SUMMARY Quadrature Amplitude Modulation (QAM) schemes are attractive in terms of bandwidth efficiency and offer a number of subchannels with different integrations via both Gaussian and Rayleigh-fading channels. Specifically, the 16-QAM phasor constellation has two, while the 64-QAM possesses three such subchannels, which become dramatically different via Rayleigh-fading channels. The analytically derived bit error rate (BER) formulae yield virtually identical curves with simulation results, exhibiting adequate BERs for the highest integrity subchannels of both 16-QAM and 64-QAM to be further reduced by forward error correction coding (FEC). However, the BERs of the lower integrity subchannels require fading compensation to reduce their values for FEC techniques to become effective. This property creates ground for a variety of carefully matched, embedded mobile transmission schemes of different complexities.

The practical implementation of such an embedded scheme is demonstrated by a low-cost, low-complexity and low-consumption 50 KBd mobile video telephone scheme offering adequate speech and image quality for channel SNRs in excess of about 20 dB via Rayleigh-fading channels.

key words: QAM theory, modulation for microcellular fading channels

1. Introduction

Bandwidth efficient Quadrature Amplitude Modulation (QAM) schemes have been extensively deployed via transmission media, where the prevailing channel impairments are linear distortions and Additive White Gaussian Noise (AWGN). This is the case via band and power limited satellite channels or cables, for example. The theory as well as implementation of such systems is well understood and the penalty of less power efficient linear amplifiers is accepted for the benefit of high bandwidth efficiency.(1)

In mobile radio and mobile satellite systems the bandwidth efficiency is at absolute premium, but at the same time they possess hostile Rayleigh-fading channels, characterised by Rayleigh envelope probability densities (PDF) and uniformly distributed random phase.(2),(3) Until quite recently, however attractive in terms of bandwidth efficiency, QAM has been thought of as unsuited for mobile channels due to the high bit error rates (BER) inflicted by the violent amplitude and phase fluctuations, even if the less power efficient Class-A amplification is found acceptable.

However, recent trends in mobile radio communica
tions resulted in the introduction of friendlier, high capacity microcellular structures with higher signal-to-noise ratios (SNR) than in conventional large-cell environments(4). Through low transmitted powers, high SNRs and moderate excess delays microcells pave the way for lightweight, possibly solar-charged, handheld portable phones of the near future.

In this friendlier propagation environment the deployment of multilevel QAM schemes with robust speech(5)–(7) and video source codecs(8) using carefully embedded forward error correction coding (FEC) rewarded us with robust mobile audio and video telephone schemes. A comparative study of various ReedSolomon and trellis coded 16- and 64-level QAM schemes utilising a number of fading-compensation techniques was given in Ref. (9).

In this contribution we focus our attention on the theoretical aspects of using the ubiquitous maximum Euclidean distance 16- and 64-level 'square' constellations, seen in Fig. 1 and 3, respectively, via Rayleigh-fading channels. For details of their practical deployment in various speech and video schemes the interested reader is referred to Refs. (5)–(9).

2. Square Constellation 16-QAM

In the well-known 16-QAM square constellation of Fig. 1 each constellation state is represented by a four-bit symbol, constituted by the in-phase bits $i_1, i_2$ and quadrature bits $q_1, q_2$. The quaternary quadrature components $I$ and $Q$ are Gray-encoded by assigning the bits 01, 00, 10 and 11 to the levels $3d, d, -d$ and $-3d$, respectively, in a similar fashion to a Karnaugh-table, where the Gray coding ensures that corrupting a transmitted phasor into any of its nearest neighbours, which is the most likely error event, results in a single bit error. QAM demodulation is carried out by independently demodulating both quadrature components $I$ and $Q$ against their respective decision boundaries seen in Fig. 1. Closer scrutiny of this figure reveals that half of the time the bits $i_1, i_2$ have a noise protection distance of $d$ from their decision boundaries, while the half of the time this distance is $3d$. As opposed to this, the bits $q_1, q_2$ are always at a protection distance of $d$ from the decision boundaries and hence are more prone to errors. Due to this property, our

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square 16-QAM scheme can be considered, as the amalgamation of two subchannels with different integrities, which we refer to as class 1 and class 2 (C1 and C2) subchannels. The demodulation process is described as regards to the C1 subchannel in analytical terms as follows:

\[
\begin{align*}
\text{if } I, Q \geq 0 & \quad \text{then } i_1, q_1 = 0 \\
\text{if } I, Q < 0 & \quad \text{then } i_1, q_1 = 1
\end{align*}
\] (1)

