A Broadband Adaptive Beamformer in Subbands with Scaled Aperture

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Abstract. This paper introduces an adaptive broadband beamforming structure which permits constant resolution over a larger frequency interval spanning several octaves. Beamformers operating in uniform subbands are grouped into units within an octave. Different octave units are applied to process signals collected from arrays of appropriately scaled aperture, therefore giving a constant overall resolution. Simulation results benefits in terms of mean square error performance and cost compared to standard uniformly and non-uniformly spaced arrays.

1. INTRODUCTION

The spatial resolution of a beamformer is reciprocally proportional to both the aperture D of the sensor array collecting the data and the frequency Ω of an impinging waveform [5]. To achieve a reasonable beamforming performance in a broadband situation, in the past recommendations have been made to limit the beamformer's operation to an octave frequency interval [10, 11].

Non-uniformly spaced arrays can increase the resolution over uniformly spaced arrays with the same number of sensor elements, additionally containing computational cost, and have been investigated for more than 40 years [7, 9]. Such arrays may be obtained, for example, by judisciously thinning a uniformly spaced sensor arrangement [8, 1], which is a non-trivial and highly complex optimisation process.

The idea of this paper is to introduce a broadband beamforming structure that can be utilised over a frequency range larger than an octave by suitably subdividing the array signals into different frequency bands. Within these frequency bands, separate beamformers with apertures matched to the considered frequency band will be operated. By guaranteeing an approximately constant product $D \cdot \Omega$, the beamformer is set to achieve a nearly uniform resolution across the entire frequency range of operation. To attain a beamformer of approximately constant resolution, we divide the array signals into different frequency bands that span no more than an octave. Within each octave, a different array aperture is used by drawing the sensor signals from a non-uniformly spaced array. Thereby, higher frequent bands will be fed from closely spaced sensors of small aperture, while the low frequent bands operate on a wider spaced array of larger aperture.

Sec. 2 will briefly introduce the analysis filter banks used for subband splitting similar to [14]. In Sec. 3, the proposed constant quality beamformer is introduced. The proposed structure will be simulated and compared to a broadband beamformer operated on data collected from an array of constant aperture in Sec. 4. Conclusions are drawn in Sec. 5.

2. SUBBAND SPLITTING FOR ADAPTIVE BEAMFORMING

To permit independent beamformers to operate on various frequency bands, the sensor signals of the array are passed through filter banks. The frequency bands produced by a filter bank can be decimated due to their band-limited spectra. However, aliasing in these so called subbands limits the performance of any subsequent adaptive processing [13], and various solutions exist to mitigate the problem. In general, additional adaptive beamforming operations have to be introduced at least between adjacent subband processors which increases the computational complexity [2]. The solution preferred here are oversampled subbands, whereby the sensor signals are split into K uniform channels decimated by a factor $N < K$ [6]. The decimation ratio N is chosen such that aliasing only affects in the stopband of the analysis filters and can be suffi-

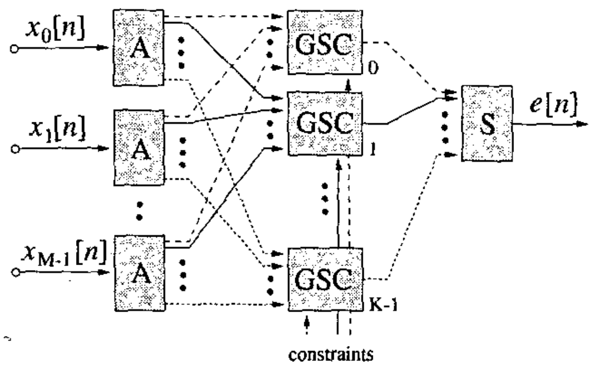

Fig. 1. Adaptive beamforming in subbands based on the generalised sidelobe canceller.

