Reducing computational effort in field optimisation problems

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Abstract *Design and optimisation of many practical electromechanical devices involve intensive field simulation studies and repetitive usage of time-consuming software such as finite elements (FEs), finite differences of boundary elements. This is a costly, but unavoidable process and thus a lot of research is currently directed towards finding ways by which the number of necessary function calls could be reduced. New algorithms are being proposed based either on stochastic or deterministic techniques where a compromise is achieved between accuracy and speed of computation. Four different approaches appear to be particularly promising and are summarised in this review paper. The first uses a deterministic algorithm, known as minimal function calls approach, introduces online learning and dynamic weighting. The second technique introduced as ES/DE/MQ – as it combines evolution strategy, differential evolution and multiquadrics interpolation – offers all the advantages of a stochastic method, but with much reduced number of function calls. The third recent method uses neuro-fuzzy modelling and leads to even further economy of computation, although with slightly reduced accuracy of computation. Finally, a combined FE/neural network approach offers a novel approach to optimisation if a conventional magnetic circuit model could also be used.*

1. Introduction

Optimal design of electromechanical devices often necessitates repetitive usage of finite element (FE) solvers or other numerically intensive field computation. A direct way of incorporating field modelling into an optimisation loop is to call the FE package every time a function evaluation is needed. Although fairly straightforward in implementation, this online approach will normally lead to unacceptable computing times, as for each set of selected design parameters a full field analysis needs to be undertaken. The number of necessary calls to the FE software escalates as the number of design variables increases; moreover, additional calls are normally required to calculate each gradient of the objective function. Although theoretically this is of no consequence, in the design office environment such an approach becomes impractical.

2. Minimal function calls approach

The minimum function calls (MFCs) approach relies on evaluating the objective function *a priori* for a number of pre-determined cases and fitting an interpolating function through the data points (Al-Khoury and Sykulski, 1998; Sykulski and Al-Khoury, 2000; Sykulski *et al.*, 2001). The optimiser then uses the interpolating function rather than calling the FE directly. In this response surface methodology (RSM) (Pahner and Hameyer, 1999) it is usual to use

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polynomial interpolating functions. Table I shows the number of coefficients in the interpolating equation for various number of variables and orders of polynomials fit. As an illustration, the second-order two variables case requires six coefficients $c_1x_1^2 + c_2x_1x_2 + c_3x_2^2 + c_4x_1 + c_5x_2 + c_6$. The fit order defines the maximum total order of any one polynomial term. For example, for third-order, x_1^3 and x_2^3 are used, but not $x_1^2x_2^2$. It should be noted that the minimum number of function evaluations needed for curve fitting is equal to the number of coefficients in the interpolating equation. For each point used in the curve fitting, a full FE simulation is required. The number of such calls is much less than if the FE simulation function was to be called directly by the optimiser. For example, using a third-order polynomial and five design variables requires 56 function calls, which will be quite acceptable in practical situations.

In the MFC approach, the position of initial points is carefully selected to be optimal in a sense that the resulting algorithms have proven stable (Sykulski *et al.*, 2001). As an example, Figure 1 shows the distribution of initial points for two variables with different orders of polynomial fit. It can be seen that points fill the search space and do not form regular arrays.

Using RSM, the computing times reduce dramatically, but care must be taken not to sacrifice accuracy. Extensive numerical experiments have shown that further significant improvements may be achieved by introducing *online learning* with *dynamic weighting* (Sykulski *et al.*, 2001).

As the optimisation process proceeds, more points become available for curve fitting and thus, the estimate of the optimum position becomes more accurate. It is therefore appropriate to apply lower weighting to points far from the predicted optimum. The weighting factor for each point is given by

$$\text{Weighting factor} = \exp(\alpha(x - x_{\text{Ref}})^2), \tag{1}$$

where x is the input vector for each point and x_{Ref} is the input vector for the best point for which a FE solution is available. The value of α is given by

	Order						
Variables	0	1	2	3	4	5	6
1	1	2	3	4	5	6	7
2	1	3	6	10	15	21	28
3	1	4	10	20	35	56	84
4	1	5	15	35	70	126	210
5	1	6	21	56	126	252	462
6	1	7	28	84	210	462	924
7	1	8	36	120	330	792	1,716
8	1	9	45	165	495	1,287	3,003
9	1	10	55	220	715	2,002	5,005
10	1	11	66	286	1,001	3,003	8,008