Taking into account the decision boundaries for the 3rd and 4th bits \( i_2, q_2 \), respectively, as shown in Fig. 1, we have:

\[
\begin{align*}
\text{if } I, Q \geq 2d & \quad \text{then } i_2, q_2 = 1 \\
\text{if } -2d \leq I, Q < 2d & \quad \text{then } i_2, q_2 = 0 \\
\text{if } -2d > I, Q & \quad \text{then } i_2, q_2 = 1.
\end{align*}
\] (2)

2.1 Demodulation in the Presence of AWGN

Upon demodulation in the C2 subchannel, a bit error will occur if the noise exceeds \( d \) in one direction or \( 3d \) in the opposite direction, where the latter probability is insignificant, and hence the C2 BER probability becomes

\[
P_{2C} = Q(d/\sqrt{N_0}/2) + \frac{1}{2} Q(3d/\sqrt{N_0}/2)
\] (5)

The C1 error probability is derived by assuming equiprobable random phasors, i.e. that the bits \( i_1, q_1 \) are half of the time at a protection distance of \( d \) from the decision boundaries, and half the time the protection distance is \( 3d \), yielding:

\[
P_{1C} = \frac{1}{2} Q(\sqrt{E_0}/\sqrt{N_0}/2) + \frac{1}{2} Q(3\sqrt{E_0}/\sqrt{N_0}/2)
\] (6)

The C1 and C2 bit error probabilities \( P_{1C} \) and \( P_{2C} \), computed from Eqs. (5) and (6) are displayed in Fig. 2, along with the average AWGN error probability \( P_C \), expressed as:

\[
P_C = (P_{1C} + P_{2C})/2.
\] (7)

Our simulation results gave practically identical curve to those in Fig. 2, where the C2 subchannel shows consistently worse performance than the C1. This performance difference becomes more pronounced in fading channels, because while the C1 performance continues to be governed by the noise due to its high average protection distance, the C2 performance suffers from the violent envelope fluctuations.

2.2 Demodulation in Rayleigh-Fading

Assuming a non-frequency-selective fading channel, where the signal bandwidth is much lower than the coherence bandwidth of the channel, the fading channel's transfer function is of the form:

\[
c(t) = \alpha(t) \cdot e^{-j\phi(t)}
\] (8)
where $\alpha(t)$ represents the Rayleigh-fading envelope and $\Phi(t)$ the phase of the channel. It is general practice to assume that the phase is uniformly distributed over $[-\pi, \pi]$. If additionally the fading is slow, so that $\alpha(t) = \alpha$ and $\Phi(t) = \Phi$ for the duration of one signalling interval, the received signal $r(t)$ at the channel's output is:

$$r(t) = \alpha(t) e^{-j\Phi(t)} m(t) + n(t),$$

where $m(t)$ is the transmitted modulated signal and $n(t)$ is the AWGN.

On rewriting Eqs. (5) and (6) derived for the C1 and C2 subchannels as a function of $E_b/N_0$ in terms of the instantaneous SNR $\gamma$ via Rayleigh-fading channels, we get:

$$P_{1R}(\gamma) = \frac{1}{2} \left[ Q\left(\sqrt{\gamma} / 5 \right) + Q\left(3\sqrt{\gamma} / 5 \right) \right],$$

and

$$P_{2R}(\gamma) = Q\left(\sqrt{\gamma} / 5 \right).$$

The instantaneous SNR $\gamma$ fluctuates with the fading envelope, and the average BER, $P_{be}$, can be calculated by finding the bit error probability at a given instantaneous SNR $\gamma$ and averaging it over all possible SNRs. Multiplying the bit error probability $P(\gamma)$ at the instantaneous SNR $\gamma$ with the occurrence probability of the specific SNR $\gamma$ expressed in terms of its probability density function (PDF) $C(\gamma)$ and then integrating this product over the range of $\gamma$, we have:

$$P_{be} = \int_{0}^{\infty} P(\gamma) \cdot C(\gamma) \, d\gamma.$$  \hfill (12)