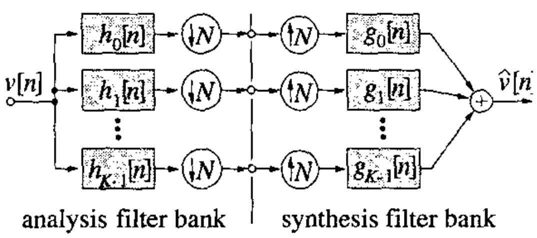

Fig. 2. Analysis and synthesis filter banks.

ciently controlled by filter bank design [3].

The resulting structure of a subband adaptive beamformer is shown in Fig. 1, where M sensor signals $x_m[n]$ are split into frequency bands by an analysis filter bank denoted by **A**. In each of these K subbands, an independent adaptive broadband beamformer, for example a generalised sidelobe canceller (GSC), is operated. Note, that the beamformers can be selected to have a temporal dimension shortened by a factor of approximately $1/N$, and are updated at an N times lower rate than a fullband processor. Hence, considerable savings may arise [14].

The output of the GSCs can be recombined to a fullband signal by means of a synthesis filter bank denoted by **S** in Fig. 1. Both blocks **A** and **S** are defined as the overall analysis and synthesis filter bank operations as given in Fig. 2.

The steady-state performance of a subband processor is generally limited by the filter bank's error in perfect reconstruction, which will introduce a distortion into the overall system of Fig. 1. Further, the alias level in the subbands will limit the adaptation of the broadband beamformer such that the possible dynamic range of subband processing is approximately given by

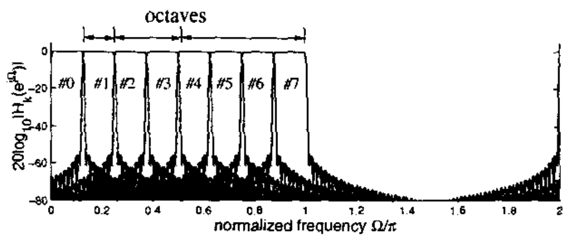

Fig. 3. Magnitude response of filter bank, only showing half the bands covering the spectrum $[0;\pi]$; the octave grouping is indicated.

the stopband attenuation of the filter banks [13]. Both perfect reconstrution and aliasing can be controlled by the filter bank design [3]. Hence implicit limitations in the subband structure can be kept below any specifications imposed by an application.

3. PROPOSED BEAMFORMING STRUCTURE

Consider the filter bank characteristic in Fig. 3 with $K = 16$ subbands. If the input to the filter bank is real valued, only the lower $K/2 = 8$ subbands need to be processed, as the remaining bands will only be complex conjugate and therefore redundant. From the depicted 8 subbands, in this case three octaves can be formed, containing subband #1, subbands #2 and #3, and subbands #4 to #7, respectively. To achieve a constant resolution, by stepping from one octave to the next-lower octave, the aperture of the array has to be doubled.

This idea is reflected in the structure shown in Fig. 4, depicting the examplary case for $M = 4$ and beamformers operating in $K/2 = 8$ subbands, whereby the array signals are drawn from a total of 8 nested sensors. For the 3 octave groups of subband beamformers (here GSCs), processor #1 operates on the lowest band, processors #2 and #3 form the second octave, and the remaining four processors are responsible for the highest octave band covered by four subbands. The aperture values for these three octave bands are $D = 3d$, $D = 6d$, and $D = 12d$. Note that the lowest band containing the non-steerable DC component is omitted from operation. As indicated in Fig. 4, not all subbands are required for processing from each sensor. For example, the last sensor recording $x_7[n]$ only contributes to the GSC #1 operating in the lowest band, where the largest aperture is utilised.

