BLOCK-BASED TURBO-CODED, TURBO-EQUALISED PARTIAL-RESPONSE MODULATION FOR DISPERSIVE MOBILE CHANNELS

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ABSTRACT

We present a novel partial response GMSK turbo equaliser, which employs high rate BCH turbo codes (TEQ-BTC) in order to achieve substantial iteration gains over non-dispersive Gaussian, narroband Rayleigh as well as over dispersive Ralyleigh channels. The proposed joint turbo decoding and equalisation scheme achieved a 0.6 dB SNR gain at a BER of 10^{-3}, compared to employing turbo decoding and equalisation in isolation over a 5-path Rayleigh fading channel.

1. INTRODUCTION

Turbo equalisation [1] was first introduced by C.Douillart, A.Picart, M.Jézéquel, P.Didier, C.Berrou and A.Glavieux in 1995 for a serially concatenated convolutional coded BPSK system. Specifically, Douillart *et al* demonstrated that the turbo equaliser was capable of mitigating the effects of Inter-Symbol Interference (ISI), provided that the channel impulse response is known. Instead of performing the equalisation and error correction decoding independently, better performance can be achieved by considering the channel's memory, when performing joint equalisation and decoding iteratively. Gertsman and Lodge [2] then showed that the iterative process of turbo equalisers can compensate for the degradations due to imperfect channel estimation. A turbo equalisation scheme for GSM was also proposed by Bauch and Franz [3], where different approaches were investigated to overcome the dispersion of the so-called *a priori* information due to the interburst interleaving scheme used in GSM. In Reference [4], Raphaeli and Zarai employed rate $R = \frac{1}{3}$ turbo codes using convolutional constituent codes in conjunction with turbo equalisers and obtained higher BER improvements due to the additional turbo coding gain, as well as due to the ISI mitigation achieved by turbo equalisation.

With the increasing demand for the digital transmission of speech or video information, current systems aim for increasing the spectral efficiency by invoking high-rate codes. This has generated interest in block turbo codes, which have been shown by Hagenauer, Offer and Papke [5] to outperform convolutional turbo codes, when the coding rate is higher than $\frac{2}{3}$. It was also observed that a rate $R = 0.981$ block turbo code using BPSK over the non-dispersive Gaussian channel can operate within 0.27 dB of the Shannon limit [6]. In Reference [7], Pyndiah presented iterative decoding algorithms for BCH turbo codes. Since BCH codes may be constructed with parameters n and k, which represent the number of coded bits and data bits, respectively, we will use the notation, BCH (n,k) to characterise the BCH codes throughout our discussion.

2. TURBO BCH-CODED TURBO EQUALISATION

The baseband transmission model employed in our simulations is illustrated in Figure 1, where the turbo equaliser of Figure 2, is used.

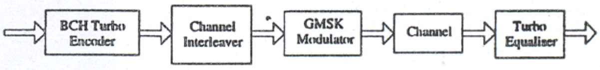

Figure 1: Baseband transmission model.

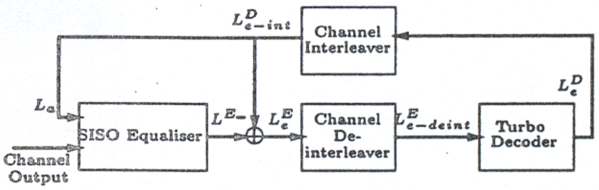

Figure 2: Turbo equaliser schematic using turbo codes. A more detailed outline of the turbo decoder is shown in Figure 3.

The structure of the turbo decoder used in conjunction with the turbo equaliser is portrayed in Figure 3, which is briefly highlighted below. The turbo equaliser using turbo BCH coding consists of two main components, namely the Soft-In/Soft-Out (SISO) equaliser and the turbo decoder, which employs two SISO decoders. In our description of the turbo equaliser, we have used the notations L^E and L^D to indicate the Log-Likelihood Ratio (LLR) from the Soft-in /

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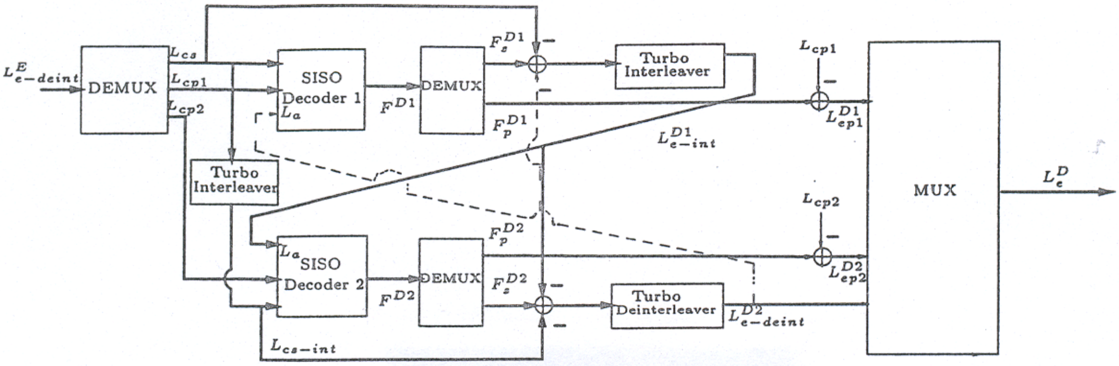

