Adaptive Minimum Bit Error Rate Beamforming Assisted Receiver for Wireless Communications

S. Chen, L. Hanzo and N.N. Ahmad

Department of Electronics and Computer Science
University of Southampton
Southampton SO17 1BJ
United Kingdom

sqc@ecs.soton.ac.uk lh@ecs.soton.ac.uk nna00r@ecs.soton.ac.uk

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Motivations

- Adaptive spatial processing with smart antenna arrays has shown real promise for substantial capacity enhancement in wireless systems.
- Adaptive beamforming is capable of separating signals transmitted on the same carrier frequency but are separated in the spatial domain.
- Classical beamforming techniques are based on minimizing the system mean square error or other related criteria.
- For a communication system, however, it is the bit error rate, not the mean square error, that really matters.
- This motivates our derivation of an adaptive beamforming technique based directly on minimizing the system bit error rate.

Simplified System Model

- The system has M users (sources), and each transmits a binary phase shift keying (BPSK) signal on the same carrier frequency $\omega = 2\pi f$.
- The baseband signal of user i with signal power A_i^2 is

$$m_i(k) = A_i b_i(k), \quad b_i(k) \in \{\pm 1\}, \quad 1 \le i \le M$$

  Source 1 is the desired user and the rest are interfering users.

- The signals at the antenna array of L uniformly spaced elements are

$$x_l(k) = \sum_{i=1}^{M} m_i(k) \exp\left(j\omega t_l(\theta_i)\right) + n_l(k) = \bar{x}_l(k) + n_l(k), \; 1 \le l \le L$$

  $t_l(\theta_i)$: the relative time delay at element l for source i,
  θ_i: the direction of arrival for source i, and
  $n_l(k)$: a complex-valued white Gaussian noise with $E[|n_l(k)|^2] = 2\sigma_n^2$.

Matrix Form of System Model

- Define the steering vector for source i

$$\mathbf{s}_i = [\exp(j\omega t_1(\theta_i)) \cdots \exp(j\omega t_L(\theta_i))]^T$$

  the system matrix

$$\mathbf{P} = [A_1\mathbf{s}_1 \cdots A_M\mathbf{s}_M]$$

  the bit vector

$$\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^T$$

  and the noise vector

$$\mathbf{n}(k) = [n_1(k) \cdots b_L(k)]^T$$

- Then, the array input vector $\mathbf{x}(k) = [x_1(k) \cdots x_L(k)]^T$ is expressed as

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k)$$

Beamformer

- The beamformer output is

$$y(k) = \mathbf{w}^H \mathbf{x}(k) = \mathbf{w}^H \bar{\mathbf{x}}(k) + \mathbf{w}^H \mathbf{n}(k) = \bar{y}(k) + e(k)$$

  where $\mathbf{w} = [w_1 \cdots w_L]^T$ is the complex-valued beamformer weight vector and $e(k)$ is Gaussian with zero mean and $E[|e(k)|^2] = 2\sigma_n^2 \mathbf{w}^H \mathbf{w}$.

- The estimate of the transmitted bit $b_1(k)$ is

$$\hat{b}_1(k) = \begin{cases} +1, & y_R(k) = \Re[y(k)] > 0, \\ -1, & y_R(k) = \Re[y(k)] \leq 0, \end{cases}$$

- The classical MMSE beamforming solution is given by

$$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{P}\mathbf{P}^H + 2\sigma_n^2 \mathbf{I}_L\right)^{-1} \mathbf{p}_1$$

  with $\mathbf{p}_1$ being the first column of $\mathbf{P}$

Signal States

- Denote the $N_b = 2^M$ possible sequences of $\mathbf{b}(k)$ as $\mathbf{b}_q$, $1 \leq q \leq N_b$. Let the first element of $\mathbf{b}_q$, corresponding to the desired user, be $b_{q,1}$.

- Then, $\bar{\mathbf{x}}(k)$ only takes values from the signal state set defined as

$$\mathcal{X} \triangleq \{\bar{\mathbf{x}}_q = \mathbf{P}\mathbf{b}_q, \ 1 \leq q \leq N_b\}$$

- Therefore, $\bar{y}(k) \in \mathcal{Y} \triangleq \{\bar{y}_q = \mathbf{w}^H \bar{\mathbf{x}}_q, \ 1 \leq q \leq N_b\}$.