Generalising this scaled aperture subband beam-

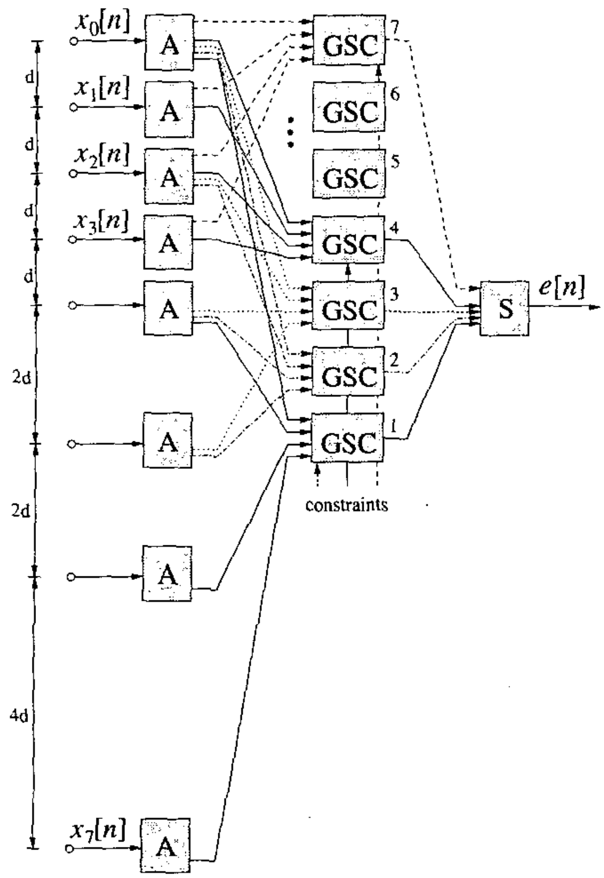

Fig. 4. Proposed broadband adaptive beamforming structure with scaled array aperture for processors in various octave bands.

former, with K uniform subbands, where K is a power of two such that $K = 2^{\kappa}$, we form κ octave bands for separate arrays and beamforming processors. The lowpass band $k = 0$ is omitted, and thereafter the subband groups of indices $k \in \{2^{\nu-1}; 2^{\nu} - 1\}$, $\nu = 0(1)(\kappa - 1)$ cover the subsequent octave frequency bands. Descending in frequency from the highest band with aperture $(M-1) \cdot d$, with each octave the aperture of the array is doubled, as mentioned before and shown in Fig. 4.

4. SIMULATION RESULTS

In the following, we simulate the proposed structure given in Fig. 4 and compare results to fixed aperture structures in both full- and subbands, as well as a non-uniformly spaced fullband array. The given array is organised as shown in Fig. 4, with a total of $M = 8$ non-uniformly spaced sensors. Fixed structures used as a benchmarker will employ the first four sensor signals with a constant aperture of $D = 3d$, and both the subband scaled aperture beamformer and the fullband non-uniformly spaced array will be based on all 8 depicted sensors.

In our simulation scenario, a broadband source of interest impinges onto the array from broadside, $\vartheta = 0°$, corrupted by a spectrally coloured broadband interferer from a direction of arrival (DOA) $\vartheta = 10°$ at -40 dB SINR and spatially and temporally uncorrelated noise at 10 dB SNR. Both broadband sources are restricted to a normalised frequency range $\Omega = \{0.125\pi; 0.875\pi\}$.

The various beamforming structures to be considered here for comparison can be characterised as follows:

Fullband fixed aperture. A fullband GSC is applied to the first $M = 4$ sensors, whereby the selected temporal dimension is $L = 140$ coefficients. A standard GSC algorithm in combination with a normalised least-mean squares (NLMS) update of the coefficients [4] is used. As a result, the system complexity is $(M \cdot L)^2 + 3M \cdot L$ multiply-accumulates (MACs), whereby the blocking matrix of dimension $ML \times ML$, the quiescent vector of length ML and the NLMS coefficient update are considered.

Fullband non-uniformly spaced. The fullband GSC is applied to all $M = 8$ non-uniformly spaced sensors and hence has a considerably larger aperture than the previous structure.

