New High-Order Filter Structures Using Only Single-Ended-Input OTAs and Grounded Capacitors

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Abstract
Despite the wealth of literature on OTA-C filters, the synthesis of high-order filter characteristics is still an active topic. In this paper the realization of voltage transfer functions based on canonical current-mode follow-the-leader-feedback (FLF) OTA-C structures are investigated. Two new structures are presented, which use only single-ended-input OTAs and grounded capacitors. The first structure has a single voltage input and multiple voltage outputs taken from different nodes, which enables it to provide simultaneous outputs of different filter functions. The second structure has a single voltage output and single voltage input distributed to different circuit nodes for a universal realization. The authors not only propose such filter structures, but also show how analytical synthesis can be used to produce filter circuits that have less active elements than those recently reported voltage-mode structures which are based on differential-input OTAs. This represents another attractive feature from chip area, and power consumption point of view. Simulation results verifying the theoretical analysis of the proposed filter structure are included.

I. Introduction
Over the last decade or so numerous voltage-mode and current-mode high-order OTA-C filter structures have been reported [1-12]. Such structures have often been developed with different design criteria in mind, including reduced number of active elements, grounded capacitors, and simple design methods. In [9], different voltage-mode structures were proposed capable of synthesising nth-order filters with both transmission poles and zeros. The filters employ 2n+2 active elements, and n grounded capacitors for canonical realizations. The active elements consist of n+2, single-ended-input OTAs, and n, differential-input OTAs. It was pointed out in [8] that single-ended-input OTAs should be employed in place of differential-input OTAs in the filter structure to avoid the generation of parasitic zeros due to the finite input parasitic capacitances of the differential-input OTAs. In [10], an OTA-C filter structure with 3n+2 single-ended-input OTAs, and only one differential-input OTA was presented, a structure that was also mentioned as a noncanonical FLF structure in [9]. Whilst this noncanonical voltage-mode FLF structure has reduced the number of differential-input OTAs, it has increased the number of single-ended-input OTAs from n+2 to 3n+2, clearly an unsuitable solution from component count point of view. In the literature, there have been a number of current-mode multiple loop feedback OTA-C filters using single-ended-input OTAs and grounded capacitors [1, 11, 12]. It would be very interesting to take the advantages of these current-mode filters to realize voltage transfer functions without increase in the component number and without use of differential input OTAs. Furthermore, the structures
of [9-12] have one input and one output which mean that there is a different filter topology for different filtering function. In some applications, however, simultaneous outputs of different high-order filtering functions may be needed.

The aim of this paper is to present two new high-order OTA-C filter structures employing only single-ended-input OTAs, and grounded capacitors, based on the canonical current-mode FLF OTA-C filter structure [11, 12], but for the realization of voltage transfer functions. The first one is capable of providing different filtering functions (LP, HP, BP) from different nodes without changing the filter topology. This is achieved without increasing the number of active elements; in fact, it has the least number of active and passive elements when compared with some recently reported voltage-mode works including [9, 10]. The second one is obtained by adding a linear input distribution OTA network to the first one. Any voltage nth-order transfer functions can be realized by the second one which simultaneously enjoys three main attractive criteria: the minimum components, only single-ended-input OTAs, and only grounded capacitors [8].

**II. New OTA-C Filter Structure for Realizing Nth-Order Low-Pass, Band-Pass, and High-Pass Filters of Voltage Transfer Function**

Fig.1 shows the proposed nth-order filter structure where \( V_{\text{in}} \) is the filter input voltage, and \( V_{\text{out}(0)}, V_{\text{out}(1)}, V_{\text{out}(2)}, V_{\text{out}(3)}, \ldots, V_{\text{out}(n-1)}, \) and \( V_{\text{out}(n)} \) are the \( n+1 \) filter voltage outputs. The settings of the \( n+1 \) filter output voltages determine the filter functions (LP, HP, BP). It can be seen that the structure employs only single-ended-input OTAs and grounded capacitors, an advantage of the current-mode FLF structure [11, 12], which is here taken for realization of voltage transfer functions.

