SUBBAND-SELECTIVE PARTIALLY ADAPTIVE BROADBAND BEAMFORMING WITH COSINE-MODULATED BLOCKING MATRIX

Wei Liu, Stephan Weiss, and Lajos Hanzo

Communications Research Group
Dept. of Electronics & Computer Science
University of Southampton, SO17 1BJ, U.K.

ABSTRACT

In this paper, a novel subband-selective generalized sidelobe canceller (GSC) with cosine-modulation for partially adaptive broadband beamforming is proposed. The columns of the blocking matrix are derived from a prototype vector by cosine-modulation, and the broadside constraint is incorporated by imposing zeros on the prototype vector appropriately. These columns constitute a series of bandpass filters, which select signals with specific direction of arrivals and frequencies. This results in highpass-type bandlimited spectra of the blocking matrix outputs, which is further exploited by subband decomposition and discarding the low-pass subbands appropriately prior to running independent unconstrained adaptive filters in each non-redundant subband. By these steps, the computational complexity of our GSC implementation is greatly reduced compared to fully adaptive GSC schemes, while performance is comparable or even enhanced due to subband decorrelation in both spatial and temporal domains.

1. INTRODUCTION

Adaptive beamforming has found many applications in various areas ranging from sonar and radar to wireless communications. It is based on a technique where, by adjusting the weights of a sensor array with attached filters, a prescribed spatial and spectral selectivity is achieved. A beamformer with M sensors receiving a signal of interest from the direction of arrival (DOA) angle θ is shown in Fig. 1. To perform beamforming with high interference rejection and resolution, arrays with a large number of sensors and filter coefficients have to be employed and the computational burden of a fully adaptive processor thus becomes considerable. A popular way to reduce the computational complexity are partially adaptive beamformers, which employ only a subset of available degrees of freedom (DOF) in the filter weight update process at the expense of a somewhat reduced performance [1].

Partially adaptive techniques have been studied widely and many ideas have been proposed such as the weight reduction transformation [2] and the eigencanceller [3]. In this paper, with a GSC [4] as the underlying structure, a novel cosine-modulated blocking matrix is proposed for partially adaptive broadband beamforming. The column vectors of this blocking matrix constitute a series of cosine-modulated bandpass filters, which separate the impinging signals and interference into components of different DOAs and frequencies. This results in bandlimited spectra of the blocking matrix outputs, and is further exploited by subband decomposition and discarding the low-pass subbands appropriately prior to running independent unconstrained adaptive filters in each non-redundant subband. By these steps the computational complexity of the system is greatly reduced and a faster convergence speed is also achieved by joint spatial and spectral decorrelation.

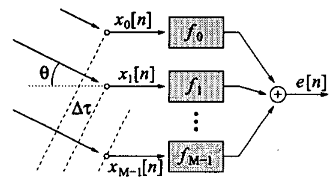

Fig. 1: A broadband beamformer with linear array.

The paper is organised as follows: Section 2 briefly reviews GSC based fully and partially adaptive broadband beamforming and then propose a special subband-selective GSC, where our novel cosine-modulated blocking matrix introduced in Sec. 3 will be applied. Finally, simulations underlining the benefit of our proposed method are discussed in Sec. 4 and conclusions drawn in Sec. 5.

2. SUBBAND-SELECTIVE GENERALISED SIDELOBE CANCELLER

2.1. Generalised Sidelobe Canceller

A linearly constrained minimum variance (LCMV) beamformer [5] performs the minimization of the variance or power of the output signal with respect to some given spatial and spectral constraints. For a beamformer with M sensors and J filter taps following each sensor, the output $e[n]$ can be expressed as

$$e[n] = \mathbf{w}^H \mathbf{x}, \tag{1}$$

where coefficients and input sample values are defined as

$$\mathbf{w} = \left[\mathbf{w}_0^T \ \mathbf{w}_1^T \ \dots \ \mathbf{w}_{J-1}^T\right]^T \tag{2}$$

$$\mathbf{w}_j = [w_{0,j} \ w_{1,j} \ \dots \ w_{M-1,j}]^T \tag{3}$$

$$\mathbf{x} = \left[\mathbf{x}^T[n] \ \mathbf{x}^T[n-1] \ \dots \ \mathbf{x}^T[n-J+1]\right]^T \tag{4}$$

$$\mathbf{x}[n-j] = [x_0[n-j] \ x_1[n-j] \ \dots \ x_{M-1}[n-j]]^T . \tag{5}$$

0-7803-7402-9/02/$17.00 ©2002 IEEE

Each vector $\mathbf{w}_j$, $j = 0(1)J-1$, contains the M *complex conjugate* coefficients sitting at the jth tap position of the M attached filters and $\mathbf{x}[n-j]$, $j = 0(1)J-1$, holds the jth data slice corresponding to the jth coefficient vector $\mathbf{w}_j$.