Subband fixed aperture. The first $M = 4$ sensor signals are decomposed into a total of $K = 16$ complex valued subbands. Due to real valued signals $x_m[n]$, half of these signals are redundant and only $K/2 = 8$ subbands decimated by $N = 14$ need to be processed. The temporal dimension L decreases approximately by a factor of N w.r.t. a fullband beamformer, but additionally 5 filter bank operations are required.

Subband scaled aperture. With its structure given in Fig. 4, only an extra 4 filter bank operations are required over the subband fixed aperture architecture. It shares the same number of subbands as the latter system.

The directivity pattern demonstrating the beamformers' gain response after convergence as a function of frequency and DOA is given in Figs. 5 and 6. Fig. 5 shows the behaviour of a subband beamformer with

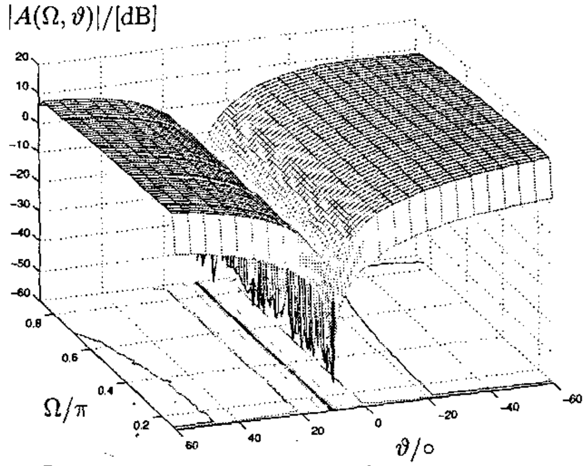

Fig. 5. Directivity pattern of subband beamformer with fixed aperture.

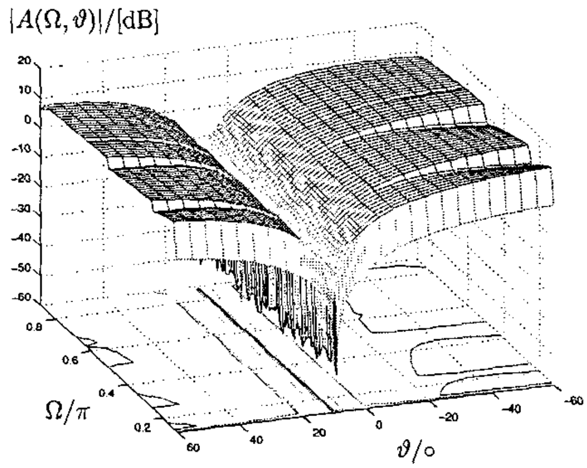

Fig. 6. Directivity pattern of subband beamformer with scaled aperture.

fixed aperture operating in the scenario outlined above. Clearly, in the direction of the inferference coming from a DOA of $-10°$, a null is placed, and the 0 dB constraint from broadside is fulfilled. Further note that the beamformer gain is smooth across the band edges of the various subbands. For the subband scaled aperture system, the octave behaviour is clearly visible from the directivity pattern in Fig. 6. Particularly for the lower frequencies it is evident that the larger aperture of the array leads to a better resolution and slightly lower overall gain compared to the standard subband beamformer in Fig. 5. Hence, the proposed structure provides an improved noise performance.

The learning characteristics of the four different beamforming structures are given in Fig. 7. The adaptation of the various beamformers is switched on at $n = 0$, and at $n = 20000$, the step size is reduced by an order of magnitude. Fig. 7 gives the mean square value of the residual error signal, which is the beamformer output subtracted from the desired broadside signal.