- Thus, $\bar{y}_R(k) = \Re[\bar{y}(k)]$ can only take values from the set

$$\mathcal{Y}_R \triangleq \{\bar{y}_{R,q} = \Re[\bar{y}_q], \quad 1 \leq q \leq N_b\}$$

  which can be divided into the two subsets conditioned on $b_1(k)$

$$\mathcal{Y}_R^{(\pm)} \triangleq \{\bar{y}_{R,q}^{(\pm)} \in \mathcal{Y}_R : \ b_1(k) = \pm 1\}$$

Bit Error Rate Expression

- The conditional probability density function of $y_R(k)$ given $b_1(k) = +1$ is

$$p(y_R| + 1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}}} \exp\left(-\frac{\left(y_R - \bar{y}_{R,q}^{(+)}\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}\right)$$

  where $\bar{y}_{R,q}^{(+)} \in \mathcal{Y}_R^{(+)}$ and $N_{sb} = N_b/2$ is the number of the points in $\mathcal{Y}_R^{(+)}$.

- Thus the BER is given by

$$P_E(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} Q\left(g_{q,+}(\mathbf{w})\right)$$

  where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp\left(-\frac{v^2}{2}\right) dv \quad \text{and} \quad g_{q,+}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{q,1})\bar{y}_{R,q}^{(+)}}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}$$

Minimum Bit Error Rate Beamformer

- The MBER beamforming solution is then defined as

$$\mathbf{w}_{\mathrm{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w})$$

- There exists no closed-form solution, but with the gradient

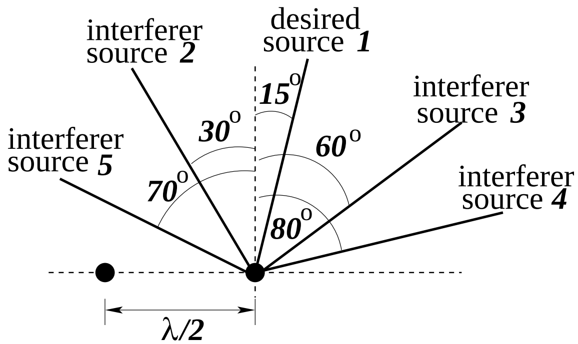
$$\nabla P_E(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}}} \sum_{q=1}^{N_{sb}} \exp\left(-\frac{\left(\bar{y}_{R,q}^{(+)}\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}\right) \mathrm{sgn}(b_{q,1}) \left(\frac{\bar{y}_{R,q}^{(+)} \mathbf{w}}{\mathbf{w}^H \mathbf{w}} - \bar{\mathbf{x}}_q^{(+)}\right)$$

  a MBER solution can be obtained iteratively using a simplified conjugated gradient algorithm.

- BER is invariant to the size of $\mathbf{w}$. Thus, if $\mathbf{w}_{\mathrm{MBER}}$ is a MBER solution, $\alpha \mathbf{w}_{\mathrm{MBER}}$ is also a MBER solution for $\alpha > 0$.

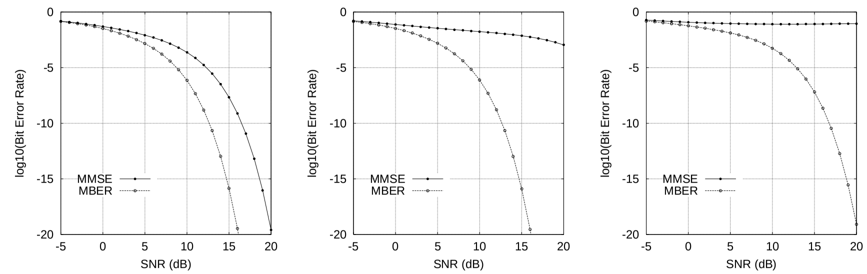
Example

Locations of the desired source and the interfering sources with respect to the two-element linear array with $\lambda/2$ element spacing, λ being the wavelength.