The random variable $\gamma = \alpha^2 E_0 / N_0$ is a transformed form of the fading envelope $\alpha$ and the PDF of $\alpha$, $C(\alpha)$ is known to be Rayleigh:

$$C(\alpha) = \left(\frac{\alpha}{\alpha_0}\right) e^{-\alpha^2 / 2\alpha_0^2},$$

where $\alpha_0$ is the variance of $\alpha$ and its second moment is given by $E(\alpha^2) = \alpha_0^2 = 2\alpha_0^2$. As $\gamma = \alpha^2 E_0 / N_0$ has two real roots, $\alpha_1$ and $\alpha_2$, but the Rayleigh distribution does not exist in the negative domain, $\alpha_2$ can be eliminated and hence the PDF of the instantaneous SNR, $C(\gamma)$, is computed as

$$C(\gamma) = C(\alpha_1) \cdot \left| \frac{d\alpha_1}{d\gamma} \right| = \frac{1}{2 \alpha_0^2} \frac{N_0}{E_0 \gamma} e^{-N_0 \gamma / 2 \alpha_0^2 E_0}.$$  \hfill (14)

We now define the average SNR $\Gamma$ as:

$$\Gamma = \bar{\gamma} = \gamma = E\left\{ \gamma^2 \right\} = \alpha^2 E_0 / N_0 = 2\alpha_0^2 E_0 / N_0$$

where we exploited that $E(\gamma^2) = \alpha^2 = 2\alpha_0^2$ for a Rayleigh PDF, then the transformed PDF, $C(\gamma)$, is given as below:

$$C(\gamma) = \frac{1}{\Gamma} e^{-\gamma / \Gamma},$$

which is known as the Chi-square distribution, with the instantaneous SNR $\gamma \geq 0$.

The C1 BER in Rayleigh-fading is then computed by substituting $C(\gamma)$ from Eq.(16) and $P_{1R}(\gamma)$ over Rayleigh-fading channels from Eq.(10), into Eq.(12), yielding:

$$P_{1R}(\Gamma') = \frac{1}{2\Gamma} \int_{0}^{\infty} \left[ Q\left(\sqrt{\gamma} / 5 \right) + Q\left(3\sqrt{\gamma} / 5 \right) \right] e^{-\gamma / \Gamma} \, d\gamma.$$  \hfill (17)

The computation of the C2 BER is more complicated, since demodulation is now carried out on an attenuated signal constellation with the modified decision boundaries given below:

- If $I, Q \geq 2\tilde{a}d$ then $i_2 = 1$
- If $-2\tilde{a}d \leq I, Q < 2\tilde{a}d$ then $i_2 = 0$
- If $-2\tilde{a}d > I, Q$ then $i_2 = 1$.

Considering the case when the transmitted bit $i_2$ or $q_2$ is a logical 0 and assuming that $\alpha < 2\tilde{a}$, the protection distance between the received phasor $\alpha \cdot d$ and the decision boundary $2\tilde{a}d$ becomes $d = (2\tilde{a}d - \alpha d)$. Upon substituting $d$ into Eq.(3) instead of $d$, we get the C2 BER for the transmission of a logical 0 as

$$P_{2,0,0} = Q\left( \frac{d}{\sqrt{N_0 / 2}} \right) = Q\left( \frac{d}{\sqrt{N_0 / 2}} \right).$$

From $E_0 = 10d^2$, $d = \sqrt{E_0 / 10}$, and whence:

$$P_{2,0,0} = Q\left( \frac{2\tilde{a} - \alpha}{\sqrt{5 N_0}} \right).$$

From $\gamma = \alpha^2 E_0 / N_0$ we get $E_0 / N_0 = \gamma / \alpha^2$, and therefore:

$$P_{2,0,0} = Q\left( \frac{2\tilde{a} - \alpha}{\alpha} \sqrt{\gamma / 5} \right) = Q\left( \frac{2\tilde{a} - \alpha}{\alpha} \sqrt{\gamma / 5} \right).$$

Upon substituting Eqs.(21) and (16) into Eq.(12) we get one component of the total C2 error probability. First the upper integration boundary of Eq.(12) is derived for $\alpha \leq 2\tilde{a}$ as

$$\gamma = \alpha^2 E_0 / N_0 \leq 4(\tilde{a})^2 E_0 / N_0.$$  \hfill (22)