Table I.
The number of necessary function calls for RSM

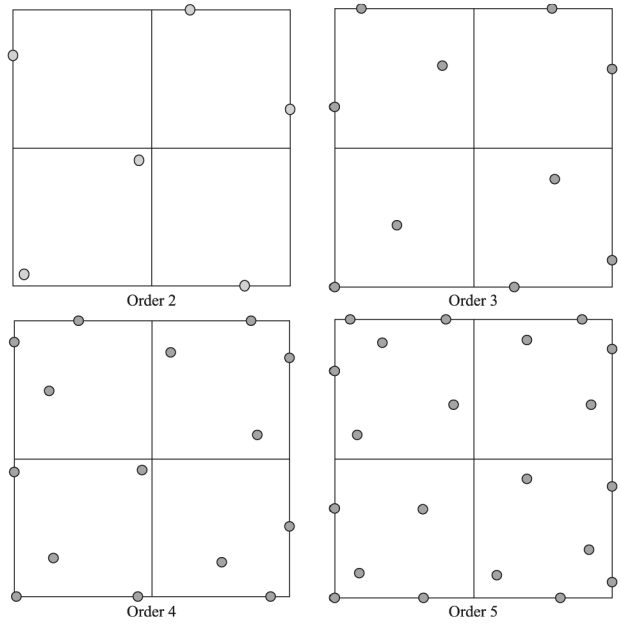

Figure 1.
Optimal positions of initial points for the case of two variables and different order of interpolating polynomial function

$$\alpha = \sinh\left(\frac{2 \times \text{No. of points}}{\text{No. of coefficients} \times \text{No. of variables}}\right). \quad (2)$$

The hyperbolic sine function is chosen because initially, all points are equally weighted, while for large number of points, the radius of the Gaussian function reduces exponentially. The rate of this exponential reduction is chosen, so that as each new point is added, approximately (on average) one point will move outside the radius. At the same time, *learning points* are added, which are not placed at the predicted optimum and thus allow the modelling of the normal gradients of the objective and constraint functions to be refined.

To illustrate the process, a brushless permanent magnet motor has been optimised for efficiency (with minimum torque constraint) in terms of magnet height, tooth width and stack length. The convergence is shown in Figure 2. It should be noted that, since every fifth point is a learning point, these points are not placed at the predicted optimum. Figure 3 shows a section through the response surface illustrating the nature of the optimisation problem. The efficiency is calculated by integrating input power and losses in a time-stepping model.

3. Evolution strategies

The deterministic approach of Section 2, despite the addition of learning points, may not be able to avoid local minima traps. If this is identified as a potential