The linearly constrained minimum variance(LCMV) problem, popularly applied to beamforming, can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \qquad \text{subject to} \qquad \mathbf{C}^H \mathbf{w} = \mathbf{f}\ , \quad (6)$$

where $\mathbf{R}_{xx}$ is the covariance matrix of observed array data in $\mathbf{x}$, $\mathbf{C}$ is a $MJ \times J$ constraint matrix and $\mathbf{f}$ is the $J \times 1$ response vector

$$\mathbf{C} = \begin{bmatrix} \mathbf{c} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{c} \end{bmatrix} \quad \text{and } \mathbf{c} \text{ is a } M \times 1 \text{ vector.} \quad (7)$$

The constraint optimisation in (6) can be conveniently solved using a GSC [4], which performs a projection of the data onto an unconstrained subspace by means of a blocking matrix $\mathbf{B}$ and a quiescent vector $\mathbf{w}_q$ (Fig. 2). Thereafter, standard unconstrained optimisation algorithms such as least mean square (LMS) or recursive least squares (RLS) algorithms can be invoked [6]. In the GSC, the multichannel signal $\mathbf{u}$ input to the optimisation process is obtained by $\mathbf{u} = \mathbf{B}^H \mathbf{x}[n]$, whereby the blocking matrix $\mathbf{B}$ must satisfy

$$\mathbf{c}^H \mathbf{B} = \mathbf{0}. \quad (8)$$

Suppose $\mathbf{B} \in \mathbb{C}^{M \times L}$. The maximum value of L should be $M-1$, which corresponds to the fully adaptive GSC. When a large number of sensors are employed, we can take a smaller value for L, i.e. $L < M - S$, resulting in a partially adaptive GSC [2], which has a reduced number of DOFs and offers reduced complexity traded off against a somewhat inferior performance.

2.2. Partially Adaptive Subband-selective GSC

Consider a unity amplitude complex input wave with angular frequency ω and DOA angle θ. Referring to Fig. 1, the waveform impinges with a time delay $\Delta\tau$ on adjacent sensors separated by d in a medium with propagation speed c. The received phasor vector at the sensor array, $\underline{X}$, is

$$\underline{X} = \left[1 e^{-j\omega\Delta\tau} \ldots e^{-j\omega(M-1)\Delta\tau}\right]^T \text{ with } \Delta\tau = \frac{d}{c}\sin\theta. \quad (9)$$

Assume that the array sensors are spaced by half wavelength of the maximum signal frequency and the temporal sampling frequency ω_s is two times the maximum signal

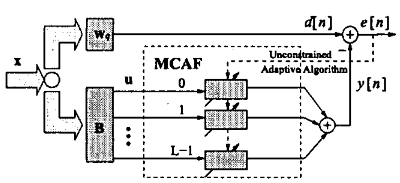

Fig. 2: A GSC structure, where adaptive optimisation is performed by a multichannel adaptive filter (MCAF).

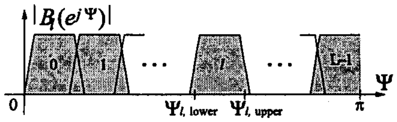

Fig. 3: Arrangement of the L column vectors in $\mathbf{B}$.

frequency, i.e. $d = \lambda_s = cT_s$, where T_s is the temporal sampling period. Then, we get $\Delta\tau = T_s \sin\theta$. Noting $\omega T_s = \Omega$, where Ω is the normalised angular frequency of the signal, the phase vector changes to

$$\underline{X} = \left[1 \quad e^{-j\Omega\sin\theta} \quad \cdots \quad e^{-j(M-1)\Omega\sin\theta}\right]^T . \quad (10)$$

Using the substitution $\Psi = \Omega\sin\theta$, the lth output of the blocking matrix, $u[l]$, $l = 0(1)L-1$, can be denoted as

$$u[l] = \mathbf{b}_l^{\mathrm{H}} \cdot \underline{X} = \sum_{m=0}^{M-1} b_l[m] \cdot e^{-jm\Psi} = B_l(e^{j\Psi}), \quad (11)$$

with $\mathbf{b}_l$ being the lth column of $\mathbf{B}$ and $B_l(e^{j\Psi}) \bullet\!\!-\!\!\circ\, b_l[m]$ is a Fourier transform pair.