By taking into account that for a Rayleigh distribution $\tilde{\alpha} = \alpha_0 \sqrt{\pi / 2}$:

$$\gamma = 4\left( \frac{\alpha_0 \sqrt{\pi / 2}}{2} \right)^2 E_0 / N_0 = 2\alpha_0^2 E_0 / N_0 = \Gamma \pi,$$

and

$$P_{2,0,0}(\Gamma') = \int_{0}^{\infty} P(\gamma) \cdot C(\gamma) \, d\gamma$$
Similarly, for $d_1 = (2\bar{a}d - \alpha d) < 0$, i.e., $\alpha > 2\bar{a}$, we have

$$P_{2,0,>2} = 1 - \left[ 1 - Q\left( \frac{d(2\bar{a} - \alpha)}{\sqrt{N_0/2}} \right) \right]$$

or

$$P_{2,1,>2/3} = 1 - Q\left( \frac{3\bar{a}}{\alpha - 2} \sqrt{\frac{\gamma}{5}} \right).$$  \hspace{1cm} (31)

Before computing the total $C_2$ bit error probability for the transmission of a logical one, we determine the integration limit for Eq. (12). Since $\alpha > 2\bar{a}/3$,

$$\gamma = a^2 \frac{E_0}{N_0} = \left( \frac{2}{3\bar{a}} \right)^2 \frac{E_0}{N_0} = \frac{4}{9} \frac{\alpha^2}{a^2} \frac{E_0}{N_0}.$$  \hspace{1cm} (32)

By exploiting for the Rayleigh distribution that $\bar{a} = a_0 \sqrt{\pi/\gamma}$, we get:

$$\gamma = \frac{4}{9} \frac{\alpha^2}{a_0^2} \frac{\pi}{\Gamma} \frac{E_0}{N_0} = \frac{\pi}{9}.$$

Therefore:

$$P_{2,1}(\Gamma') = P_{2,1,>2/3}(\Gamma') + P_{2,1,>2/3}(\Gamma'),$$  \hspace{1cm} (34)

or

$$P_{2,1}(\Gamma') = \frac{1}{T} \int_{0}^{\pi/2} Q\left( \frac{3\bar{a}}{\alpha - 2} \sqrt{\frac{\gamma}{5}} \right) e^{-\pi r/d} d\gamma$$

and

$$P_{2,0}(\Gamma') = \frac{1}{T} \int_{0}^{\pi/2} \left[ -Q\left( \frac{3\bar{a}}{\alpha - 2} \sqrt{\frac{\gamma}{5}} \right) \right] e^{-\pi r/d} d\gamma.$$  \hspace{1cm} (35)

For random transmitted data the overall $C_2$ subchannel error probability is given by:

$$P_{2,0}(\Gamma') = \frac{1}{2} \left[ P_{2,1}(\Gamma') + P_{2,0}(\Gamma') \right].$$  \hspace{1cm} (36)

When we computed the $C_1$ and $C_2$ bit error probabilities from Eqs. (17) and (36), respectively, we received nearly identical curves to those, depicted in Fig. 2 for our simulations. Observe that the BER difference between the $C_1$ and $C_2$ subchannels becomes more profound than via AWGN channels and the $C_1$ BER is sufficiently low to be further reduced by forward error correction coding (FEC), while the $C_2$ performance requires fading-compensation to be deployed to mitigate the channel impairments.

3. Square Constellation 64-QAM

In 64-level QAM (64-QAM) systems the phasors are represented by 6-bit symbols, Gray-coded to minimise the decoded error probability, as seen in Fig. 3. The 6-bit complex phasors are decomposed into the octal $I$ and $Q$ components. The amplitudes $7d, 5d, 3d, d, -d, -3d, -5d$ and $-7d$ of the $I$ and $Q$ components are assigned the 3-bit Gray codes $011, 010, 000, 001, 101, 100, 110$ and $111$, respectively, again, similarly to a Karnough table. Lastly, the three $I$ and $Q$ bits are interleaved to give a 6-bit QAM symbol represented by $i_1, q_1, k, q_2, k, q_0$. Similarly to 16-QAM, demodulation is carried out using the decision boundaries shown in Fig. 3 and the equations below:
Fig. 3 64-Level QAM constellation.

