RESIDUAL NUMBER SYSTEM ASSISTED CDMA: A NEW SYSTEM CONCEPT

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ABSTRACT

In this paper a new, so-called residue number system (RNS) based direct sequence-code division multiple access (DS-CDMA) scheme employing M-ary orthogonal modulation is proposed. In conventional M-ary orthogonal modulation, the complexity of the receiver increases exponentially with the number of bits per symbol. However, using this technique, a linear, rather than exponential receiver complexity dependence on the number of bits per symbol holds. Hence, powerful nonbinary error control schemes such as Reed-Solomon codes over a large Galois Field can be efficiently incorporated. Furthermore, the error correction properties of the RNS can be exploited in order to improve the performance of the system.

1. INTRODUCTION

In M-ary orthogonal-based DS-CDMA it is possible to improve the bandwidth-efficiency by increasing the number of bits per symbol [1]. However, the number of bits per symbol, $\beta = \log_2 M$, is logarithmically proportional to the number of orthogonal sequences M [1]. The complexity of the receiver increases linearly with M and exponentially with the number of bits per symbol β. Hence, the achievable bandwidth-efficiency gain is limited by the maximum number of orthogonal sequences and by the acceptable complexity of the receiver. However, it was argued in [4] that when using the so-called residue number system (RNS) whereby a set of orthogonal signals are transmitted in parallel as proposed by Yang, *et al.* in [2–4], a linear, rather than exponential receiver complexity dependence on β is valid. Hence, the RNS technique allows symbol-by-symbol transmission in conjunction with powerful nonbinary error control schemes, such as Reed-Solomon (RS) codes over a large Galois Field, for example GF(255), without imposing an excessive complexity on the receiver. Furthermore, by exploiting the error correction properties of the RNS, an improvement in the system performance can be achieved.

In this contribution – in contrast to references [2–4], which assume that each of the orthogonal signals propagated through statistically independent radio channels, – we will examine the performance of such a system, when the signals are orthogonally code multiplexed and transmitted in the form of a single composite signal, as in a multi-code DS-CDMA system.

This paper is organized as follows. We will commence with a brief introduction to RNS-based M-ary orthogonal signalling in DS-CDMA in Section 2, as proposed by Yang, *et al.* in [2–4]. In Section 3, computer simulations are presented. Finally, our concluding remarks are given in Section 4.

2. FUNDAMENTALS OF RESIDUE NUMBER SYSTEM BASED M-ARY ORTHOGONAL MODULATION

RNS-based communications systems were first proposed in References [2–4]. Let $\{m_1, m_2, \ldots, m_v\}$, where $m_1 < m_2 < \ldots m_v$, be the set of so-called information moduli involved in the transformation from the conventional binary weighted number system to its equivalent residue number system. This transformation algorithm is referred to here as the residue number system transform (RNST). Conversely, the transformation from the residue number system to the equivalent weighted number system is termed as the inverse residue number system transform (IRNST). Several well-known algorithms, such as the Chinese remainder theorem [5] and the mixed radix conversion approach [6], have been proposed in order to carry out the IRNST. It can be shown

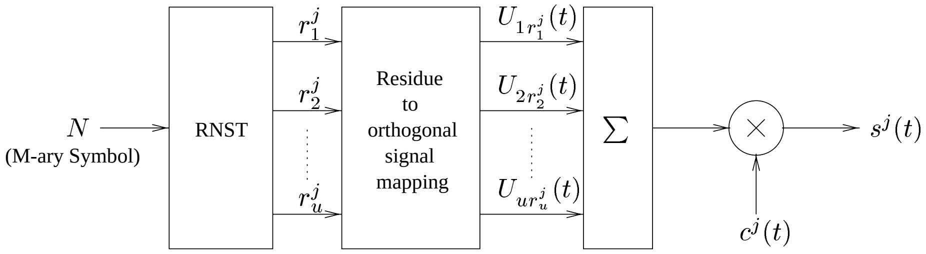

Figure 1: The RNS-based orthogonal communication system's ith user's transmitter block diagram

that if N, representing an M-ary information symbol to be transmitted, is in the range of $0 \leq N < \prod_{i=1}^{v} m_i$, then the RNST is realized by uniquely representing N as a v-tuple residue digit sequence $(r_1, r_2, \ldots, r_v)$, where $r_i = N \bmod m_i$ for $i = 1, 2, \ldots, v$ [7]. **The range of $0 \leq N < \prod_{i=1}^{v} m_i$ is also referred to as the dynamic range of the RNS, in which symbols can be uniquely represented by their residues $r_1, r_2, \ldots, r_v$.**