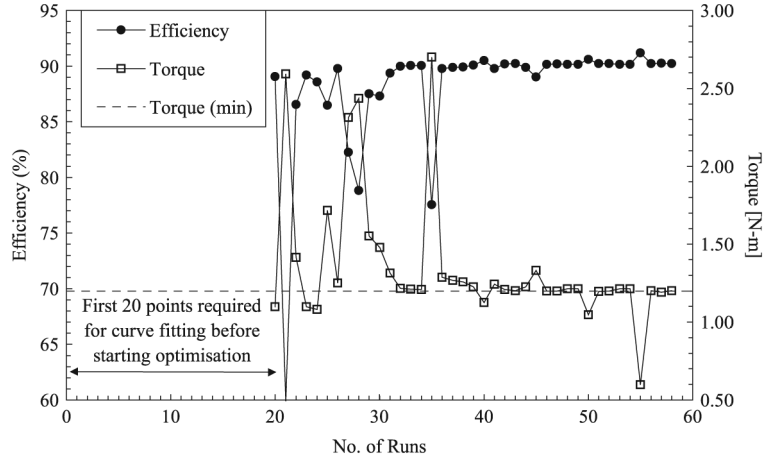

Figure 2.
Convergence of efficiency and torque

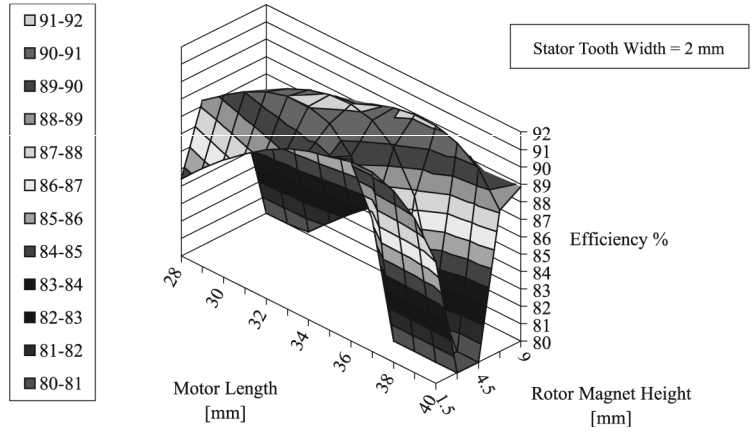

Figure 3.
Brushless PM motor optimisation response surface

problem, then stochastic techniques may offer a better choice. Most such techniques are very expensive in terms of number of necessary function evaluations and thus, impractical. Some more recent methods, however, look more promising and one such technique introduced originally in the work Farina and Sykulski (2001) is reported here. It uses a combination of evolution strategy, differential evolution and multiquadrics interpolation (ES/DE/MQ) as shown in Figure 4.

Consider a C-core where the pole faces are to be shaped to achieve homogeneous magnetic field in a rectangular region in the centre of the air gap. The field at 35 points on a regular grid is evaluated and the objective function is computed

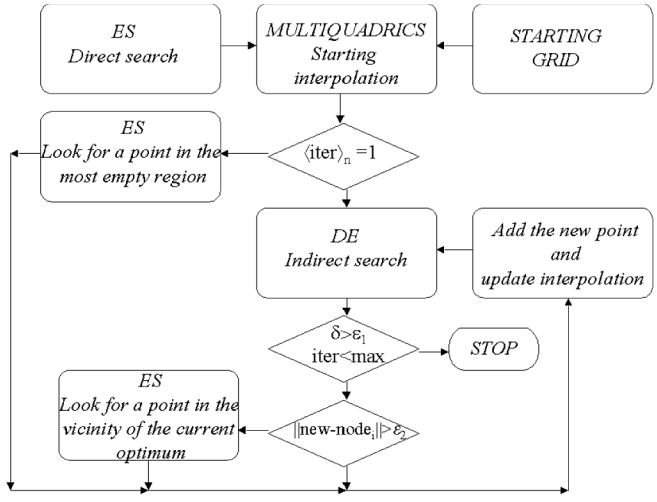

Figure 4.
Flowchart of the *ES/DE/MQ* method (Farina and Sykulski, 2001)

$$F_C = \max_{i=1,35} |B_0 - B_i| (B_0)^{-1}, \tag{3}$$

where B_i are magnetic field values on the grid and B_0 is the value at the centre. The design variables are the coordinates of the six points (x_1-x_6) defining the shape of the pole face. The geometry, design constraints and the control grid are shown in Figure 5.

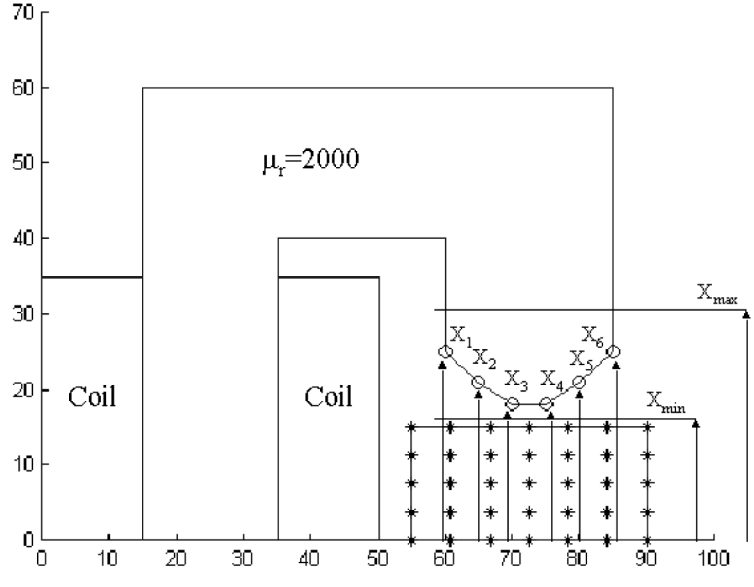

Figure 5.
The geometry of the C-core shaped magnet

The results of the optimisation are compared with optimum configurations obtained with standard techniques on the real objective function (full direct search). Three standard strategies (one ES and two versions of DE1 and DE2) and a gradient based algorithm (GBA) have been considered. The latter is the Matlab *FMINCON* optimisation function.

As shown in the last column in Table II the number of objective function calls is greatly reduced (it is even notably smaller than for the direct method GBA), whereas the value of the objective function is similar to ES and DE2 results and better than those obtained with DE1 and GBA. The optimal configuration is shown in Figure 6.

