transmission burst can be used as the modem mode switching criterion. We define the average frame LLR magnitude as follows:

$$\mathcal{L}\text{average} = \frac{\sum_{i=0}^{\mathcal{F}} \mathcal{L}(u_i)}{\mathcal{F}}, \quad (4)$$

where $\mathcal{F}$ is the number of data bits per frame and u_i is the ith data bit in the transmission burst. Figure 1 portrays the average transmission burst LLR magnitude fluctuation over the frame-invariant fading channel versus the frame index for binary phase shift keying (BPSK) at the output of the RBF DFE over a two-path Rayleigh fading channel. The average frame LLR magnitude before decoding is slowly varying and it is relatively predictable for a number of consecutive data burst. Therefore the average frame LLR magnitude of the *current* transmitted burst can be used to

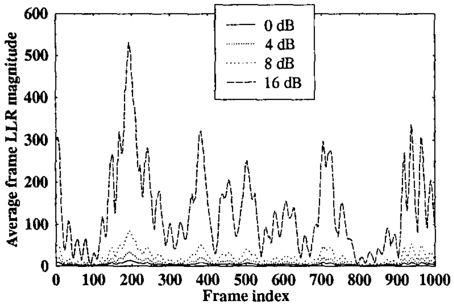

Figure 1: Average frame LLR magnitude **before decoding** versus frame index for BPSK at the output of the RBF DFE over the two-path Rayleigh fading channel of Table 2. Perfect channel impulse response (CIR) estimation is assumed and error propagation in decision fedback is ignored. The transmission burst used is shown in Figure 2.

select the modulation mode for the *next* transmission burst based on a set of switching LLR magnitude thresholds corresponding to the $\mathcal{M}$-QAM regime used, where $\mathcal{M} = 2, 4, 16$ and 64. The turbo decoder iteratively improves the BER of the decoded bits. Since the average frame LLR magnitude before and after decoding has an approximately linear relationship, as demonstrated by Figure 3 for 1 and 6 decoder iterations, the average probability of error for the decoded frame can be inferred from the average frame LLR magnitude provided by the RBF equaliser and fed into the turbo decoder instead of those provided by the turbo decoder. Thus, this parameter can also be used as the switching criterion of the turbo coded scheme. The advantage of using this method – rather than using the average frame LLR magnitude after decoding as the switching criterion – is that turbo decoding can be skipped for received 'no transmission' (NO TX) data bursts and we only need to measure the quality of the equalised NO TX data sequence using the RBF DFE in order to predict the modulation mode to be used by the next transmission burst. The NO TX mode is utilized for adaptive schemes with transmission blocking, when dummy symbols are transmitted instead of a valid data sequence.

The average frame LLR magnitude obtained from the RBF DFE is compared to a set of switching LLR magnitude corresponding to the modulation mode of the equalised data burst. Consequently, a modulation mode is selected for the next transmission, assumming reciprocity of the uplink and downlink. This implies that the similarity of the average

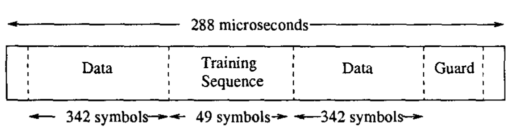

Figure 2: Transmission burst structure of the FMA1 non-spread data mode as specified in the FRAMES proposal [10].

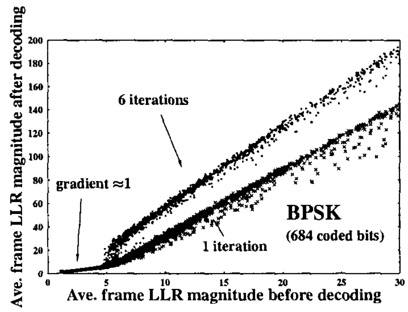

Figure 3: The average frame LLR magnitude after turbo decoding versus the average frame LLR magnitude before turbo decoding for BPSK. The turbo coding parameters are given in Table 1 and the turbo interleaver size is 494 bits. The transmission burst used is shown in Figure 2.