The filter structure of Fig. 1 is obtained as follows. The transfer functions of an nth-order filter with low-pass, band-pass, and high-pass responses at different outputs can be written as:
\[ V_{\text{out}(i)} = V_{\text{in}} \left( \frac{a_i s^i}{\Delta} \right) \text{, for } i = 0, 1, 2 \ldots, n-1, n, \]

where \( \Delta = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \ldots + a_1 s + a_0. \) \hfill (1)

Cross multiplying the transfer function of Eq. (1) as \( i = 0 \), and re-arranging the result, we obtain

\[ V_{\text{out}(0)} \left( \frac{a_n s^n}{a_0} \right) = V_{\text{in}} - V_{\text{out}(0)} - V_{\text{out}(0)} \sum_{i=1}^{n-1} \left( \frac{a_i s^i}{a_0} \right) \]

From Eq. (1), we have

\[ V_{\text{out}(i)} = V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \text{ (i = 1, 2, 3, \ldots, n)} \] \hfill (3)

\[ V_{\text{out}(i)} = V_{\text{out}(0)} \left( \frac{a_i s^i}{a_0} \right) = V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \text{ (i = 1, 2, 3, \ldots, n)} \] \hfill (4)

Substituting Eq. (4) into Eq. (2) gives

\[ V_{\text{in}} - V_{\text{out}(0)} = \sum_{i=1}^{n} V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \]

Eq. (5) is a voltage relationship. In order to be consistent with the input-and-output current relationship of an OTA, i.e., \( V_+ - V_- = I_{\text{out}} \), we multiply each side of Eq. (5) by an equal transconductance of unity value (1), leading to

\[ \left( V_{\text{in}} - V_{\text{out}(0)} \right) l = \sum_{i=1}^{n} V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) l = \sum_{i=1}^{n} V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \left( \frac{\left( a_i s^i \right) l}{a_{i-1}} \right) \]

Then, Eq. (3) can be derived as

\[ V_{\text{out}(i)} = V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \left( \frac{\left( a_i s^i \right) l}{a_{i-1}} \right) \text{ (i = 1, 2, 3, \ldots, n)}, \]

namely,

\[ V_{\text{out}(i)} l = V_{\text{out}(i-1)} \left( \frac{a_i s^i}{a_{i-1}} \right) \left( \frac{\left( a_i s^i \right) l}{a_{i-1}} \right) \text{ (i = 1, 2, 3, \ldots, n)}. \] \hfill (7)

Substituting Eq. (7) into Eq. (6) yields
\[
(V_{in} - V_{out(0)})(1) = \sum_{i=1}^{n} V_{out(i)}(1)
\]  
(8)

By using the active element, OTA, whose characteristic relationship is \( I_{out} = g_m V_{in} \), and the grounded capacitor, whose admittance is \( sC \), to implement Eq. (7), a fundamental OTA-grounded capacitor structure is obtained by a grounded capacitor (with capacitance \( \frac{a_i}{a_{i-1}} \)) connected with the output terminal of a single-ended-input OTA (with transconductance unity) having an input voltage \( V_{out(i)} \) and an output voltage \( V_{out(i-1)} \) across the grounded capacitor. Next, we implement Eq. (8) using the output currents of the OTAs and Kirchhoff's current law. The combination of the individual circuit yields the circuit shown in Fig. 1, in which all the coefficient values are normalized.

In summary, the proposed synthesis method has decomposed the \( n \)-th order transfer function (Eq. (1)) into \( n+1 \) transfer functions including \( n \), 1\(^{\text{st}}\)-order transfer functions (Eq. (7)) and one constraint equation (Eq. (8)). To illustrate the synthesis method, consider the structure generation of a 4\(^{\text{th}}\)-order filter. The synthesis method uses Eqs. (7) and (8). Based on these equations, when \( n=4 \), the 4\(^{\text{th}}\)-order OTA-C filter has the following three transfer functions

\[
\frac{V_{out(0)}}{V_{in}} = \left( \frac{a_0}{\Delta} \right) \quad \text{(Low-pass)}; \quad \frac{V_{out(2)}}{V_{in}} = \left( \frac{a_2s^2}{\Delta} \right) \quad \text{(Band-pass)}; \quad \frac{V_{out(4)}}{V_{in}} = \left( \frac{a_4s^4}{\Delta} \right) \quad \text{(High-pass)};
\]

where \( \Delta = a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 \)

(9)

It should be noted that, in general, a voltage follower may be necessary at the output of the filter to buffer the output and avoid the effects of load capacitance or resistance changing the response of the filter.