Residue digit error correction and/or detection can be incorporated into the transformation in order to protect the data through the introduction of redundancy. In this case, an additional set of so-called redundant moduli $\{m_{v+1}, \ldots, m_u\}$, where $u > v$ and $m_1 < \ldots < m_v < m_{v+1} < \ldots < m_u$, is involved in the transformation, such that N is now represented as a u-tuple residue digit sequence $(r_1, r_2, \ldots, r_v, r_{v+1}, \ldots, r_u)$. This u-tuple residue digit sequence constitutes the so-called redundant residue number system (RRNS). A RRNS(u, v) code is capable of detecting $(u - v)$ or less residue digit errors and correct up to $[(u - v)/2]$ residue digit errors. Furthermore, according to the properties of the RNS arithmetic, the symbol N can be recovered from any v out of the u number of residue digits and their relevant moduli. This implies that by discarding $d = u - v$ number of residue digits which, according to some performance metric, are considered to be the most likely to be in error, then the original symbol can still be recovered from the remaining v residue digits and their relevant moduli, provided that the remaining v residue digits are correct.

2.1. Transmitter Model

The transmitter block diagram of the RNS-based orthogonal communication system for the jth user is shown in Fig. 1. The information symbol N of duration T_s is transformed by the RNST block to its equivalent residue digit sequence $(r_1, r_2, \ldots, r_u)$, which are then mapped to the corresponding orthogonal sequences $[U_{1r_1}(t), U_{2r_2}(t), \ldots, U_{ur_u}(t)]$ and code multiplexed for transmission. For practical reasons, the number of bits per symbol N is given by

$$\beta = \left\lfloor \log_2 \left(\prod_{i=1}^{v} m_i \right) \right\rfloor . \tag{1}$$

Hence the practical dynamic range of N is in the region of $0 \leq N < 2^{\beta}$. As opposed to conventional M-ary orthogonal modulation, which required 2^{β} orthogonal sequences for transmission [1], the total number of orthogonal sequences M_{RNS} required for the RNS-based system is given by [4] :

$$M_{RNS} = \sum_{i=1}^{u} m_i, \tag{2}$$

where the set of orthogonal sequences can be written as

$$\left\{U_{10}(t), U_{11}(t), \ldots, U_{1(m_1-1)}(t); U_{20}(t), \ldots, U_{2(m_2-1)}(t); \ldots; U_{u0}(t), \ldots, U_{u(m_u-1)}(t)\right\}, \tag{3}$$

and the orthogonal sequences $U_{10}(t), \ldots, U_{1(m_1-1)}(t)$ are used for example to convey the residue digit r_1. The rate of these orthogonal sequences can be assumed to be equal to the chip rate used in the system. Each user is assigned the same set of orthogonal sequences, namely that given in (3). A long user-specific scrambling code is then used to identify each user. Hence, the lowpass equivalent transmitted signal for the jth user is given by

$$s^j(t) = \sum_{i=1}^{u} \sqrt{\frac{P}{u}} U_{ir_i^j}(t) c^j(t), \tag{4}$$

where $c^j(t)$ is the jth user's scrambling code and P is the average transmit power. After transmitting through a frequency-selective slowly-fading Rayleigh channel, the uplink received signal $r(t)$ consists of a composite sum of all the delayed, phase shifted, attenuated replicas of the users' respective transmitted signals given in (4).

Hence, the equivalent lowpass representation of $r(t)$ can be written as

$$r(t)=\sum_{k=1}^{K}\sum_{l=1}^{L_P}\sqrt{\frac{P}{u}}\alpha_l^k e^{-j\phi_l^k}s^k(t-\tau_l^k)+n(t), \tag{5}$$

where α_l^k, ϕ_l^k and τ_l^k are the lth path's amplitude, phase and delay for the kth user and $n(t)$ is the AWGN with a single-sided power spectrum density of N_0. It is assumed that there are K number of active users in the system and there are L_P resolvable paths for each user.