if $I, Q \geq 0$ then $i_1, q_1 = 0$

if $I, Q < 0$ then $i_1, q_1 = 1$, (37)

for the most significant bits, and

if $I, Q \geq 4d$ then $i_2, q_2 = 1$

if $-4d \leq I, Q < 4d$ then $i_2, q_2 = 0$

if $-4d > I, Q$ then $i_2, q_2 = 1$, (38)

for the next most significant bits, and finally for the least significant bits

if $I, Q \geq 6d$ then $i_3, q_3 = 1$

if $2d \leq I, Q < 6d$ then $i_3, q_3 = 0$

If $-2d \leq I, Q < 2d$ then $i_3, q_3 = 1$

if $-6d \leq I, Q < -2d$ then $i_3, q_3 = 0$

if $-6d > I, Q$ then $i_3, q_3 = 1$. (39)

As seen for 16-QAM before, the position of the bits in the 6-bit QAM symbol has a major effect as regards to their error probabilities. For example, the $i_1, q_1$ C1 bits can have a protection distance of $d$, $3d$, $5d$ or $7d$ from the C1 decision boundary, giving an average 'protection distance' of $16d/4 = 4d$. This average protection distance for the C2 $i_2, q_2$ bits is $2d$, while for the C3 $i_3, q_3$ bits it is $d$. Clearly, our 64-QAM modem is constituted by three subchannels of different integ-
Assuming perfect coherent detection let us consider first the C3 subchannel's BER with its decision boundaries of $-6d$, $-2d$, $2d$ and $6d$, where each QAM symbol has a different error probability, depending on its position in the constellation of Fig. 3. For example, bit $i_3$ of phasor $P_5$, which is a logical 1, will be corrupted, if a noise sample with an amplitude in excess of $d$ is added to it, yielding a decoding to $P_6$. However, when it is carried by a noise vector having larger than $7.5d$ amplitude over the C3 decision boundary at $6d$, the bit-decision becomes error-free again, with the erroneous decoding in $P_8$. In the opposite direction the decisions are error-free, until the noise amplitude reaches $-3d$, where it becomes interpreted as phasor $P_3$. For noise amplitudes in the range between $-3d$ and $-7d$ there are erroneous decisions, but once the $-6d$ decision boundary is exceeded, due to a negative noise sample larger than $-7d$, the received sample falls into the error-free $P_5$ domain again, giving the erroneous decoded phasor $P_6$. Based on these arguments, the $i_3$ bit-error probability of the $P_5$ phasor is given by:

$$P_{e5} = Q[d/\sqrt{N_0/2}] + Q[3d/\sqrt{N_0/2}] - Q[5d/\sqrt{N_0/2}] - Q[7d/\sqrt{N_0/2}]$$

with $N_0$ being the one-sided Gaussian noise spectral density. The average symbol energy for 64-QAM is found to be $E = 42d^2$, whence substituting $d = \sqrt{E/42}$ into Eq.(40), and introducing the average SNR $\gamma = E/N_0$ gives:

$$P_{e5} = Q[\sqrt{\gamma/21}] + Q[3\sqrt{\gamma/21}] - Q[5\sqrt{\gamma/21}] - Q[7\sqrt{\gamma/21}]$$

As another example, we consider phasor $P_8$, where the situation is entirely different, because its C3 bit $i_3$ is never corrupted by positive noise samples. On the other hand, for negative noise samples it cycles through erroneous and error-free zones, when the noise increases past $-d$, $-5d$, $-9d$ and $-13d$, as seen in Fig. 3, yielding an error probability of:

$$P_{e8} = Q[\sqrt{\gamma/21}] + Q[9\sqrt{\gamma/21}] - Q[5\sqrt{\gamma/21}] + Q[13\sqrt{\gamma/21}]$$

Based on a similar approach, the error probabilities $P_{e6}$ and $P_{e7}$ are computed as follows:

$$P_{e6} = Q[\sqrt{\gamma/21}] + Q[3\sqrt{\gamma/21}] - Q[5\sqrt{\gamma/21}] + Q[9\sqrt{\gamma/21}]$$

$$P_{e7} = Q[\sqrt{\gamma/21}] + Q[3\sqrt{\gamma/21}] - Q[7\sqrt{\gamma/21}] + Q[11\sqrt{\gamma/21}]$$