This hybrid *ES/DE/MQ* method has been shown to be able to avoid local minima traps for a number of test functions and achieves a significant reduction in the number of necessary function calls, making the approach suitable for computationally intensive FE design/optimisation problems. Moreover, the quality of the resultant optimum is comparable to, or better than, those obtained using other methods.

4. Neuro-fuzzy modelling

This recent technique employs the neuro-fuzzy modelling (NFM) (Rashid *et al.*, 2001b) and uses optimisation based on the genetic algorithm (GA) and the sequential quadratic programming (SQP) method (Rashid *et al.*, 2000a, b, 2001a). In the NF/GA/SQP approach, an n-dimensional hyper-space is sampled initially using a grid structure or a suitable design of experiment (DoE)

Table II.
Comparative optimisation results for a C-core

	Starting	Optimum	n
DE1	9 random	0.0803	720
DE2	13 random	0.0704	881
ES	0.7532/0.4344/0.6411	0.0642	450
GBA	0.7532	0.0855	188
ES/DE/MQ	0.7532	0.0718	118

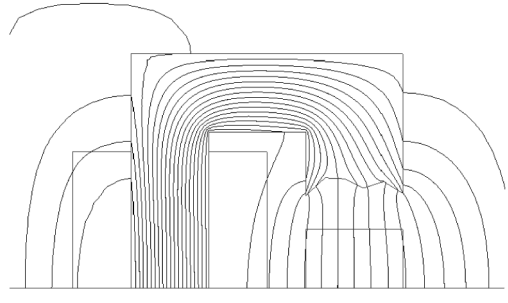

Figure 6.
C-core optimal configuration

orthogonal array (Fowlkes and Creveling, 1995) if the number of variables is high. The model data is subsequently employed to create a neuro-fuzzy model, which provides an approximation of real function. The notion of membership functions (MFs) is introduced which can be described by Gaussian, generalised bell or other curves. During the supervised training process, the parameters of each MF are modified using the back-propagation algorithm and the consequent parameters are established using least squares, ultimately providing an approximation of the system under investigation. This empirical model then effectively replaces the actual function generator, in this case the FE solver, easing the computational cost when applying the optimisation routine. This comprises a GA to identify the locality of the global optimum followed by the SQP method to isolate it accurately. The latter is possible due to the extraction of derivative information from the neuro-fuzzy model.

In order to minimise the cost of sampling, the hyper-surface is iteratively refined by addition of the perceived optimum, a number of genetically sampled points and a number of random samples for explorative purposes to the model data-set. The grid is also reset after a number of iterations to concentrate on the area of interest. The process is repeated until the stopping criterion is met. That is, when convergence to an optimal point occurs, given by the infinity norm between the successive perceived optimum points or on reaching the maximum number of iterations or sample points (Rashid *et al.*, 2001b).

Consider a magnetiser problem with six design parameters (Gallardo and Lowther, 1999; Mohammed and Uler, 1997), as shown in Figure 7. The design objective is to model the pole face, using the six free nodes, to realize a sinusoidal field along the chord *AB*. Results are obtained for the unconstrained problem in which all node vectors are assumed feasible and the constrained case in which certain vectors are assumed infeasible to avoid non-smooth designs. In practice, this means that the gradients of each of the five chords in Figure 7 must remain negative. Thus, additional constraints, other than those pertaining to the problem bounds, are imposed (Rashid *et al.*, 2001b), and poor regions of design space are discarded.

The basic objective function is given by:

$$f = \sum_{k=1}^{59} (B_{\text{desired},k} - B_{\text{calculated},k})^2 \tag{4}$$

where

$$B_{\text{desired},k} = B_{\max} \sin(90^\circ - k)$$

with $1 \leq k \leq 59$.

The results for unconstrained optimisation are summarised in Table III and compared with the ES/DE/MQ method of Section 3, as well as with standard

evolutionary strategies and MATLAB's GBA (similar comparison to that of Table II). In the NF/GA/SQP approach, the initial design space is sampled using an orthogonal experimental design array yielding 27 samples (Uler and Mohammed, 1996), complemented with 23 randomly selected samples to give an initial data-set of 50 points. Sampling in subsequent iterations is composed of the pseudo optimum, a number of genetic samples and a number of random samples. In the ES/DE/MQ approach, a pseudo-grid using an initial node set of 64 points ($2^{n\text{dof}}$) is employed, where each of the six points P_i assumes two possible values given by the range limits or constraints. Results from constrained optimisation are described in Table IV.