When the beamformer is constrained to receive the signal of interest from broadside, the blocking matrix has to suppress any component impinging from $\theta = 0$. Therefore, at $\Psi = 0$ the response of the $\mathbf{b}_l$ has to be zero. Now we arrange these column vectors on the interval $\Psi \in [0;\pi]$ as shown in Fig. 3,

$$B_l(e^{j\Psi}) = \begin{cases} 1 & \text{for } \Psi \in [\Psi_{l,\text{lower}}; \Psi_{l,\text{upper}}] \\ 0 & \text{otherwise} \end{cases} . \quad (12)$$

From (12), we can get

$$B_l(e^{j\Omega\sin\theta}) = \begin{cases} 1 & \text{for } \Omega \in [\Psi_{l,\text{lower}}; \pi] \\ 0 & \text{otherwise} \end{cases} . \quad (13)$$

By this arrangement, the blocking matrix cannot only decompose the received signals and interference in the spatial domain, but also in the temporal domain—the column vectors simultaneously perform a temporal high-pass filtering operation. With increasing l, these filters are associated with a tighter and tighter highpass spectrum and the last output $(L-1)$ only contains the ultimate highpass component. Thus, if we decompose each of these highpass signals into subbands in a similar way as in [7], the subband signals in the corresponding lowpass subbands will be zero and can be omitted from the subsequent subband adaptive processing. The subband setup is shown in Fig. 4. The blocks labelled A perform analysis operations, splitting the signal into K frequency bands by a K−channel filter bank with decimation ratio N. Within each subband, an independent unconstrained multichannel adaptive filter(MCAF) is operated, and a synthesis filter bank, labelled S, recombines the different subsystem outputs to a fullband beamformer output $e[n]$.

Now we analysis the computational complexity of this system. For the subband decomposition and adaptation itself, it requires only K/N^2 (K/N^3) of the operations required for a fullband adaptive algorithm with a complexity of $\mathcal{O}(L_a)$ $(\mathcal{O}(L_a^2))$, where L_a is the total number of coefficients in the fullband realisation [7]. If sufficiently selective column vectors $\mathbf{b}_l$ can be designed, the first ($k =$

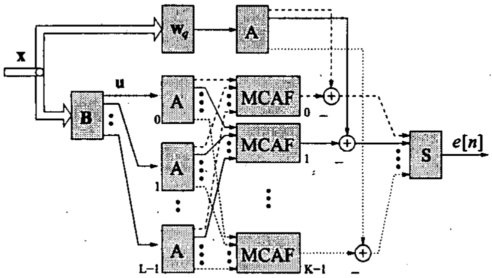

Fig. 4: A Subband-selective GSC, where an independent MCAF is applied to each subband.

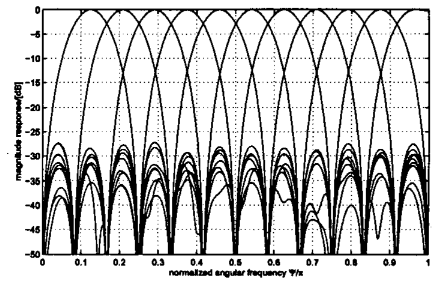

Fig. 5: A design example for a 28 × 11 blocking matrix.

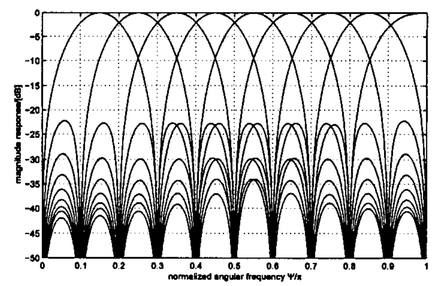

Fig. 6: Frequency responses of 19 × 9 blocking matrix.