The error probabilities $P_{e1}$, $P_{e2}$, $P_{e3}$, $P_{e4}$ are equivalent to those given by $P_{e6}$, $P_{e7}$, $P_{e6}$, $P_{e5}$, respectively, and the same holds for all corresponding phasors in the columns of the phasor diagram of Fig. 3. Furthermore, the $q_3$ bit error probability is identical to that of $i_3$, if independent random sequences are transmitted. Averaging the C3 bit error probabilities yields:

$$P_{c3}(\gamma) = Q[\sqrt{\gamma/21}] + \frac{3}{4} Q[3\sqrt{\gamma/21}] - \frac{3}{4} Q[5\sqrt{\gamma/21}] - \frac{1}{4} Q[7\sqrt{\gamma/21}]$$

$$+\frac{1}{4} Q[9\sqrt{\gamma/21}] - \frac{1}{4} Q[11\sqrt{\gamma/21}] - \frac{1}{4} Q[13\sqrt{\gamma/21}]$$

where we observe that the last three terms in Eq. (45) represent extremely unlikely events as the Gaussian noise sample must exceed the $9d$ protection distance of the best protected C1 bit. Therefore these terms are neglected in our further calculations. The $C_1$ and $C_2$ BER in AWGN is derived using our previous approach to yield:

$$P_{c2}(\gamma) = \frac{1}{2} Q[\sqrt{\gamma/21}] + \frac{1}{2} Q[3\sqrt{\gamma/21}] + \frac{1}{4} Q[5\sqrt{\gamma/21}] + \frac{1}{4} Q[7\sqrt{\gamma/21}]$$

$$+ \frac{1}{4} Q[9\sqrt{\gamma/21}] + \frac{1}{4} Q[11\sqrt{\gamma/21}]$$

The $P_{c1}$, $P_{c2}$ and $P_{c3}$ error probability vs. channel SNR performances evaluated using Eqs.(45) - (47) are plotted in Fig. 4, which differ from our simulation results only by a fraction of a dB. In harmony with our

![Fig. 4 C1, C2, and C3 BER via AWGN and Rayleigh channels vs. SNR.](image-url)
expectations, there is consistent, but modest BER advantage when using the C1 subchannel. All subchannels have BERs < 10^{-3} for SNR values in excess of 23 dB and hence they are sufficiently low for speech transmission. As we will see in the next section, the higher protection distance of the C1 subchannel withstands the envelope fading and the prominance of the C1 subchannel becomes more pronounced via Rayleigh channels, as we experienced in case of 16-QAM.

3.2 64-QAM Demodulation in Rayleigh-Fading

Adopting an approach similar to that used for the 16-QAM in Sect. 2, the formula for the C1 BER via Rayleigh channels can be obtained directly by substituting the Gaussian channel result of Eq. (47) into Eq. (12) along with \( C(\gamma) \) in Eq. (16), to arrive at:

\[
P_{C1}(\Gamma) = \frac{1}{4\Gamma} \int_{0}^{\infty} [Q(\sqrt{\gamma}/21) + Q(3\sqrt{\gamma}/21)] \\
+ Q(5\sqrt{\gamma}/21) + Q(7\sqrt{\gamma}/21)] e^{-\gamma/d}\gamma.
\]  

(48)

Clearly, the C1 decision boundaries constituted by the coordinate axis are unaltered by the fading, but the probability of bit error for the C2 and C3 cannot be derived without considering the effects of fading on each phasor and decision boundary individually. Here we remind the reader that although the received phasors are attenuated by the instantaneous fading \( a \), demodulation is carried out with reference to the expected value \( 2\alpha \).

Considering \( P_6 \) the modified protection distance \( d_1 \) measured from the decision boundary \( 4\alpha \) is seen from Fig. 3 to be \( d_1 = 4\alpha \delta - 3\alpha d \), which, when overcome by noise, leads to erroneous decisions. If \( d_1 < 0 \), then in the absence of noise the decisions are always erroneous, while in the presence of noise the phasors can be carried back into the error-free decision zone. In both cases the error probability is given in terms of the \( Q(\gamma) \) function:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{4\alpha}{\alpha} - 3\right)\sqrt{21}\right].
\]  

(49)

As regards to phasor \( P_3 \) the modified protection distance from the C2 decision boundary \( 4\alpha \delta \) is \( d_1 = 4\alpha \delta - \alpha d \), where the weight of the instantaneous attenuation \( \alpha \) is now three times less than for \( P_6 \), giving:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{4\alpha}{\alpha} - 1\right)\sqrt{21}\right].
\]  