Definitions: SNR$= A_1^2/2\sigma_n^2$, SIR$_i = A_1^2/A_i^2$ for $i = 2, \cdots, M$.

Bit Error Rate Comparison



(a) (b) (c)

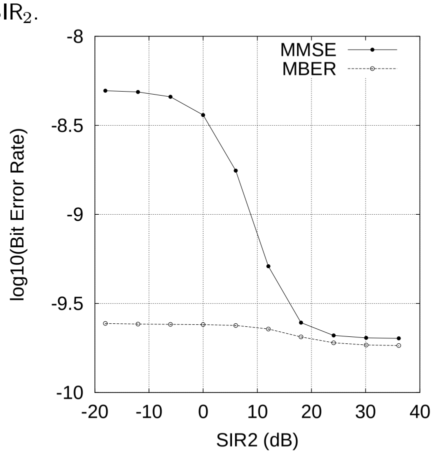
(a): SIR$_i = 0$ dB, $i = 2, 3, 4, 5$;

(b): SIR$_2 = -6$ dB and SIR$_i = 0$ dB, $i = 3, 4, 5$;

(c): SIR$_i = -6$ dB, $i = 2, 3, 4, 5$;

Near-Far Effect

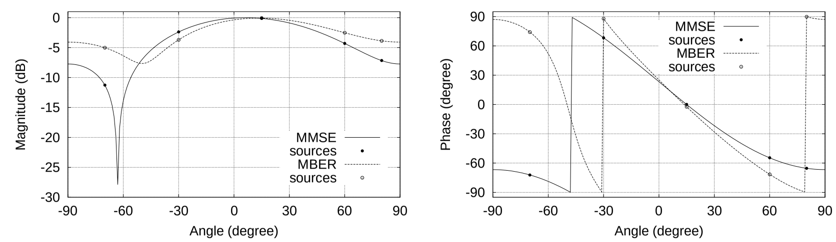
The near-far effect to bit error rate performance. SNR$= 10$ dB, SIR$_i = 24$ dB for $i = 3, 4, 5$, varying SIR$_2$.



- The MBER solution appears to be more robust to the near-far effect.

Beam Pattern Comparison

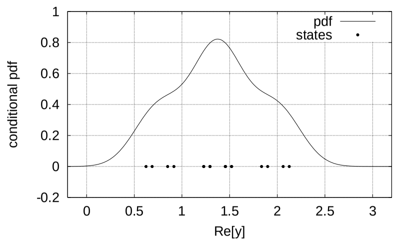
SNR$= 10$ dB, SIR$_i = 0$ dB, $i = 2, 3, 4, 5$.



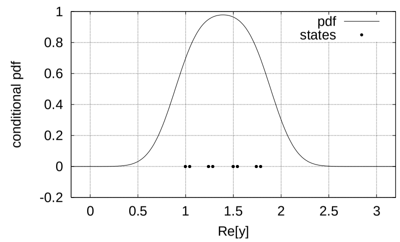
- Let $F(\theta)$ be the normalized DFT of the beamformer weight vector.
- Traditionally, the magnitude of $F(\theta)$ is used to judge the performance of a beamformer.
- Magnitude response along can be misleading, as in this case.
- At the four angles for the four interfering sources, the phase responses of the MBER solution are much closer to $\pm\frac{\pi}{2} \Rightarrow$ a much better response of $y_R(k) = \Re[y(k)]$.

Probability Density Function Comparison

Conditional probability density function of beamformer given $b_1(k) = +1$ and subset $\mathcal{Y}_R^{(+)}$. SNR$= 10$ dB, SIR$_i = 0$ dB, $i = 2, 3, 4, 5$.