Figure 3: The structure of the turbo decoder in Figure 2, which is used for turbo equalisation.

Soft-out equaliser and decoder, respectively, while the subscript e is used to represent the so-called extrinsic values. The terminology is justified by the fact that the extrinsic information concerning a given bit in turbo detection algorithms does not originate directly from the channel's output information concerning the bit. More explicitly, the extrinsic information accrues from the surrounding channel output bits and it was imposed by the code constraints, but again, it explicitly excludes the received channel output concerning the bit, which may be of low confidence level due to channel effects. When this extrinsic information related to the bit concerned is passed on to the second component decoder, it has to be considered as additional information originating 'from outside' the second codeword, hence providing extra information concerning the codeword, which is an independent source due to the randomising effect of the turbo interleaver at the commencement of iterations, but becomes more correlated during the consecutive iterations. This is why in the context of the second component decoder this quantity is referred to as *extrinsic information*, which is used synonymously to *a priori* information. We have used the Log-MAP algorithm [8] in these SISO blocks, since the Log-MAP algorithm achieves optimal performance, despite having reduced computational complexity compared to the original MAP algorithm [9].

At the commencement of the iterative detection the SISO equaliser of Figure 2 calculates the Log Likelihood Ratio (LLR) of the receiced coded bits c_i at the channel's output. Subsequently, the extrinsic information L^D_{e-int} is subtracted from the equaliser's output LLR L^E and the resultant L^E_e is channel-deinterleaved to give $L^E_{e-deint}$. This is then passed to the turbo decoder, as illustrated in Figure 2, where the demultiplexer of Figure 3 separates $L^E_{e-deint}$ into the LLR of the source bits, which is denoted by L_{cs} and the LLR of the parity bits from encoder 1 and 2, namely L_{cp1} and L_{cp2}, respectively. For turbo coded systems which employ puncturing, the demultiplexer will also perform 'depuncturing', corresponding to inserting zeros in the punctured bit positions.

In contrast to conventional turbo decoding, no iterations are performed within the turbo decoder of the proposed turbo decoder / equaliser scheme, hence SISO decoder 1 does not receive *a priori* information from SISO decoder 2, and the *a posteriori* LLR F^{D1} at its output is calculated by using the LLR of the source bits L_{cs} and that of the parity bits L_{cp1} only, as seen in Figure 3. Hence, we express the *a posteriori* LLR of the coded bits as:

$$F^{D1} = \left\{ \ln \left(\frac{P(c_i = +1 | L_{cp1} \wedge L_{cs})}{P(c_i = -1 | L_{cp1} \wedge L_{cs})} \right) \right\} . \qquad (1)$$

The extrinsic information generated by decoder 1 is calculated by subtracting the LLR value L_{cs} from the *a posteriori* LLR F_s^{D1} of the source bits. This differs from Raphaeli and Zarai's turbo equaliser implementation [4], which does not take into account the LLR values L_{cs} from the SISO equaliser, when determining the extrinsic information of the source bits. This extrinsic information was then passed through the turbo interleaver of Figure 3 and becomes the *a priori* information L_a of decoder 2. Hence decoder 2 can evaluate the *a posteriori* LLR, F^{D2} by using the *a priori* information L_a from decoder 1, the LLR of the interleaved source bits L_{cs-int} and the parity bits L_{cp2}. The *a posteriori* LLR of the coded bits is expressed as:

$$F^{D2} = \left\{ \ln \left(\frac{P(c_i = +1 | L_a \wedge L_{cp2} \wedge L_{cs-int})}{P(c_i = -1 | L_a \wedge L_{cp2} \wedge L_{cs-int})} \right) \right\} . \qquad (2)$$