(50)

In case of \( P_7 \) the situation is considerably worse, since the modified protection distance from the C2 decision boundary at \( 4\alpha \delta \) is seen from Fig. 3 to be \( d_1 = 4\alpha \delta - 5\alpha d \), which results in:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{4\alpha}{\alpha} - 5\right)\sqrt{21}\right].
\]  

(51)

The most vulnerable phasor \( P_8 \) has the lowest protection distance of \( d_1 = 4\alpha \delta - 7\alpha d \) and highest error probability of:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{4\alpha}{\alpha} - 7\right)\sqrt{21}\right].
\]  

(52)

The components of the C2 error probabilities in Eqs. (49)-(52) have to be substituted in Eq. (12) together with Eq. (16) to give the average C2 BER of:

\[
P_{C2}(\gamma) = Q\left[\left(\frac{6\alpha}{\alpha} - 3\right)\sqrt{21}\right] + Q\left(3\alpha - 2\alpha\right)\sqrt{21}.
\]  

(53)

The computation of the C3 subchannel performance is based on a similar approach, but for each individual phasor there two modified protection distances, \( d_1 \) and \( d_2 \). For example, for the phasor \( P_6 \) we find \( d_1 = 6\alpha \delta - 3\alpha d \) and \( d_2 = 3\alpha d - 2\alpha d \), representing the distances from the C3 decision boundaries at \( 6\alpha \delta \) and \( 2\alpha d \), respectively. Under noisy conditions the decisions are erroneous in one of two scenarios. Either, if the modified protection distance \( d_1 \) is exceeded by the noise samples, which holds for negative \( d_1 \) values as well, or if the phasor \( P_6 \) is carried across the C3 decision boundary at \( 2\alpha d \) by large negative noise samples. This happens, whenever the noise value resides below the level \( -d_2 \). Based on these arguments, the C3 bit error probability for \( P_6 \) is computed as:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{6\alpha}{\alpha} - 3\right)\sqrt{21}\right] + Q\left(3\alpha - 2\alpha\right)\sqrt{21}.
\]  

(54)

As regards to phasor \( P_3 \) the modified protection distances are \( d_1 = 2\alpha d - \alpha d \) and \( d_2 = 6\alpha \delta - \alpha d \), respectively. The decisions are erroneous, if \( d_1 < 0 \), which is exceeded by noise, but noise samples larger than \( d_2 \) carry the phasors back into another error-free decision zone, leading to:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{6\alpha}{\alpha} - 3\right)\sqrt{21}\right] - Q\left(3\alpha - 2\alpha\right)\sqrt{21}.
\]  

(55)

For phasor \( P_7 \) the modified protection distances are given by \( d_1 = 6\alpha \delta - 5\alpha d \) and \( d_2 = 5\alpha d - 2\alpha d \), yielding:

\[
P_{C6}(\gamma) = Q\left[\left(\frac{6\alpha}{\alpha} - 5\right)\sqrt{21}\right].
\]  

(56)
Lastly, for phasor $P_b$ we have $d_1 = 7\alpha d - 6\alpha d$ and $d_2 = 7\alpha d - 2\alpha d$, where errors are caused by noise samples below the level $-d_1$, while for noise values below $-d_2$, the phasors are carried to another error-free zone across the decision boundary at $2\alpha d$, whence:

$$P_{eb}(\gamma) = Q\left(\left(1 - \frac{6\alpha}{a}\right)\sqrt{\frac{\gamma}{21}}\right) - Q\left(\left(1 - \frac{2\alpha}{a}\right)\sqrt{\frac{\gamma}{21}}\right).$$

(57)

Assuming random input sequences and equiprobable phasors $P_1 \cdots P_4$, we can substitute Eqs. (54)-(57) along with Eq. (16), into Eq. (12) to derive the average C3 bit error probability vs. channel SNR $\Gamma^*$, which is given by:

$$P_{eb}(\Gamma) = \frac{1}{4\Gamma^*} \left\{ \int_0^{\alpha} \left[ Q\left(\frac{6\alpha}{a} - 3\right)\sqrt{\frac{\gamma}{21}} \right] e^{-\gamma/\Gamma^*} d\gamma + \int_0^{\alpha} \left[ Q\left(\frac{3 - 2\alpha}{a}\right)\sqrt{\frac{\gamma}{21}} \right] e^{-\gamma/\Gamma^*} d\gamma - \int_0^{\alpha} \left[ Q\left(\frac{6\alpha}{a} - 1\right)\sqrt{\frac{\gamma}{21}} \right] e^{-\gamma/\Gamma^*} d\gamma - \int_0^{\alpha} \left[ Q\left(\frac{2\alpha}{a} - 1\right)\sqrt{\frac{\gamma}{21}} \right] e^{-\gamma/\Gamma^*} d\gamma \right\}. \quad (58)$$