2.2. Receiver Model

The receiver block diagram invoked for detecting the kth user's RNS-based signal is shown in Fig. 2. According

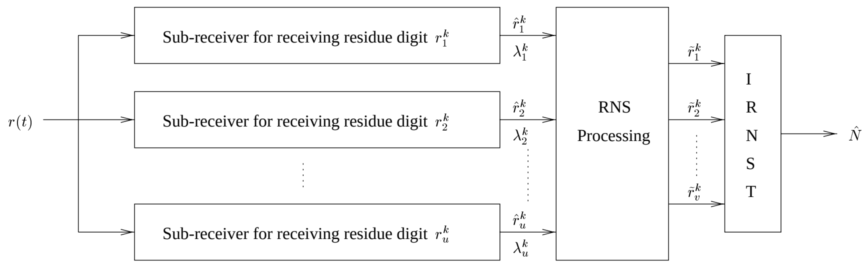

Figure 2: The RNS-based orthogonal communication system's kth user's receiver block diagram

to the properties of the RNS arithmetic [4], the residue digits' operations belonging to the different moduli are mutually independent. Hence, the receiver can be broken down into u number of independent 'sub-receivers' with each sub-receiver dedicated to receiving one residue digit. Each sub-receiver has a coherent RAKE structure, with an order of diversity equal to L_P. Fig. 3 shows the block diagram of a sub-receiver designed for receiving the residue digit, r_i^k. It is assumed that each branch in the sub-receiver is synchronized to the respective path of the desired signal. The received signal $r(t)$ is first descrambled by the desired user's scrambling code $c^k(t)$ as shown in Fig. 3. In each sub-receiver, the descrambled signal is then multiplied with the conjugate of the respective path parameters $\alpha_l^k e^{j\phi_l^k}$ where $l=1,2,\ldots,L_P$ which, without loss of generality, is assumed to be perfectly estimated. The signal is then correlated with each of the m_i orthogonal sequences, i.e. with $\{U_{i0}(t), U_{i1}(t), \ldots, U_{i(m_i-1)}(t)\}$. The resulting outputs $U_{i0}^k(l),\ldots,U_{i(m_i-1)}^k(l)$, where $l=1,2,\ldots,L_P$, are then maximally combined with their multipath replicas as shown in Fig. 3. Finally a decision as to which residue digit was transmitted is made, denoted as $\hat{r}_i^k$ based on maximum likelihood detection (MLD). If residue digit discarding is employed, a side information, denoted by λ_i^k is also specified. This side information is the result of the so-called ratio statistic test (RST), defined as [2]:

$$\lambda_i^k=\frac{{}^1\max_i\left\{\hat{U}_{i0}^k,\hat{U}_{i1}^k,\ldots,\hat{U}_{i(m_i-1)}^k\right\}}{{}^2\max_i\left\{\hat{U}_{i0}^k,\hat{U}_{i1}^k,\ldots,\hat{U}_{i(m_i-1)}^k\right\}}, \tag{6}$$

where ${}^1\max_i\{\cdot\}$ and ${}^2\max_i\{\cdot\}$ represent the maximum and the 'second maximum' of the correlator outputs of $\left\{\hat{U}_{i0}^k,\hat{U}_{i1}^k,\ldots,\hat{U}_{i(m_i-1)}^k\right\}$, respectively, as illustrated in Fig. 3. **The rationale behind introducing the RST as a performance metric is that if the highest correlator output is significantly higher than the second highest, then our decision is a confident one.** The residue digit error correction, detection and discarding schemes are handled by the RNS processing block of Fig. 2. If no redundancy is introduced, then the detected residue digits $\hat{r}_1^k, \hat{r}_2^k, \ldots, \hat{r}_v^k$ of all the sub-receivers will be directly inverse transformed by the IRNST block of Fig. 2 in order to recover the equivalent M-ary symbol $\hat{N}$.

As mentioned previously, when redundancy is incorporated, the scheme is capable of detecting up to $(u-v)$ or less residue digit errors and correct up to $(u-v)/2$ residue digit errors [2]. RNS-based error detection can be used in conjunction with an outer RS coding layer, in order to mark the decoded symbol as an erasure for the outer RS decoder. If only residue digit discarding is employed, then the RNS processing block will simply discard $u-v$ unreliable residue digits, corresponding to the lowest λ_i^k values, $i=1,2,\ldots,u$. Assuming that $\hat{r}_{v+1}^k,\ldots,\hat{r}_u^k$ are discarded, then the remaining residue digits $\tilde{r}_1^k, \tilde{r}_2^k, \ldots, \tilde{r}_v^k$ are inverse transformed by the IRNST to the equivalent M-ary symbol in Fig. 2.