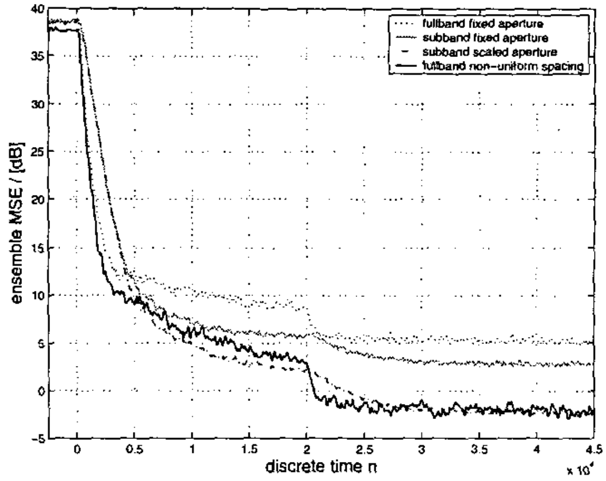

Fig. 7. Learning curves of beamformers showing the ensemble mean square value of the residual error.

structure	MAC/sample	rel. cost
fullband fixed	315280	100.00%
fullband non-uniform	1257760	400.00%
subband fixed	14034	4.45%
subband scaled	14250	4.52%

Tab. 1. Computational complexities in terms of multiply-accumulates (MACs), and relative relatation to a fullabnd fixed quadratude array.

Although the fullband structures exhibit an initially faster convergence, the subband structures in general show a better steady-state performance. The non-uniformly spaced fullband beamformer reaches similar performance levels as achieved by the proposed scaled aperture array, however the number of sensors are effectively doubled.

The power spectral densities of the various steady-state errors are presented in Fig. 8. For the fixed aperture arrays with $M = 4$ sensors, the low resolution at low frequencies makes interference cancellation difficult, and the remaining error is large. The non-uniform array shows a good performance, although the best cancellation is achieved at mid-frequency range. Finally, the scaled aperture array gives a fairly even

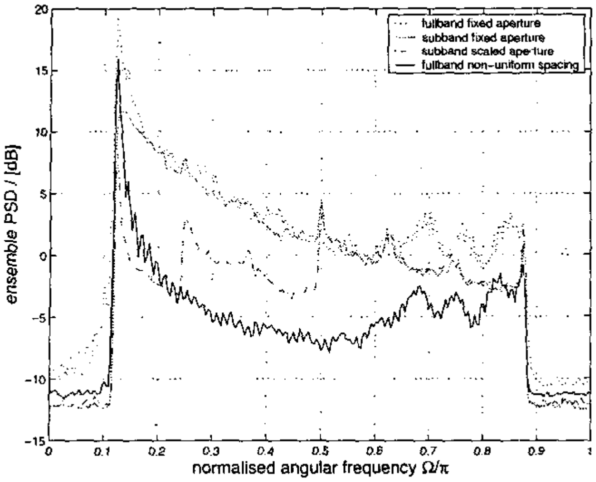

Fig. 8. Power spectral density of the residual error signal after convergence.

distribution, whereby the octave structure in the PSD plot is visible, as within an octave, the higher frequency ranges are better suppressed.

Based on the MSE in Fig. 7, it seems that no significant overall performance advantage of the subband scaled array over a non-uniformly spaced beamformer has been achieved. If however the computational cost of the various structures is considered, whereby a filter bank operation is given by $\frac{1}{N}(4K\ log_2K + 4K + L_p)$ multiply-accumulate (MAC) operations [12] with L_p being the length of the filter bank filters, then due to the cost comparison in Tab. 1., a clear advantage arises for the scaled aperture array.

5. CONCLUSIONS

The poor resolution of broadband arrays at low frequencies has motivated the introduction of a scaled aperture array, which is designed to have a reasonably constant resolution across the frequency range of operation. Simulation results demonstrate the correct performance and desired characteristics of the system, whereby similarly dimensioned beamformers with fixed aperture are considerably outperformed. A fullband non-uniform array with a larger number of overall sensors achieved similar performance results, however at a considerably higher computational cost.

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