### III. New OTA-C Filter Structure for Realizing Nth-Order Universal Voltage Transfer Functions

In the above section, we proposed a new filter structure, shown in Fig. 1, for realizing \( n \)-th order OTA-C low-pass, band-pass, and high-pass filters. Eq. (1), corresponding to Fig. 1, shows that there are \( n+1 \) different-order transfer functions which can be realized at \( n+1 \) different nodes in Fig. 1, respectively. The general \( n \)-th order voltage transfer function, shown as below,

\[
\frac{V_{out}}{V_{in}} = \frac{b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \ldots + b_2 s^2 + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_2 s^2 + a_1 s + a_0}
\]

(10)

is the linear combination of the \( n+1 \) different transfer functions shown in Eq. (1), i.e.,

\[
\frac{V_{out}}{V_{in}} = \frac{b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \ldots + b_2 s^2 + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_2 s^2 + a_1 s + a_0} = \sum_{i=0}^{n} \left( \frac{b_i}{a_i} \right) \left( \frac{a_i s^i}{\Delta} \right)
\]

where \( \Delta = a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 \).

(11)
Two synthesis approaches to realize the above relationship are proposed as follows.

(i) Using the linear combination method to perform the synthesis, multiply both sides of Eq. (11) by $V_{in}$ and obtain

$$
V_{out} = (1) \times \sum_{i=0}^{n} \left[ \left( \frac{b_i}{a_i} \right) \left( \frac{a_i s^i}{\Delta} \right) V_{in} \right]
$$

(12)

Then, we take each nodal voltage in the circuit structure of Fig. 1, i.e., $V_{out(i)} (= \frac{a_i s^i}{\Delta} V_{in})$ in which $i = 0, 1, 2 \ldots, n-1,$ and $n,$ as the input voltage of an extra OTA with the transconductance $b_i/a_i.$ Join all of the output terminals of the $n+1$ extra OTAs and connect the summing point with an equivalent grounded resistor realized by a single-ended-input OTA with unity transconductance. The realized circuit structure uses $2n+4$ single-ended-input OTAs and $n$ grounded capacitors.

(ii) A more effective synthesis approach is explained as follows. Multiply both sides of Eq. (11) by $V_{in}$ and obtain the following other form (different from Eq. (12))

$$
V_{out} = \sum_{i=0}^{n} \left[ \left( \frac{b_i}{a_i} \right) \left( \frac{a_i s^i}{\Delta} \right) V_{in} \right]
$$

(13)
The physical meaning of the above relationship is “to insert different weights of the input voltage signal, \( V_{in} \), into each node in the filter structure shown in Fig. 1 and then obtain the output voltage signal”. According to this approach \([1, 9, 12]\), giving a forward signal, with a weight of input voltage signal, from input voltage node to each inner node in the filter structure shown in Fig. 1, we obtain the other new OTA-C filter structure, shown in Fig. 2, for realizing the general nth-order voltage transfer function shown in Eq. (10). The realized circuit structure uses 2n+2 single-ended-input OTAs and n grounded capacitors.

The circuit structure (Fig. 2) of approach (ii) uses two fewer single-ended-input OTAs than that (Fig. 3) of approach (i) and is recommended to realize the general nth-order voltage transfer function shown in Eq. (10). Note that, in Fig. 2, all 2n+2 OTAs have single-ended input and all n capacitors are grounded. The numbers of active OTAs and grounded capacitors are the minimum numbers \([9]\) to realize such a general nth-order voltage transfer function shown in Eq. (10).

In Fig. 1, although we let all the tranconductances of the OTAs be unity and let the capacitances of n grounded capacitors be \( a_1/a_0 \), \( a_2/a_1 \), \( a_3/a_2 \) \( \ldots \ldots \), \( a_{n-1}/a_{n-2} \), and \( a_n/a_{n-1} \), in fact, all the values of the transconductances and the capacitances can be given flexibly. The restriction of the values of the transconductances and the capacitances shown in Fig. 1 is used for being consistent with the derivation process and the original given transfer functions shown in Eq. (1). Similarly, in order to simplify and clarify the network analysis of Fig. 2, we let the value of each grounded capacitor be unity although, as a matter of fact, it can be given by any different required value. Circuit analysis yields the following transfer function of Fig. 2:

\[
\frac{V_{out}}{V_{in}} = \sum_{i=0}^{n} s^i B_i + \sum_{i=0}^{n} s^i \left[ \sum_{j=0}^{n-i} g_{bij} \left( \prod_{k=0}^{n-i-j} g_{a(i-j-k)} \right) \right] + g_{bd} \left( \prod_{j=0}^{n-i} g_{a(j)} \right) = \sum_{i=0}^{n} s^i B_i + \sum_{i=0}^{n} s^i A_i
\]