frame LLR magnitude of consecutive data bursts can be exploited, in order to set the next modulation mode. The modulation modes utilized in our system are BPSK, 4QAM, 16QAM, 64QAM and NO TX. The modulation mode is switched according to the average frame LLR magnitude as follows:

$$\text{Mod. Mode} = \begin{cases} \text{NO TX} & \text{if } \mathcal{L}\text{average} \leq \mathcal{L}_2^{\mathcal{M}} \\ \text{BPSK} & \text{if } \mathcal{L}_2^{\mathcal{M}} < \mathcal{L}\text{average} \leq \mathcal{L}_4^{\mathcal{M}} \\ \text{4QAM} & \text{if } \mathcal{L}_4^{\mathcal{M}} < \mathcal{L}\text{average} \leq \mathcal{L}_{16}^{\mathcal{M}} \\ \text{16QAM} & \text{if } \mathcal{L}_{16}^{\mathcal{M}} < \mathcal{L}\text{average} \leq P_{64}^{\mathcal{M}} \\ \text{64QAM} & \text{if } \mathcal{L}_{64}^{\mathcal{M}} < \mathcal{L}\text{average}, \end{cases} \tag{5}$$

where $\mathcal{L}_i^{\mathcal{M}}, i = 2, 4, 16, 64$ are the switching LLR magnitude thresholds corresponding to the $\mathcal{M}$-QAM mode. The LLR magnitude switching thresholds corresponding to $\mathcal{M}$-QAM, $\mathcal{L}_i^{\mathcal{M}}, i = 2, 4, 16, 64$, can be obtained by estimating the average frame LLR magnitude degradation/improvement, when the modulation mode is switched from $\mathcal{M}$-QAM to a higher/lower number of modulation levels.

The RBF equaliser based on the Bayesian equaliser solution has a high computational complexity due to evaluating nonlinear exponential functions and due to the high number of additions/subtractions and multiplications/divisions required. Hence we propose computing the output of the RBF network in logarithmic form by using the Jacobian logarithm [9], in order to avoid the computation of exponentials and to reduce the number of multiplications performed. We refer to our RBF equaliser using Jacobian logarithm as the *Jacobian logarithmic RBF equaliser*. We will present this idea in more detail in the next section and in our simulations we will use this reduced complexity equaliser.

Component code	BCH(31,26)
Code rate	0.722
Turbo interleaver type	Random
Component decoders	Log-MAP
Number of iterations	6

Table 1: The turbo coding parameters

2.1. Jacobian Logarithmic RBF Equaliser

The Jacobian logarithm is defined by the relationship [9]:

$$\begin{aligned} J(\lambda_1, \lambda_2) &= ln(e^{\lambda_1} + e^{\lambda_2}) \\ &\approx max(\lambda_1, \lambda_2) + ln(1 + e^{-|\lambda_1 - \lambda_2|}) \\ &\approx max(\lambda_1, \lambda_2) + f_c(|\lambda_1 - \lambda_2|) \end{aligned} \quad (6)$$

where $f_c(\cdot)$ is the so-called correction function. The correction function $f_c(x) = ln(1 + e^{-x})$ has a dynamic range of $ln(2) \geq f_c(x) > 0$, and it is significant only for small values of x. Thus, $f_c(x)$ can be tabulated in a look-up table, in order to reduce the computational complexity. The correction function $f_c(\cdot)$ only depends on $|\lambda_1 - \lambda_2|$, therefore the look-up table is one-dimensional and only a few values have to be stored.

The Jacobian logarithm of Equation 6 can be extended, in order to cope with a higher number of exponential summations, such as in $ln\left(\sum_{k=1}^{n} e^{\lambda_k}\right)$. Reference [9] showed that this can be done by nesting the $J(\lambda_1, \lambda_2)$ operation as follows:

$$ln\left(\sum_{k=1}^{n} e^{\lambda_k}\right) = J(\lambda_n, J(\lambda_{n-1}, \ldots J(\lambda_3, J(\lambda_2, \lambda_1)) \ldots)). \quad (7)$$

Having characterised the Jacobian logarithm, we will now decribe, how this operation can be used in order to reduce the computational complexity of the RBF equaliser.