0) MCAF would be a single channel adaptive filter drawing its low frequency input solely from the first branch of **B**. The second ($k = 1$) MCAF block in Fig. 4 will most likely only cover some of the lower branches of **B**, while finally only the last MCAF ($K - 1$) consists of L non-sparse channels. Thus, under ideal conditions, the dimensionality of the MCAFs can be reduced by half, with a proportional decrease in computational complexity. Considering the whole subband-selective GSC, it only requires $\frac{LK}{2(M-1)N^2}$ or $\frac{LK}{2(M-1)N^3}$ of computations of the traditional fully adaptive GSC schemes.

3. COSINE-MODULATED BLOCKING MATRIX

In our subband-selective GSC, the blocking matrix plays a central role and the column vector design with a good band-selective property is of great importance.

We may design each of the column vectors independently subject to the constraint (8) [8]. In order to reduce the design and implementation complexity of the blocking matrix, we here propose a cosine-modulated version. All column vectors come from a prototype vector by cosine-modulation and the broadside constraint is guaranteed by imposing zeros appropriately on the prototype vector.

Assume the prototype vector is $h[m]$, $m = 0(1)M{-}1$. It is shifted along the frequency axis by $\frac{(2l+3)\pi}{2L+2}$ and $-\frac{(2l+3)\pi}{2L+2}$, respectively and properly added to obtain the lth column vector $b_l[m]$, $l = 0(1)L{-}1$

$$b_l[m] = h[m]\cos\left[\frac{\pi}{2L+2}(2l+3)(m - \frac{M-1}{2}) - (-1)^l\frac{\pi}{4}\right]. \tag{14}$$

To comply with the broadside constraint $B_l(e^{j\Psi})|_{\Psi=0} = 0$, the frequency response $H(z)$ of $h[m]$ should have one zero at each point of $\omega_l = \pm\frac{(2l+3)\pi}{2L+2}$. If we factorize $H(z)$ into two parts

$$\begin{aligned} H(z) &= P(z)Q(z), \qquad \text{with} \\ Q(z) &= \prod_{l=0}^{L-1}(1 - e^{j\frac{2l+3}{2L+2}\pi}z^{-1})(1 - e^{-j\frac{2l+3}{2L+2}\pi}z^{-1}), \end{aligned} \tag{15}$$

then the broadside constraint will be automatically satisfied for all the column vectors and the free parameters contained in $P(z)$ can be used to optimize its frequency response. By this factorization, the design of the blocking matrix becomes an unconstrained optimization problem of the prototype vector. The objective function we minimize is

$$\Phi = \int_{\omega_s}^{\pi} \|H(e^{j\Psi})\|^2 d\Psi, \tag{16}$$

where ω_s is the stopband cutoff frequency. The optimization problem can be solved conveniently by invoking a nonlinear optimisation software package, such as the subroutines BCONF/DBCONF in the IMSL library [9]. A design example for the blocking matrix with $M = 28$ sensors, and $L = 11$ column vectors is given in Fig. 5.

Note that for $S - 1$ order derivative constraints in the blocking matrix [10], we can replace $Q(z)$ by $Q(z)^S$ in (15), but too many DOFs will be sacrificed and a satisfying performance may not be achieved for small-scale arrays.

4. SIMULATIONS AND RESULTS

The following simulations are based on a beamformer setup with $M = 19$ sensors and $J = 70$ taps for each attached sensor filter. The GSC is constrained to received a signal of interest from broadside, which is white Gaussian with unit variance. The beamformer should adaptively suppress a broadband interference signal covering the frequency interval $\Omega = [0.15\pi; 0.85\pi]$ from $\theta = 20°$ and with a singal to interference ratio(SIR) of -24 dB. Additionally, all sensors receive spectrally and spatially uncorrelated noise at 20 dB SNR. The dimension of **B** is 19×9 and its frequency response is shown in Fig. 6. We employ a 12-channel oversampled GDFT filter banks [11] with decimation factor $N = 10$. Our partially adaptive cosine-modulated subband-selective GSC is compared with two

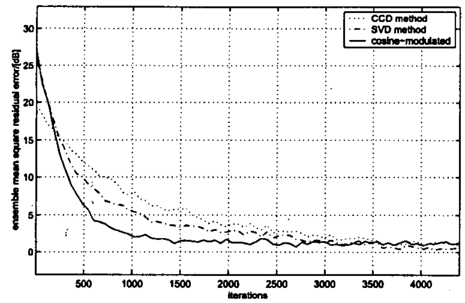

Fig. 7: Learning curves for simulation I (stepsize=0.30).