Having acquired F^{D2}, the same procedure can be applied to it, as to decoder 1, in order to generate the extrinsic information, which is then passed through the turbo deinterleaver. Instead of passing the de-interleaved extrinsic information $L^{D2}_{e-deint}$ back to decoder 1, this is now passed to the multiplexer MUX. Considering the other two inputs of MUX, besides the extrinsic information $L^{D2}_{e-deint}$ of the source bits, MUX also processes the extrinsic information of the parity bits from both decoders, namely L^{D1}_{ep1} and L^{D2}_{ep2}, respectively. The extrinsic information L^{D1}_{ep1} is simply derived by substracting the input LLR L_{cp1} from the output F_p^{D1} of decoder 1. The same applies to decoder 2 upon exchanging the corresponding indices. The multiplexer MUX selects the extrinsic information L^D_e of the coded bits and after interleaving passes it to the input of the SISO equaliser as *a priori* L_a information, which is shown in Figure 2. This completes the first iteration. In subsequent iterations, the turbo equaliser achieves better performance, until a saturation point, where further iterations will not give substantial improvement or until the chosen termination criterion is satisfied [10]. Due to lack of space here we curtail our algorithmic discourse and the reader is referred to the references cited for a more detailed discussion on the operation of the various algorithms involved.

Simulation Parameters	
Modulation	GMSK: $B_n = 0.3$, transmission frequency=900 MHz, bit rate = 270.833 Kbit/s
BCH turbo encoder	Rate=0.72 BCH (31,26) turbo code No puncturing, random turbo interleaver with a depth of 9880 bits
BCH (31,26) decoder 1 and 2	Log-MAP algorithm
GMSK equaliser	Log-MAP algorithm
Channel Interleaver	171-row, 80-column block interleaver with a depth of 13960 bits
Channel	a. Non-dispersive Gaussian channel b. Narrowband fading channel with a Doppler frequency of 41.7 Hz c. 5-path dispersive fading channel, where all weights have equal power and are symbol spaced, with a Doppler frequency of 41.7 Hz

Table 1: Simulation parameters for the turbo equaliser using the BCH(31,26) turbo decoder.

3. RESULTS AND DISCUSSION

In our simulations we assumed the perfect knowledge of the channel impluse response. Here, we compared the coded BER performance of the turbo equaliser using BCH turbo codes over the non-dispersive Gaussian channel, Rayleigh fading narrowband channel and a 5-path dispersive Rayleigh fading channel, in order to study the performance of turbo equalisers in mitigating the Controlled InterSymbol Interference (CISI) introduced by the partial-response GMSK modem and by the dispersive fading channel, as compared to separate GMSK equalisation and turbo decoding. The parameters used in our simulations are tabulated in Table 1. With reference to Figure 4, we see that the turbo equaliser outperforms the turbo decoder by about 1.5 dB at BER= 10^{-5} after 7 iterations. We are able to obtain this improvement, since the turbo equaliser mitigates the CISI due to the partial-response GMSK modem and the channel-induced ISI in each iterative step, while BCH turbo decoding in isolation does not.

In a narrow-band Rayleigh fading channel, where the Doppler frequency is 41.7 Hz, as shown in Table 1, after

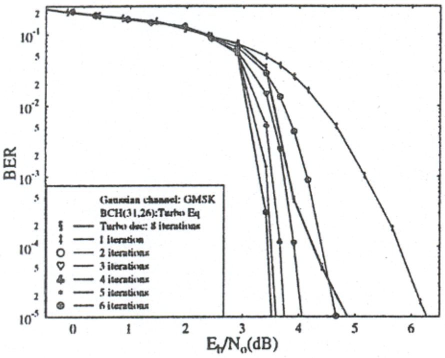

Figure 4: Performance of GMSK turbo equaliser using BCH (31,26) turbo codes and the simulation parameters specified in Table 1, over non-dispersive Gaussian channel after 6 iterations, compared to the BCH (31,26) turbo decoder using 8 iterations.

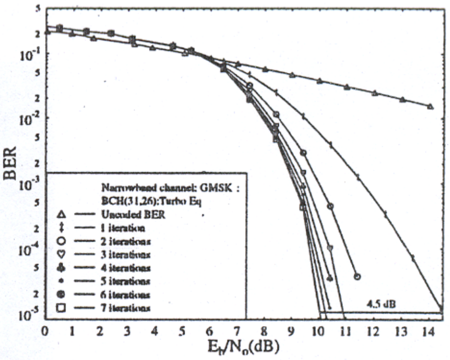

Figure 5: Performance of GMSK turbo equaliser using BCH (31,26) turbo codes and the simulation parameters specified in Table 1, over Rayleigh fading narrowband channels, after 7 iterations.