It is very satisfying to see that both methods achieve good results with significant reduction in the number of function calls compared with more

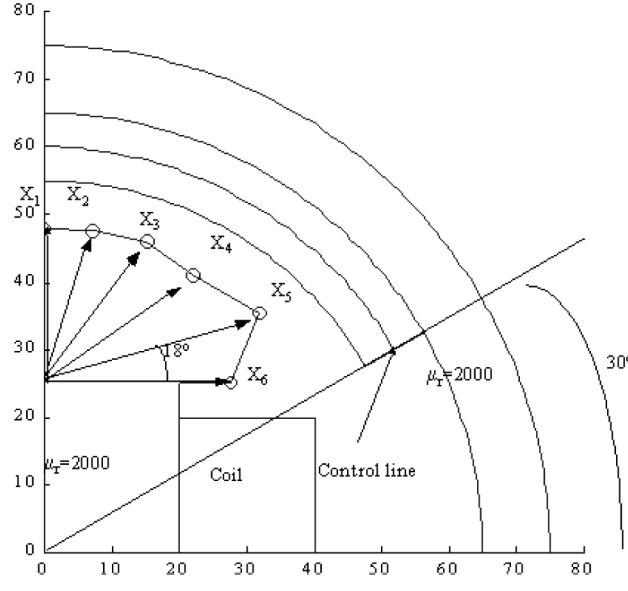

Figure 7.
The magnetiser model

	Starting	Optimum	*n*
DE1	11 Random	1.235E-5	987
DE2	11 Random	5.423E-5	1,035
ES	1.457E-3	1.187E-5	433
ES	9.486E-2	1.318E-4	351
GBA	1.457E-3	1.238E-4	41
GBA	9.486E-2	2.433E-4	281
ES/DE/MQ	1.457E-3	1.961E-5	234
ES/DE/MQ	9.486E-2	2.125E-5	206
NF/GA/SQP	–	6.570E-5	189

Table III.
Unconstrained optimisation results for magnetiser

standard methods. The introduction of constraints seems to be particularly effective for NF/GS/SQP approach, improving the profile of the pole face and taking significantly fewer samples as expected. The DE/ES/MQ algorithm gives slightly better results in both cases, but requires more samples, surprisingly even more in the constrained case. The optimal shape obtained with the unconstrained ES/DE/MQ method is shown in Figure 8.

The success of both the methods lies in their ability to search unexplored regions of space whilst exploiting available knowledge to identify more accurately regions of minima. On an average the DE/ES/MQ method finds a slightly better solution at the cost of a greater number of function evaluations. Both methods, however, require the number of function calls much lower than would be expected using conventional stochastic methods (Gallardo and Lowther, 1999; Mohammed and Uler, 1997; Uler and Mohammed, 1996), and this is where the benefits of such approaches lie, in improving the efficiency of the optimisation process whilst maintaining solution accuracy.

5. Combined FEs/neural networks

There is growing interest in the ways in which the performance of a specific device could be modelled using a neural network. Such a network learns the

Table IV. Constrained optimisation results for magnetiser

	Optimum	n
ES/DE/MQ	1.58E-5	246
NF/GA/SQP	4.65E-5	155

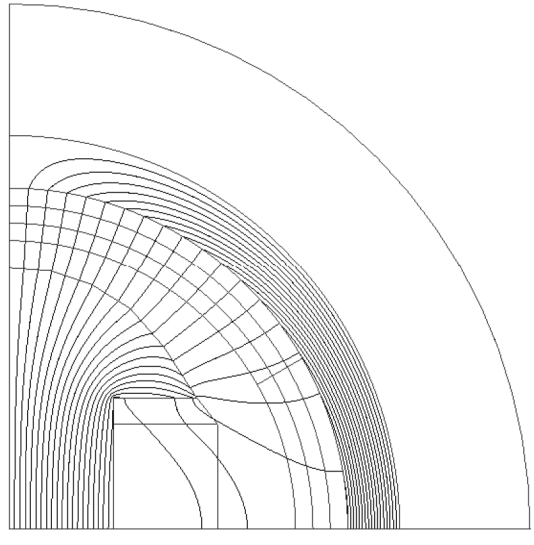

Figure 8. Magnetiser optimal configuration obtained from ES/DE/MQ (Farina and Sykulski, 2001)