The overall response of the M-hidden nodes RBF network is given by [3]:

$$f_{RBF}(\mathbf{v}_k) = \sum_{i=1}^{M} w_i exp(-|\mathbf{v}_k - \mathbf{c}_i|^2/\rho), \quad (8)$$

where $\mathbf{c}_i, i = 1, \ldots, M$ are the RBF centers and ρ is the RBF width. Expressing Equation 8 in a logarithmic form and invoking the Jacobian logarithm, we obtain:

$$\begin{aligned} ln(f_{RBF}(\mathbf{v}_k)) &= ln(\sum_{i=1}^{M} w_i exp(-\|\mathbf{v}_k - \mathbf{c}_i\|^2/\rho)) \\ &= ln(\sum_{i=1}^{M} exp(ln(w_i)) exp(-\|\mathbf{v}_k - \mathbf{c}_i\|^2/\rho)) \\ &= ln(\sum_{i=1}^{M} exp(\lambda_{ik})) \\ &= J(\lambda_{Mk}, J(\lambda_{(M-1)k}, \ldots J(\lambda_{2k}, \lambda_{1k}) \ldots)), \end{aligned} \quad (9)$$

where $\lambda_{ik} = ln(w_i) - \|\mathbf{v}_k - \mathbf{c}_i\|^2/\rho$ and $\|\cdot\|$ denotes the Euclidean norm. By introducing the Jacobian logarithm, every summation of two exponentials and each weight multiplication operation in Equation 8 is subtituted by an addition, a subtraction, a table look-up and a comparison operation, thus reducing the associated computational complexity. Most of the computation load comes from computing the Euclidean norm term $\|\mathbf{v}_k - \mathbf{c}_i\|^2$, which will depend on the number of RBF centers and the size of the vectors $\mathbf{c}_i$ and $\mathbf{v}_k$ or – in other words – the equaliser order.

Figure 4 shows the turbo coded BER performance of the Jacobian RBF DFE compared to that of the RBF DFE over

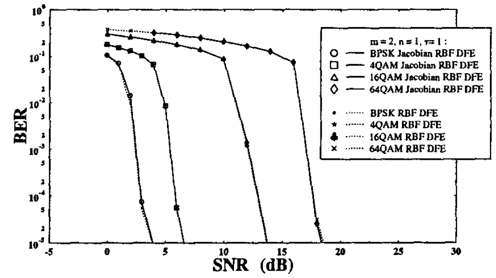

Figure 4: BER versus SNR performance for the Jacobian logarithmic RBF DFE with turbo coding over the dispersive two-path AWGN channel having a transfer function $H(z) = 0.707 + 0.707z^{-1}$ for various QAM schemes. The turbo coding parameters are given in Table 1.

a dispersive two-path additive white Gaussian noise channel having the transfer function $H(z) = 0.707 + 0.707z^{-1}$. Both equalisers have a feedforward order of $m = 2$, feedback order of $n = 1$ and a decision delay of $\tau = 1$. The parameters of the turbo codec are given in Table 1 and a turbo interleaver of size 9984 bits is used. Both equalisers are shown to give a similar turbo coded performance, since the Jacobian logarithm method is capable of providing a good approximation of the equalised channel output LLRs. Another advantage of the Jacobian logarithmic version of the RBF DFE is that in providing soft values for the turbo decoder, the Jacobian RBF DFE output is already in the logarithmic form and thus we can dispense with taking the logarithm in order to obtain the bit LLRs. Let us now consider the performance of the proposed scheme.

3. SIMULATION RESULTS

Transmission Frequency	1.9GHz
Transmission Rate	2.6MBd
Vehicular Speed	30 mph
Normalised Doppler Frequency	3.3×10^{-5}
Channel weights	$0.707 + 0.707z^{-1}$

Table 2: Simulation parameters for two-path Rayleigh fading channel scenario.

Mod. Mode	BPSK	4QAM	16QAM	64QAM
Interleaver Size	684	1368	2736	4104

Table 3: Corresponding random interleaver size for each modulation mode.

The simulation parameters are listed in Table 2, noting that we analysed the joint AQAM and RBF DFE scheme in conjunction with turbo coding over a two-path Rayleigh fading channel. The wideband fading channel was frame-invariant, implying that during a transmission frame the channel impulse response was considered time-invariant. The transmission burst used in our treatise is shown in Figure 2. The RBF DFE used in our simulations had a feedforward order of $m = 2$, feedback order of $n = 1$, decision delay of $\tau = 1$ and it utilised the Jacobian logarithm for reducing its computational complexity. We used the binary BCH(31, 26) turbo component code and the size of the random turbo interleaver used was varied according to the modulation mode used – as given in Table 3 – in order to enable burst-by-burst decoding.