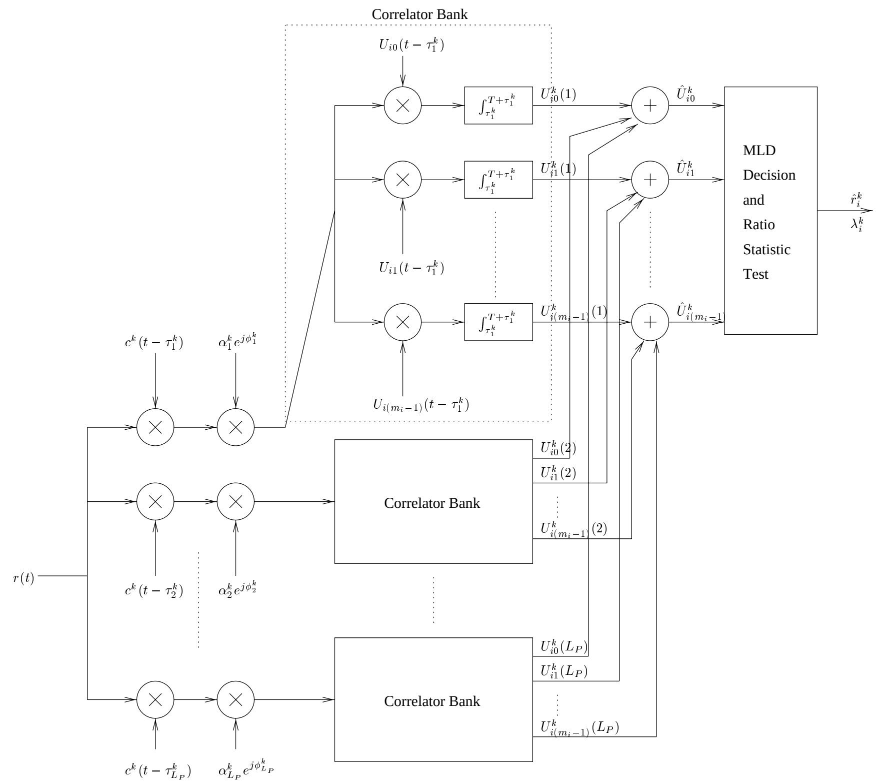

Figure 3: Block diagram of a kth user's sub-receiver receiving residue digit r_i^k, with L_P diversity branches

3. SIMULATION RESULTS

The average bit error probability of the discussed system model was evaluated based on system simulations. In the simulations, a set of 256 chips per symbol duration Walsh functions were used as the orthogonal sequences, while the signature sequences were randomly generated. The channel was modelled as a slowly varying frequency selective multipath Rayleigh fading. The discrete channel impulse response is assumed to be made up of L_P resolvable paths having an equal average power. The path phases and path delays were assumed to be uniformly distributed in $[0, 2\pi)$ and $[0, T_s)$, respectively. The simulation results are shown in Fig. 4–5.

The BER performance of the RNS-based orthogonal DS-CDMA scheme is shown in Fig. 4 using moduli $(5, 7, 8)$. As shown in Fig. 4, the BER of the multi-user transmission scenario is unacceptable for speech services with no error coding. Hence, in order to improve the BER performance, outer RS coding is invoked before the RNS conversion. The performance of this scheme is also shown in Fig. 4 using the RS(255,223) code. As can be seen, the BER performance has improved to an acceptable level for $K = 5$.

The error detection properties of the redundant RNS can be exploited by the outer RS channel coding, in order to improve the BER performance. The simulation results are shown in Fig. 5. There is an improvement of 1 dB for $K = 5$ at a BER of 10^{-3}, when using residue digit error detection.

Fig. 6 shows the performance of another redundant RNS-based DS-CDMA system in conjunction with RS coding, but using a different set of moduli, namely $(16, 17, 19)$ with 19 being the redundant modulus. This set of moduli also provides INTEGER$[\log_2(16 \cdot 17 \cdot 19)] = 8$ bits per symbol, but requiring more orthogonal signals, i.e. $16 + 17 + 19 = 52$ orthogonal signals as compared to using $(5, 7, 8, 9)$, which only needs $5 + 7 + 8 + 9 = 29$ orthogonal signals. Moreover, as it can be seen in Fig. 6, the BER performance of the set $(16, 17, 19)$ is better than that of the set $(5, 7, 8, 9)$. Hence, there is a trade-off between the receiver complexity and the achievable BER performance.