shape of the hyper-surface and provides a fast evaluation of any point in it. Typically, the neural network is trained in a batch mode, prior to the optimisation process – essentially "off-line" (Arkadan and Chen, 1994; Ratner *et al.*, 1996). A recent attempt has been made to construct a system which can provide "online" training, i.e. a network which is capable of learning and modifying its behaviour as it is used (Seguin *et al.*, 1999). Such a network has major benefits over a static system in that it can handle a large number of variations of a device and track developments in design related to material changes and manufacturing processes. A diagram of the system is shown in Figure 9. This differs from a conventional system in that the numerical analysis (FE) component and the neural network exist in parallel and data can flow either way from the device model to determine the performance parameters. Each time, a set of performance parameters is generated, the data are fed back to provide a new training set for the neural network. Initially, as in the earlier proposed systems, the network is trained off-line on a device typical of the class of problems to be handled. The decision on which approach to take to generate the performance parameters is made within the device model by an intelligent system which contains a description of the current capabilities of the neural network and relates these to the problem being considered.

The neural network component of the architecture shown in Figure 9 consists of two parts. The first is intended to produce the actual values of the parameters for the specified device in a manner similar to that described by Arkadan and Chen (1994) and Ratner *et al.* (1996); the second part indicates the sensitivity of the device to changes in the inputs. This latter information is then used to guide the optimiser. The sensitivity prediction part of the system is described by Seguin *et al.* (1999) and is based on a knowledge-based network (Dandurand and Lowther, 1999), which implements a set of simple rules derived from a magnetic circuit. This is then corrected by the addition of an error prediction network trained on numerical examples. An example of a simple C-core actuator has been used (Figure 10).

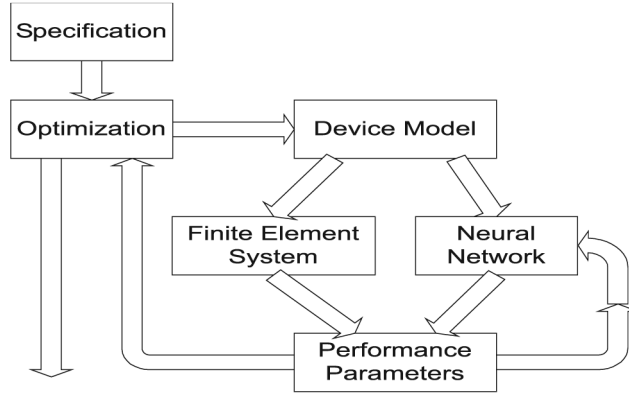

Figure 9.
Design process using online neural network

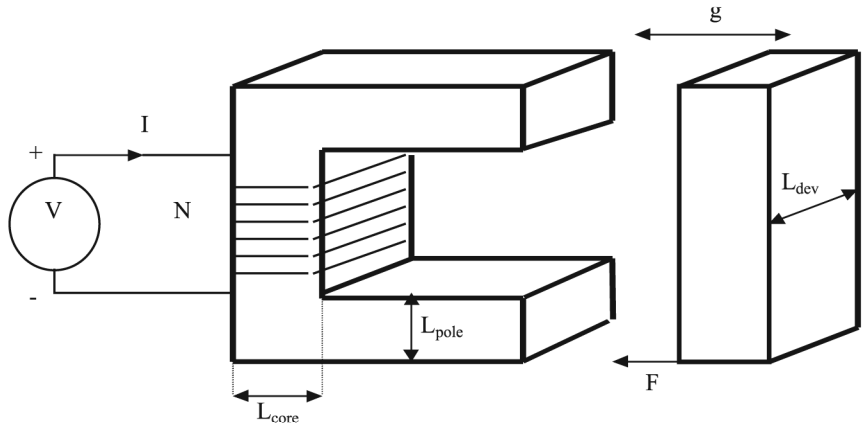

Figure 10.
A simple C-core actuator (Seguin *et al.*, 1999)

First, a conventional magnetic circuit model of the device is developed to create a set of sensitivity rules which guide the optimisation. Such a model is necessarily simplified and the effects of non-linearity and leakage ultimately need to be included. These may be considered as local perturbations on the underlying magnetic circuit structure. Thus, an efficient route to achieve a fast and accurate prediction of the device performance is to measure the error between the magnetic circuit prediction and the numerical analysis. This error can be determined online and can be learnt by a second neural network operating in concert with the knowledge-based system. In order to achieve this, the *error correcting network* needs to have the capability to correct the error "locally" within the design space and a *radial basis function* network has been found to be well suited to perform this task. A series of tests were performed with the objective to minimise the error as the device was driven into saturation and the fringing and non-linearity effects became more important. In this sense, the neural network system can take over from a full numerical (FE) analysis once it has been trained thus, providing either a designer or an optimisation system with extremely fast turnaround times on design modifications.