All \( g_{ai} \), \( i = 0, 1, 2 \ldots, n-1, n \), can be found exactly, i.e.,

\[
g_{an} = a_n, \quad \text{and} \quad g_{ai} = \frac{a_i}{a_{i+1}} \quad \text{for} \quad i = 0, 1, 2 \ldots, n-2, n-1, n
\]

by solving the following \( n+1 \) equations

\[
\prod_{j=0}^{n-i} g_{a(n-j)} = a_i, \quad i = 0, 1, 2 \ldots, n-1, n.
\]

And all \( g_{bi} \), \( i = 0, 1, 2 \ldots, n-1, n \), can be found exactly by solving the following another \( n+1 \) equations

All \( a_i \), \( i = 0, 1, 2 \ldots, n-1, n \), can be found exactly, i.e.,

\[
g_{an} = a_n, \quad \text{and} \quad g_{ai} = \frac{a_i}{a_{i+1}} \quad \text{for} \quad i = 0, 1, 2 \ldots, n-2, n-1, n
\]

by solving the following \( n+1 \) equations

\[
\prod_{j=0}^{n-i} g_{a(n-j)} = a_i, \quad i = 0, 1, 2 \ldots, n-1, n.
\]
\[ g_{bn} = b_n \]
\[ \sum_{j=0}^{1} g_{bj} \left( \prod_{k=0}^{n-1-j} g_{ai(j+k)} \right) = b_i , \ i = 1, 2 \ldots, n-1. \tag{17} \]
\[ g_{bo} \left( \prod_{j=0}^{n-1} g_{bj} \right) = b_0 \]

In any cases, if the calculated \( g_{bj} = 0 \), then the corresponding OTA should be eliminated; and if the calculated \( g_{bj} \) is a negative value, then the two input terminals of the corresponding OTA should be interchanged.

The relative sensitivities, \( S_Y^X = \left( \frac{x}{Y} \frac{\partial Y}{\partial x} \right) \), are the values of filter characteristic parameters with respect to circuit elements. If we let the values of all capacitors from the right side in Fig. 2 be \( C_0, C_1, C_2 \ldots, C_{n-2}, \) and \( C_{n-1} \), respectively, as we divide both numerator and denominator of its transfer function by \( (C_0 C_1 C_2 \ldots C_{n-2} C_{n-1}) \), the transfer function of Fig. 2 is similar to Eq. (14) by replacing \( g_{ai} \) with \( g_{ai}/C_1 \) for \( i = 0, 1, 2 \ldots, n-2 \) and \( n-1 \). Due to space limitation, only key sensitivity results are quoted without derivation. For example, the denominator coefficient sensitivities to \( g_{ai} \) where \( i = 0, 1, 2 \ldots, n-1, n \), are 1 or 0. The numerator coefficient sensitivities to \( g_{bi} \) are

\[ S_{g_{bi}}^{B_{si}} = 0 \text{ for } i = 0, 1, 2 \ldots, n-1, \ S_{g_{si}}^{B_{si}} = 1, \]
\[ S_{g_{bi}}^{B_{si}} = 0 \text{ for } i = 1, 2 \ldots, n-1, n, \ S_{g_{si}}^{B_{si}} = 1, \]
\[ S_{g_{bi}}^{B_{si}} = \frac{g_{bj} \left( \prod_{k=0}^{n-1-i} g_{ai(j+k)} \right)}{\sum_{j=0}^{1} g_{bj} \left( \prod_{k=0}^{n-1-j} g_{ai(j+k)} \right)} \text{ for } i = 1, 2, 3 \ldots, n-1 \text{ and } j = 0, 1, 2 \ldots, i. \tag{18} \]

Detailed numerator and denominator coefficient sensitivities with respect to \( g_{ai}, g_{bi}, \) and \( C_i \) are in the range from -1 to 1 provided that all \( g_{ai} \) and \( g_{bi} \) are positive.

**IV. Comparison with Previous Works and Simulation Results**

Table 1 shows a comparison in terms of number of OTAs and types of OTAs between the proposed filter structures (Figs.1 and 2), and those recently reported in [9] and [10].
Table 1 Comparison of recently reported nth-order OTA-based filters

<table>
<thead>
<tr>
<th>Parameters Cases</th>
<th>Number of OTAs</th>
<th>Types of OTAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nth-order proposed filter (Fig. 1)</td>
<td>n+2</td>
<td>Single-input (n+2)</td>
</tr>
<tr>
<td>Nth-order proposed filter (Fig. 2)</td>
<td>2n+2</td>
<td>Single-input (2n+2)</td>
</tr>
<tr>
<td>Nth-order general filter in [9]</td>
<td>2n+2</td>
<td>Single-input (n+2), Double-input (n)</td>
</tr>
<tr>
<td>Nth-order general filter in [10]</td>
<td>3n+3</td>
<td>Single-input (3n+2), Double-input (1)</td>
</tr>
</tbody>
</table>