(a) MMSE

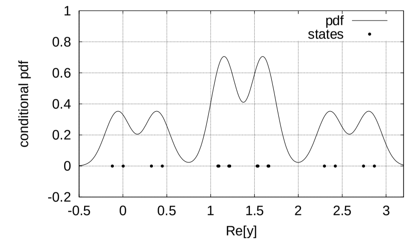


(b) MBER

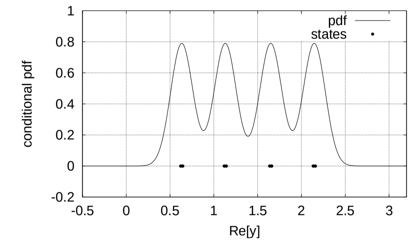
- The beamformer weight vector is normalized to a unit length, so that the BER is mainly determined by the minimum distance of the subset $\mathcal{Y}_R^{(+)}$ to the decision threshold $y_R = 0$.
- This minimum distance is much larger in the case of the MBER beamformer.

Probability Density Function Comparison

Conditional probability density function of beamformer given $b_1(k) = +1$ and subset $\mathcal{Y}_R^{(+)}$. SNR$= 15$ dB, SIR$_i = -6$ dB, $i = 2, 3, 4, 5$.



(a) MMSE



(b) MBER

- The beamformer weight vector is normalized to a unit length.
- Note that $\mathcal{Y}_R^{(+)}$ and $\mathcal{Y}_R^{(-)}$ are no longer linearly separable in the case of the MMSE beamformer, and this explains the high BER floor of the MMSE beamformer in slide 10 (c).

Block-Data Adaptive MBER Algorithm

- Given a block of K training samples $\{\mathbf{x}(k), b_1(k)\}$, a Parzen window estimate of the beamformer p.d.f. is

$$\hat{p}(y_R) = \frac{1}{K\sqrt{2\pi\rho_n^2 \mathbf{w}^H\mathbf{w}}} \sum_{k=1}^{K} \exp\left(-\frac{(y_R - y_R(k))^2}{2\rho_n^2 \mathbf{w}^H\mathbf{w}}\right)$$

  where the kernel width ρ_n is related to the noise standard deviation σ_n.

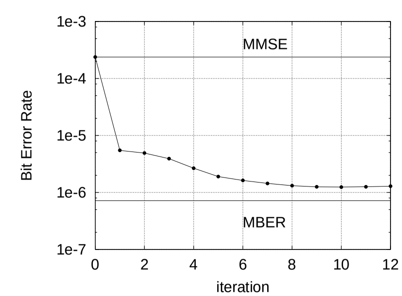
- From this estimated p.d.f., the estimated BER is given by:

$$\hat{P}_E(\mathbf{w}) = \frac{1}{K}\sum_{k=1}^{K} Q\left(\hat{g}_k(\mathbf{w})\right) \quad \text{with} \quad \hat{g}_k(\mathbf{w}) = \frac{\mathrm{sgn}(b_1(k)) y_R(k)}{\rho_n \sqrt{\mathbf{w}^H\mathbf{w}}}$$

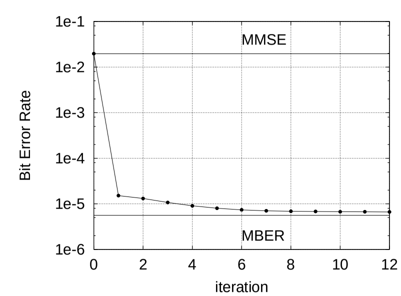
- Upon substituting $\nabla P_E(\mathbf{w})$ by $\nabla \hat{P}_E(\mathbf{w})$ in the conjugate gradient updating mechanism, a block-data based adaptive algorithm is obtained.

Convergence of Block Adaptive Algorithm

Convergence rate of the block-data based adaptive MBER algorithm for a block size of $K = 200$. The initial weight vector is set to $\mathbf{w}_{\mathrm{MMSE}}$.