6. Pareto optimisation

The design of electromechanical devices has to be put in the context of general trends and developments of optimisation methods (Neittaanmaki *et al.*, 1996; Russenschuck, 1996). The role of *multi-objective optimisation* (Deb, 2001; Schatzer *et al.*, 2000; Thiele and Zitzler, 1999) is increasing as practical designs usually involve conflicting requirements. Traditionally, such problems are often converted into *single-objective* tasks with *a priori* application of some knowledge or imposition of a decision (for example, through *weighting factors*), but it is argued that information can easily be lost in the process and some existing "optimal" solutions may even be mathematically impossible to achieve. Instead the application of pareto optimal front (POF) approximation is

advocated. The mathematical theory of Pareto multi-objective optimisation may be somewhat complicated (Deb, 2001, Miettinen, 1999) but some basic definitions and properties are easily explained using a special case of two objective functions being minimised as shown in Figure 11.

A multi-objective problem may be *convex* or *non-convex*, *discontinuous*, *deceptive* or *multimodal*, and there are various ways of treating such conditions. The important point is that a result is not a single solution, but a set of possible (and in some sense acceptable) solutions given by various combinations of design parameters (the design domain search space is not shown here, but could consist of a number of variables). The final decision about the choice of the design is therefore made *a posteriori* and any point on the POF may be considered optimal. Such information is clearly more helpful to a designer than a result from a single-objective model.

A comprehensive treatment of POF approximations for multi-objective shape design optimisation may be found in the work of Farina (2002), including several practical examples (air-cored solenoid, electrostatic micromotor, single-phase reactor and inductor for transverse flux heating).

7. Conclusions

In this paper, it has been argued that optimisation methods have achieved a status of a mature tool which can be applied efficiently to practical design problems requiring accurate, but time-consuming, field simulations. There are a vast number of methods and techniques of optimisation and the difficulty is that the choice of the "best" one is problem dependent. In this paper, attention has been drawn on methods particularly suitable to computationally intensive design problems, such as those which arise when a FE (or similar) method has to be used for accurate prediction of performance. Most of such methods are

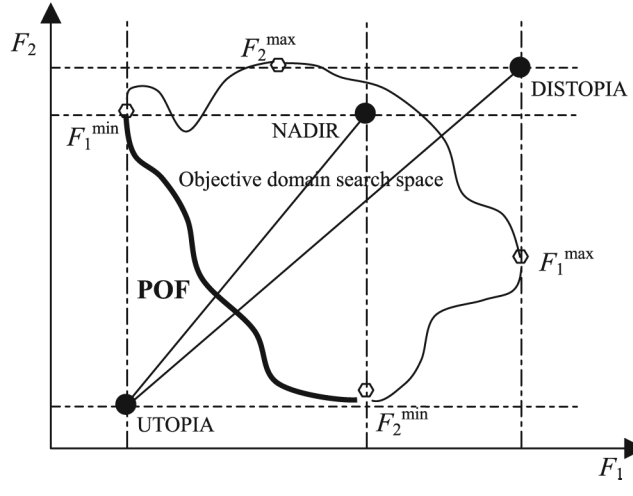

Figure 11.
Example of objective domain search space showing the POF and UTOPIA, DISTOPIA and NADIR points

based on RSM. If local minima traps are considered not to be a problem, a deterministic method such as MFCs approach is recommended. Particular combinations of evolutionary strategies and GAs have also been designed and reported here for increasing the chances of finding the global optimum. Some recent work on application of neural networks also looks promising. Finally, the importance of multi-objective optimisation has been stressed.

References

Al-Khoury, A.H. and Sykulski, J.K. (1998), "Automation of finite element aided design of brushless PM motors", *Proceedings of ICEM'98*, 2-4 September 1998, Istanbul, Turkey, pp. 614-8.

Arkadan, A.A. and Chen, Y. (1994), "Artificial neural network for the inverse electromagnetic problem of system identification", *Southeastcon '94*, April 1994, Miami, USA, pp. 162-4.

Dandurand, F. and Lowther, D.A. (1999), "Electromagnetic device performance identification using knowledge based neural networks", *IEEE Transactions on Magnetics*, Vol. 35 No. 3, pp. 1817-20.

Deb, K. (2001), *Multi-objective Optimization Using Evolutionary Algorithms*, Wiley, Chichester.