In our experiments, we obtained the LLR magnitude degradation/improvement from the average frame LLR magnitude of every modulation mode used, under the same instantaneous channel scenario. In order to achieve a certain target BER performance we have to map the average frame LLR magnitude to the expected BER of the decoded burst. The BER of the decoded burst can be estimated from the average probability of error for the bits in the decoded burst. We refer to this estimate as the *short-term BER*, which is defined as:

$$\text{Short-term BER} = \frac{\sum_{n=1}^{\mathcal{F}} P_{error}(u_i)}{\mathcal{F}}, \qquad (10)$$

where $\mathcal{F}$ is the number of bits per burst and $P_{error}(u_i)$ is the probability of error for the detected bit, as defined in Equation 3. We opt for using the average of the bit LLRs instead of the average of the bit error probability as the switching criterion in order to avoid the computation of the error probability for every bit based on Equation 3.

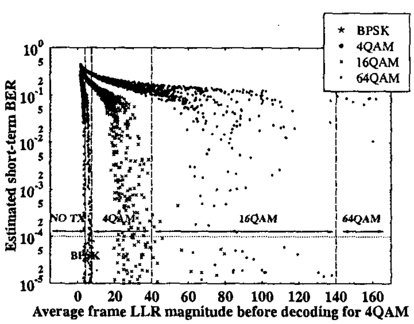

Figure 5: The estimated short-term BER for all the possible modulation modes versus the average frame LLR magnitude of 4QAM over the two-path Rayleigh fading channel of Table 2. The turbo coding parameters are given in Table 1. The turbo interleaver size is fixed according to the modulation mode used as shown in Table 3.

Figure 5 shows the short-term BER of every modulation mode used **after** turbo decoding, versus the average frame LLR magnitude of 4QAM **before** decoding. In order to obtain the target BER of 10^{-4}, Figure 5 demonstrates how each switching LLR magnitude $\mathcal{L}_i^4$ is obtained. For example, if the average LLR magnitude of the received 4QAM transmission burst, $\mathcal{L}_{\text{average}}$ is in the range of $100 > \mathcal{L}_{\text{average}} \geq 40$, the modulation mode is switched to 16QAM for the next transmission burst, since the BER of this 16QAM transmission burst is estimated to be below the channel decoded target BER of 10^{-4}. Note that due to the 'spreading' of the average frame LLR magnitude versus the short-term BER curve – especially for higher level QAM modes – the switching threshold is estimated from the mean of this dynamic range. Using the above method, the switching LLR magnitude thresholds were obtained for the target BER of 10^{-4} for all legitimate mode switching combinations, as listed in Table 4.

	$\mathcal{L}_2^{\mathcal{M}}$	$\mathcal{L}_4^{\mathcal{M}}$	$\mathcal{L}_{16}^{\mathcal{M}}$	$\mathcal{L}_{64}^{\mathcal{M}}$
NO TX	8.0	17.0	90.0	380.0
BPSK	8.0	17.0	90.0	380.0
4QAM	4.0	7.5	40.0	140.0
16QAM	2.0	3.0	11.5	55.0
64QAM	1.7	2.2	6.2	30.0

Table 4: The switching LLR magnitude thresholds $\mathcal{L}_i^{\mathcal{M}}$ of the Jacobian AQAM RBF DFE scheme with turbo coding for the target BER of 10^{-4} over the two-path Rayleigh fading channel of Table 2.

Figure 6 shows the performance comparison of the AQAM/Jacobian RBF DFE scheme with turbo coding for data quality transmission (target BER of 10^{-4}) with its constituent turbo coded fixed modulation schemes. Figure 6 also shows the BER and BPS performance of the AQAM/RBF DFE scheme *without* turbo coding. The number of BPS for the coded BPSK, 4QAM, 16QAM and 64QAM modes is 0.72, 1.44, 2.89 and 4.33, respectively. Comparing the coded AQAM schemes with the coded fixed modulation schemes shown in Figure 6, the BPS and BER performance of the adaptive scheme is superior in comparison to the individual constituent fixed modulation modes.