In summary, the C1, C2 and C3 subchannel performances over Rayleigh-fading channels are given by Eqs. (48), (53) and (58), respectively. The equivalent BER vs. channel SNR curves are also plotted in Fig. 4, which again coincide within a fraction of a dB with our simulation results. The BER difference between the three classes becomes more pronounced than via the AWGN channel portrayed in Fig. 2. On the same note, the BER of the C1 subchannel is well suited for FEC-coded speech or image transmission, while the C2 and C3 performances have to be improved by fading-compensating automatic gain control and optional diversity reception for FEC techniques to become efficient in such a system.

4.1 A Low-complexity Mobile Video Telephone Scheme

Based on our theoretical discourse on 16-QAM properties here we propose a practical low-cost, low-complexity and low power consumption mobile video telephone scheme for friendly microcellular channels. In this environment the SNR is sufficiently high for bandwidth-efficient 16-QAM to be deployed to moderate the excessive video bandwidth requirement of our light-weight hand-held portable design, since now cost, complexity and consumption are at premium. With the above philosophy in mind we contrived and simulated the following minimalist system, seen in Fig. 5. The CCITT 32 kbps adaptive differential pulse code modulation (ADPCM) voice codec ensures a high mean opinion score (MOS) of four at very low complexity and low bandwidth efficiency, as well as low robustness against channel errors. The low-complexity, 160 kbps, 10 frames/sec, inter-frame predictive two dimensional DPCM (2DDPCM) video subchannel integrities are consistently different via both AWGN and Rayleigh-fading channels, but their discrepancy is more dramatic via the fading channel. The subchannels with lower average protection distances have prohibitively high residual BERs, as far as their mitigation by practical FEC techniques is considered. This is because due to the received signal envelope fluctuations the multilevel QAM constellation collapses towards the origin of the coordinate system, causing excessive BERs for the low-protection subchannels. The high-protection subchannels typically withstand the channel fading better and hence their behaviour is mostly governed by the AWGN. Automatic gain control (AGC) techniques with and without channel-sounding side-information for fading compensation are reported in Refs. (5)-(9) allowing the reduced residual BERs to be cleaned up by appropriate FEC methods, rendering QAM an attractive method for bandwidth-efficient communication via mobile transmission media. In addition, the different subchannels offer a fertile ground for attractive robust and bandwidth-efficient embedded speech and video transmission schemes.
The codec operates on $128 \times 128$ pixels images and provides acceptable image quality for this small screen-size at high robustness against channel errors. The speech and video bits are sorted in two sensitivity classes by the 'Mapper' and FEC coded by the low-complexity RS (12, 8, 2) Reed-Solomon encoder operating over the Galois Field $GF(16)$. This encoder codes eight 4-bit symbols into 12 symbols and is capable of correcting two such symbol errors. The 'Interleaver' is provided to help randomise the bursty channel errors and its output stream is directed by the 'Demapper' to the appropriate 16-QAM subchannel. The 16-QAM BER performance is improved by a fade-tracking AGC and second order diversity\(^{(6)}\).\(^{(8)}\). The received bit-stream is then FEC-, voice-and video-decoded to reconstruct the input speech and video signals. The ADPCM speech segmental SNR (SSNR) and 2DDPCM peak SNR (PSNR) objective quality measures were evaluated for various channel SNRs in the flat Rayleigh fading channel for a pedestrian mobile speed of 3 mph and a propagation frequency of 900 MHz. Our results are seen in Fig. 6. Both the speech and video performances are virtually unimpaired for SNRs in excess of 20 dB and start to degrade below that. Observe that the adaptive DPCM speech codec rapidly collapses, when the channel injects errors, while the inherently lower quality video performance decays gracefully. The overall signalling rate is about 50 KBd, a value easily accommodated by the coherence bandwidth of typical microcellular mobile channels.

References

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