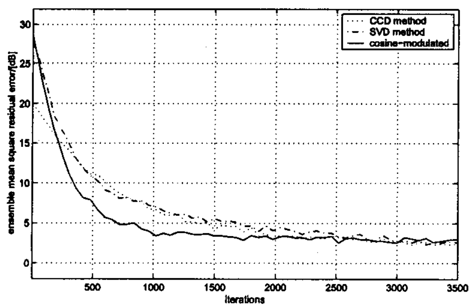

Fig. 8: Learning curves for simulation II (stepsize=0.40).

other fully adaptive GSCs based on the different building of the blocking matrix–the cascaded columns of difference (CCD) method [12] and the singular value decomposition (SVD) method [13]. The performance measure is the mean square value of the residual error, i.e. the output $e[n]$ minus the signal of interest impinging from broadside. Two simulation results are shown based on multichannel normalised LMS algorithm with a step sizes of 0.3 and 0.4, respectively. From these simulations, we can see that with the increasing of stepsize, all the GSCs converge faster, but also with a higher steady-state error. Compared with its fully-adaptive counterparts, our new method achieves a much higher convergence speed because of the combined decorrelation in both spatial and spectral domains. Because only part of the DOFs are employed, the steady-state error of our method is a little higher, which is a price we pay for the low computational complexity.

5. CONCLUSIONS

A subband-selective generalized sidelobe canceller for partially adaptive broadband beamforming with cosine-modulated blocking matrix has been proposed. In this structure, the column vectors of the blocking matrix are derived from a prototype vector by cosine-modulation and these vectors constitute a series of bandpass filters, which decompose the impinging signals into components of specific DOA angles and frequencies and lead to band-limited spectra of blocking matrix outputs. Subband methods are employed to remove their redundancy by discarding the corresponding lowpass subbands. The combination of partial adaptation, subband decomposition and discarding permit a considerably reduced computational complexity. As demonstrated in simulations, our approach also has the additional benefit of faster convergence for LMS-type adaptive algorithms employed in the GSC.

6. REFERENCES

[1] D. J. Chapman, "Partial Adaptivity for Large Arrays," *IEEE Transactions on Antennas and Propagation*, vol. 24, no. 9, pp. 685–696, September 1976.

[2] B. D. Van Veen and R. A. Roberts, "Partially Adaptive Beamforming Design via Output Power Minimization," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, pp. 1524–1532, 1987.

[3] B. D. Van Veen, "Eigenstructure Based Partially Adaptive Array Design," *IEEE Transactions on Antennas and Propagation*, vol. 36, no. 1, pp. 357–362, January 1988.

[4] L. J. Griffith and C. W. Jim, "An Alternative Approach to Linearly Constrained Adaptive Beamforming," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 1, pp. 27–34, January 1982.

[5] O. L. Frost, III, "An Algorithm for Linearly Constrained Adaptive Array Processing," *Proceedings of the IEEE*, vol. 60, no. 8, pp. 926–935, August 1972.

[6] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Englewood Cliffs, 2nd edition, 1991.

[7] W. Liu, S. Weiss, and L. Hanzo, "Subband Adaptive Generalized Sidelobe Canceller for Broadband Beamforming," in *Proc. IEEE Workshop on Statistical Signal Processing*, Singapore, August 2001.

[8] W. Liu, S. Weiss, and L. Hanzo, "A Novel Method for Partially Adaptive Broadband Beamforming," in *Proc. IEEE Workshop on Signal Processing Systems*, Antwerp, Belgium, September 2001.

[9] Visual Numerics Inc., *IMSL Fortran Numerical Libraries.*

[10] K.C. Huarng and C.C. Yeh, "Performance Analysis of Derivative Constraint Adaptive Arrays with Pointing Errors," *IEEE Transactions on Antennas and Propagation*, vol. 40, pp. 975–981, August 1992.

[11] S. Weiss and R. W. Stewart, *On Adaptive Filtering in Oversampled Subbands*, Shaker Verlag, Aachen, Germany, 1998.

[12] N. K. Jablon, "Steady State Analysis of the Generalized Sidelobe Canceler by Adaptive Noise Canceling Techniques," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 330–337, March 1986.

[13] K. M. Buckley and L. J. Griffith, "An Adaptive Generalized Sidelobe Canceller with Derivative Constraints," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 311–319, March 1986.