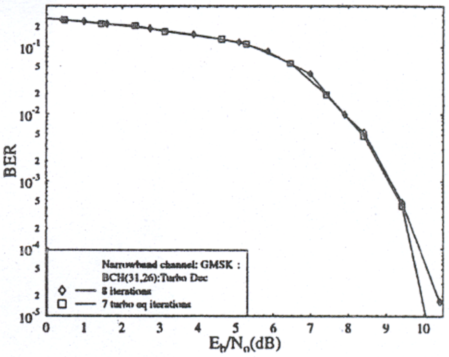

Figure 6: Performance of BCH (31,26) turbo decoded GMSK using the simulation parameters specified in Table 1, after 8 iterations over Rayleigh fading narrowband channels, compared with the GMSK turbo equaliser using BCH (31,26) turbo codes, after 7 iterations.

7 iterations, the iteration gain obtained by using the turbo equaliser is approximately 4.5 dB at BER $= 10^{-5}$, as shown in Figure 5. However, we observed that the performance of the BCH(31,26) turbo decoder, shown in Figure 6, is comparable to that of the turbo equaliser using BCH(31,26) turbo codes, up to $E_b/N_o = 9.5$ dB. An SNR improvement of 0.5 dB is obtained at BER $= 10^{-5}$. The performance of the turbo equaliser using BCH(31,26) turbo codes is not significantly better than that of the stand-alone BCH(31,26) turbo decoder, since in the narrowband Rayleigh-fading channel the main cause of the signal degradation comes from the phase rotation and amplitude fluctuation in the main channel path, since there is no channel-induced ISI added to the CISI of the GMSK signal. Although the SISO equaliser in the turbo equaliser structure of Figure 2 is capable of mitigating the effects of CISI in the GMSK signal, it is unable to compensate for the severe signal attenuations, when the channel is in a deep fade, hence suffering from burst errors.

It is interesting to note that when we perform stand-alone BCH(31,26) turbo decoding, we still receive extrinsic information from the SISO equaliser, like in the turbo equalisation scheme. Since the equaliser cannot compensate for the severe signal attenuation, when the channel suffers from a deep fade, performing the turbo decoding independently from the SISO equaliser will not inflict a severe BER performance degradation compared to the turbo-equalised BCH scheme. The performance of the turbo equaliser using BCH(31,26) turbo codes over the dispersive 5-path channel is about 0.6 dB better, than that of the BCH(31,26) turbo decoder at BER $= 10^{-3}$, as demonstrated in Figure 7. Over this channel the SISO equaliser mitigates the CISI of the GMSK modem, as well as the channel-induced ISI. Furthermore, the equaliser benefits from the inherent multipath diversity, since not all channel paths are attentuated simultaneously, in contrast to the narrowband Rayleigh fading channel. Therefore, by taking account of the latency in the channel, as well as the modulation-induced signal spreading, when performing the decoding, we are able to attain an improved BER performance in comparison to operating the decoding process independently of the equalisation.

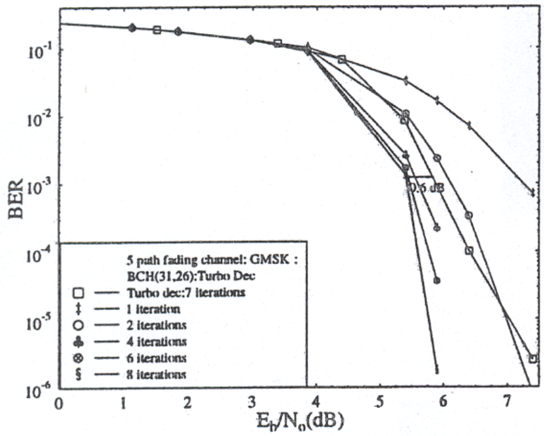

Figure 7: Performance of the BCH(31,26) turbo decoder using the parameters specified in Table 1, after 8 iterations over a 5-path Rayleigh fading channel, compared with the turbo equaliser using BCH(31,26) turbo codes, after 8 iterations.

4. CONCLUSION

In conclusion, we have characterised the performance of a partial response GMSK turbo equaliser, which employs high-rate BCH turbo codes, in order to increase the BER performance of communications systems. Over non-dispersive Gaussian channels we showed that by jointly considering the latency introduced by partial-response GMSK modulation and turbo decoding, we obtained a 1.5dB lower SNR requirement at BER $= 10^{-5}$ compared to the situation, where turbo decoding was performed in isolation from the equalisation. However, over narrowband Rayleigh fading channels the SISO equaliser was unable to compensate for the severe fading-induced impairments of the GMSK signal and hence no significant BER improvement was obtained in comparison to isolated turbo decoding. In contrast, over the dispersive 5-path channel, the turbo equaliser using turbo BCH codes outperformed the isolated schemes by about 0.6 dB at BER$=10^{-3}$.

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