4. SUMMARY

In M-ary orthogonal DS-CDMA based on the residue number system, the required number of correlators is determined by the sum of the moduli. The performance can be improved by exploiting the properties of the RNS through error correction and/or erasure. The results presented in this paper are based on a typical DS-CDMA system. Here we curtail our discussions on RNS-based orthogonal schemes, noting that the proposed techniques are quite attractive in implementational terms, requiring however further research, which we intended

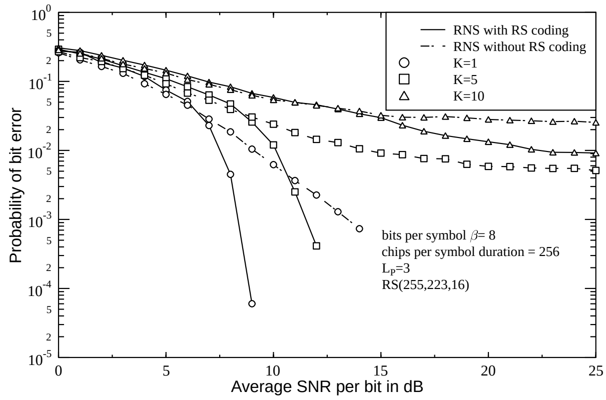

Figure 4: Multi-user BER performance for the **nonredundant** RNS-based orthogonal DS-CDMA system using the **moduli** $(\mathbf{5}, \mathbf{7}, \mathbf{8})$, **with and without RS(255,223,16) coding** for $\boldsymbol{K} = \mathbf{1}, \mathbf{5}$ and **10 users**.

to stimulate with the aid of the above brief discussions.

5. REFERENCES

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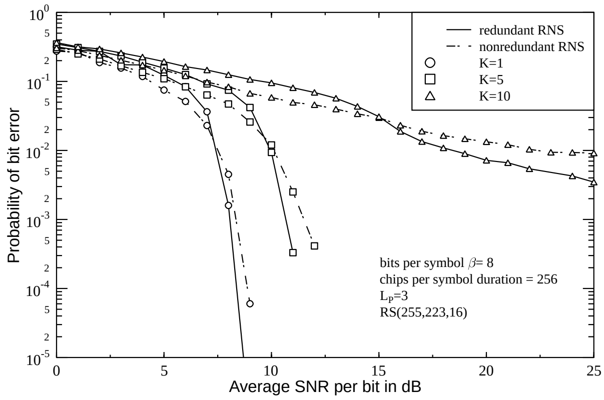

Figure 5: Multi-user BER performance for the **redundant** RNS-based orthogonal DS-CDMA system using **moduli $(5, 7, 8, 9)$ (9 being the redundant modulus)**, concatenated with the outer RS(255,223,16) code for $\boldsymbol{K = 1, 5}$ and **10 users**. **One residue digit discarding** is employed and the performance is compared with a nonredundant scenario using moduli $(5, 7, 8)$.

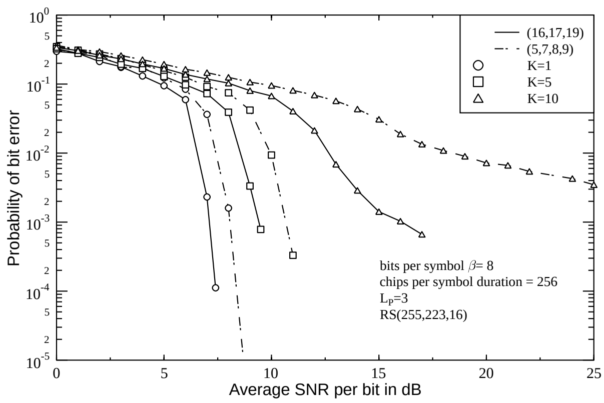

Figure 6: Multi-user BER performance for the **redundant** RNS-based orthogonal DS-CDMA system using **moduli $(16, 17, 19)$ (19 being the redundant modulus)**, concatenated with the outer RS(255,223,16) code for $\boldsymbol{K = 1, 5}$ and **10 users**. **One residue digit discarding** is employed and the performance is compared with another scenario using moduli $(5, 7, 8, 9)$ with 9 being the redundant modulus.