Farina, M. (2002), "Cost-effective evolutionary strategies for Pareto optimal front approximation to multi-objective shape design optimization of electromagnetic devices", PhD dissertation, Department of Electrical Engineering, University of Pavia, Italy.

Farina, M. and Sykulski, J.K. (2001), "Comparative study of evolution strategies combined with approximation techniques for practical electromagnetic optimisation problems", *IEEE Transactions on Magnetics*, Vol. 37 No. 5, pp. 3216-20.

Fowlkes, W.Y. and Creveling, C.M. (1995), *Engineering Methods for Robust Product Design*, Addison-Wesley, New York.

Gallardo, J.A. and Lowther, D.A. (1999), "The optimisation of electromagnetic devices using niching genetic algorithms", *COMPEL*, Vol. 18 No. 3, pp. 285-97.

Miettinen, K. (1999), *Nonlinear Multiobjective Optimisation*, Kluwer, Dordrecht, The Netherlands.

Mohammed, O.A. and Uler, F.G. (1997), "A hybrid technique for the optimal design of electromagnetic devices using direct search and genetic algorithms", *IEEE Transactions on Magnetics*, Vol. 33 No. 2, pp. 1931-4.

Neittaanmaki, P., Rudnicki, M. and Savini, A. (1996), *Inverse Problems and Optimal Design in Electricity and Magnetism*, Oxford Science Publications, Oxford.

Pahner, U. and Hameyer, K. (1999), "Adaptive coupling of differential evolution and multiquadrics approximation for the tuning of the optimization process", *Proceedings of COMPUMAG Sapporo*, 25-28 October 1999, pp. 116-7.

Rashid, K., Ramirez, J.A. and Freeman, E.M. (2000a), "Hybrid optimisation in electromagnetics using sensitivity information from a neuro-fuzzy model", *IEEE Transactions on Magnetics*, Vol. 36 No. 4, pp. 1061-5.

Rashid, K., Ramirez, J.A. and Freeman, E.M. (2000b), "A general approach for extracting sensitivity analysis from a neuro-fuzzy model", *IEEE Transactions on Magnetics*, Vol. 36 No. 4, pp. 1066-70.

Rashid, K., Ramirez, J.A. and Freeman, E.M. (2001a), "Optimisation of electromagnetics devices using sensitivity information from clustered neuro-fuzzy models", *IEEE Transactions on Magnetics*, Vol. 37 No. 5, pp. 3575-8.

Rashid, K., Farina, M., Ramirez, J.A., Sykulski, J.K. and Freeman, E.M. (2001b), "A comparison of two generalized response surface methods for optimisation in electromagnetics", *COMPEL*, Vol. 20 No. 3, pp. 740-52.

Ratner, I., Ali, H.O., Pteriu, E.M. and Eatherley, G. (1996), "Neural network modelling of electromagnetic field problems", *International Workshop on Neural Networks for Identification, Control, Robotics and Signal/Image Processing*, August 1996, Venice, Italy, pp. 387-91.

Russenschuck, S. (1996), "Synthesis, inverse problems and optimization in computational electromagnetics", *IJNM*, Vol. 9 No. 1/2, pp. 45-57.

Schatzer, Ch., Binder, A. and Muller, W. (2000), "A new approach for solving vector optimization problems", *IEEE Transactions on Magnetics*, Vol. 36 No. 4, pp. 1071-5.

Seguin, J., Dandurand, F., Lowther, D.A. and Sykulski, J.K. (1999), "The optimization of electromagnetic devices using a combined finite element/neural network approach with online training", *COMPEL*, Vol. 18 No. 3, pp. 266-74.

Sykulski, J.K. and Al-Khoury, A.H. (2000), "A system for interactive design and optimisation of brushless PM motors", *COMPEL*, Vol. 19 No. 2, pp. 664-8.

Sykulski, J.K., Al-Khoury, A.H. and Goddard, K.F. (2001), "Minimal function calls approach with online learning and dynamic weighting for computationally intensive design optimisation", *IEEE Transactions on Magnetics*, Vol. 37 No. 5, pp. 3423-6.

Thiele, L. and Zitzler, E. (1999), "Multiobjective evolutionary algorithms a comparative case study and the strength Pareto approach", *IEEE Transactions on Evolutionary Computation*, Vol. 3 No. 4, pp. 257-71.

Uler, G.F. and Mohammed, O.A. (1996), "Ancillary techniques for the practical implementation of GAs to the optimal design of electromagnetic devices", *IEEE Transactions on Magnetics*, Vol. 32 No. 3, pp. 1194-7.