Referring to Figure 6, the coded BPS was better than that of the uncoded scheme for the channel SNR range of 0 to 26dB with a maximum gain of 4dB at a channel SNR of 0dB. However, at high SNRs the BPS performance is limited by the coding rate of the system to a maximum BPS of $\frac{23}{31} \cdot 6 = 4.33$. The coded AQAM system also exhibited a superior BER performance, when compared to the uncoded

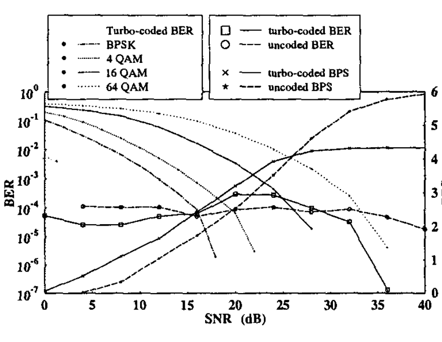

Figure 6: The BER and BPS performance of the joint AQAM/Jacobian RBF DFE with turbo coding for data-quality transmission using the parameters listed in Table 2. The modem mode switching levels used for this scheme are listed in Table 4. The turbo coding parameters was given in Table 1. The turbo interleaver size is fixed according to the modulation mode used as shown in Table 3.

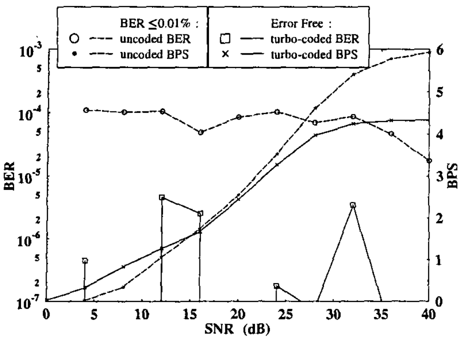

Figure 7: The BER and BPS performance of the joint AQAM/Jacobian RBF DFE with turbo coding designed for error-free transmission using the parameters listed in Table 2. The modem mode switching levels used for this scheme are listed in Table 5. The turbo coding parameters are given in Table 1. The turbo interleaver size is fixed according to the modulation mode used as shown in Table 3.

system for the channel SNR range of 0 to 16dB and in the range above 28dB. The coded AQAM system failed to achieve the target BER of 10^{-4} for the SNR range of 16dB to 28dB. This is because the LLR magnitude threshold was set too low using our suggested method by estimating the threshold from the midpoint or average of the spread LLR magnitude range at high channel SNRs shown in Figure 5. This problem can be solved by increasing the thresholds corresponding to the 4QAM and 16QAM modes, which are predominant in the high channel SNR range.

Figure 7 shows the BER and BPS performance of the error-free turbo coded AQAM/Jacobian RBF DFE scheme with the more conservative, increased LLR magnitude switching thresholds listed in Table 5. The BER and BPS performance of the uncoded AQAM/RBF DFE system is also given in the figure for comparison. The BPS performance of the error-free coded system was better than that of the uncoded AQAM system for the channel SNR range of 0 to 15dB, as evidenced by Figure 7. However, the BPS performance was limited by the coding rate of the system to a maximum value of 4.33 at high channel SNRs.

4. CONCLUSION

In this paper we have demonstrated the application of turbo BCH coding in conjunction with AQAM over a wideband fading channel. The system exhibited a better BPS performance, when compared with the uncoded AQAM/RBF DFE system at low to medium channel SNRs, as evidenced by Figure 6. The same figure also shows an improved coded BER performance at higher channel SNRs. A virtually error-free turbo coded AQAM scheme was also characterised in Figure 7.

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	$\mathcal{L}_2^{\mathcal{M}}$	$\mathcal{L}_4^{\mathcal{M}}$	$\mathcal{L}_{16}^{\mathcal{M}}$	$\mathcal{L}_{64}^{\mathcal{M}}$
NO TX	10.0	30.0	280.0	1000.0
BPSK	10.0	30.0	280.0	1000.0
4QAM	8.0	12.0	100.0	350.0
16QAM	3.0	5.0	30.0	120.0
64QAM	2.5	3.0	13.0	70.0

Table 5: The switching LLR magnitude thresholds $\mathcal{L}_i^{\mathcal{M}}$ of the Jacobian AQAM RBF DFE scheme with turbo coding for error-free transmission over the two-path Rayleigh fading channel of Table 2.

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