Another parameter which needs to be compared with one another is the span (or spread) of filter component values which may be large for high-order filters. A wider span of filter component values leads to a worse implementation carried out in CMOS which will create the quadratic current ratios. To give insight into what the component spread of the new filter structure shown in Fig.1, Table 2 provides the comparison of the fourth-order Butterworth high-pass (HP), low-pass (LP), and band-pass (BP) filters designed using the proposed filter (Fig.1) and those reported in [9, 10]. The Table gives the component spread for equal transconductance (g) and equal capacitance (C) designs. It can be seen that the proposed filter has the same component spread (equal g design) as those in [9, 10] except the case for band-pass in [9]. In the case of equal C design, the filter in [10] has the least component spread, and our filter has better component spread than that in [9].

Table 2 Component spread comparison of the fourth-order HP, LP, and BP Butterworth filters

<table>
<thead>
<tr>
<th>Spread Case</th>
<th>Equal g design</th>
<th>Equal C design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C&lt;sub&gt;min&lt;/sub&gt;</td>
<td>C&lt;sub&gt;max&lt;/sub&gt;</td>
</tr>
<tr>
<td>HP (Fig. 1)</td>
<td>0.383</td>
<td>2.613</td>
</tr>
<tr>
<td>HP [10]</td>
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</tr>
<tr>
<td>HP [9]</td>
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<tr>
<td>LP [9]</td>
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<td>2.613</td>
</tr>
<tr>
<td>BP (Fig. 1)</td>
<td>0.383</td>
<td>2.613</td>
</tr>
<tr>
<td>BP [10]</td>
<td>0.383</td>
<td>2.613</td>
</tr>
<tr>
<td>BP [9]</td>
<td>0.383*</td>
<td>2.613*</td>
</tr>
</tbody>
</table>

*The transconductance spread is two (not unity).
To verify the theoretical analysis of the proposed filter structure shown in Fig. 1, a third-order low-pass and high-pass OTA-C filter has been simulated using H-Spice with the UMC05 level-49 parameters, with W/L = 5µ/1µ for NMOS and 10µ/1µ for PMOS transistors, and the component values: (i) $g_{0,1,2,3,4}=56\mu S$, $I_b=8.852\mu A$, $C_1=18pF$, $C_2=9pF$, and $C_3=4.5pF$ for the responses with $f_o=0.990MHz$, using $V_{dd}=2.5V$, and $V_{ss}=-2.5V$, (ii) $g_{0,1,2,3,4}=62.8\mu S$ ($I_b=10.495\mu A$), $C_1=10pF$, $C_2=5pF$, and $C_3=2.5pF$ for the responses with $f_o=1.999MHz$, using $V_{dd}=2.5V$, and $V_{ss}=-2.5V$. The OTAs were implemented using the CMOS circuit given in [13]. Fig. 3 shows the simulated low-pass and high-pass responses of the filter. As can be seen there is a close agreement between theory and simulation, for example, the simulated 3dB frequencies are 0.94MHz (high-pass) and 1.04MHz (low-pass), compared to 0.990MHz in the ideal case, and 1.83MHz (high-pass) and 2.02MHz (low-pass), compared to 1.999MHz in the ideal case. Other filters with different order and responses were also simulated and found to perform as theory predictions.

V. CONCLUSION

This paper has presented two new high-order filter structures (shown in Figs. 1 and 2) that employ only single-ended-input OTAs and grounded capacitors. It has been shown how decomposing analytically an nth-order transfer function into $n+1$ simple realisable transfer functions using OTA-C circuits produces the first filter structure that employs less active elements than some of the recently reported methods and offers simultaneous multiple outputs. The proposed first filter structure has $n+2$ single-ended-input OTA, and $n$ grounded capacitors for a given filter order, $n$. Realizing the general transfer function by using the more effective synthesis approach different from the linear combination of each output signal of the first filter structure leads to the second filter structure which can realize any kind of voltage transfer functions employing the minimum components, only $2n+2$ “single-ended-input” OTAs, and only $n$ “grounded” capacitors.

ACKNOWLEDGEMENTS

The authors would like to thank the anonymous reviewers for their constructive comments.

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Fig.3. Amplitude-frequency response of third-order low-pass and high-pass filters