(a)

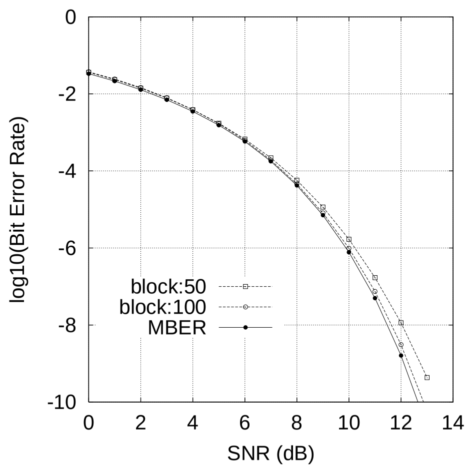


(b)

(a): SNR$= 10$ dB, SIR$_i = 0$ dB for $i = 2, 3, 4, 5$, adaptive gain $\mu = 1.0$ and $\rho_n^2 = 6\sigma_n^2 = 0.3$. (b): SNR$= 10$ dB, SIR$_3 =$SIR$_4 = 0$ dB, SIR$_2 =$SIR$_5 = -6$ dB, adaptive gain $\mu = 0.5$ and $\rho_n^2 = 2\sigma_n^2 = 0.1$.

Effect of Block Size

Effect of block size on the performance of the block-data based adaptive MBER algorithm for $\text{SIR}_2 = -6$ dB and $\text{SIR}_i = 0$ dB, $i = 3, 4, 5$.



Least Bit Error Rate Algorithm

- Consider a single-sample p.d.f. estimate of the beamformer output

$$\tilde{p}(y_R, k) = \frac{1}{\sqrt{2\pi}\rho_n} \exp\left(-\frac{(y_R - y_R(k))^2}{2\rho_n^2}\right)$$

- This leads to a single-sample BER estimate $\tilde{P}_E(\mathbf{w}, k)$.
- Using the instantaneous stochastic gradient

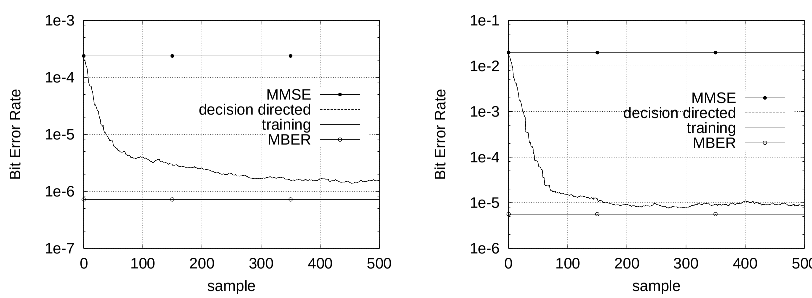
$$\nabla \tilde{P}_E(\mathbf{w}, k) = -\frac{\text{sgn}(b_1(k))}{2\sqrt{2\pi}\rho_n} \exp\left(-\frac{y_R^2(k)}{2\rho_n^2}\right) \mathbf{x}(k)$$

- leads to the LBER algorithm

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \frac{\text{sgn}(b_1(k))}{2\sqrt{2\pi}\rho_n} \exp\left(-\frac{y_R^2(k)}{2\rho_n^2}\right) \mathbf{x}(k)$$

Learning Curves of LBER Algorithm

Learning curves of the LBER algorithm averaged over 20 runs, the initial weight vector is set to $\mathbf{w}_{\text{MMSE}}$, solid curve is for training and dashed curve for decision-directed adaptation with $\hat{b}_1(k)$ substituting $b_1(k)$ (two curves are indistinguishable).



(a) (b)

(a): SNR$=10$ dB, $\text{SIR}_i = 0$ dB for $i = 2, 3, 4, 5$, $\mu = 0.03$ and $\rho_n^2 = 8\sigma_n^2 = 0.4$.
(b): SNR$=10$ dB, $\text{SIR}_3 = \text{SIR}_4 = 0$ dB, $\text{SIR}_2 = \text{SIR}_5 = -6$ dB, $\mu = 0.02$ and $\rho_n^2 = 4\sigma_n^2 = 0.2$.

Conclusions and Future Works

- An adaptive minimum bit-error-rate beamforming technique has been developed, assuming BPSK modulation scheme, narrow-band channel, and narrow-band beamformer.
- It has been demonstrated that the MBER solution can offer substantial performance improvement over the classical MMSE solution, in terms of lower achievable BER.
- A LMS-style stochastic gradient adaptive MBER algorithm, called the LBER, has been shown to perform well.
- Future works will investigate the extension to other modulation schemes, such as QPSK, and consider wideband channel and wideband beamformer.