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The modelling of VLF Trimpis using both finite element and 3D Born modelling

K. Baba

Department of Electronic Engineering, Chubu University, Kasugai Aichi, Japan

D. Nunn and M. Hayakawa

Department of Electronic Engineering, University of Electro-Communications, Chofu, Tokyo, Japan

Abstract.
This paper investigates the numerical modelling of VLF Trimpis produced by a D region inhomogeneity on the Great Circle Path. Two different codes are used. The first is a 2D finite element method (FEM) code, whose solutions are valid in the non-Born limit. The second is a 3D model that invokes the Born approximation. The predicted Trimpis from these codes compare closely, thus confirming the validity of both models. The modal scattering matrices have a comparable structure, and indicate strong scattering between the dominant TM modes. Analysis of the scattering matrix from the FEM code delineates the Born regime. For a LIE with a radius of 100kms, the Born approximation becomes invalid at an electron density perturbation of about 8 el/cc.

1. Introduction

The Trimpi phenomenon is a well-known geophysical problem in VLF wave propagation that has been extensively studied both experimentally and theoretically [Nunn, 1997]. Trimpis comprise transient perturbations in amplitude and phase of sub-ionospheric CW VLF signals, ~0.2 dB and 5 degs, lasting ~100 secs.

The so-called 'Classic Trimpis' are believed to be due to lightning whistlers, which interact at the equator with radiation belt keV electrons, and cause them to precipitate into the D region of the nightime ionosphere, giving localised patches [Helliwell, 1973]. These patches or LIE's (Localised ionisation enhancements) may be inferred to occupy a height range of 65-85 kms, with horizontal dimensions of order 40-150 kms [Dowden and Adams, 1989].

There is a need for accurate modelling of the mechanism producing Trimpis, as this can resolve the question of the physical mechanisms creating the LIE's. Modelling will enable us to use Trimpi observations from multiple transmitters and receivers, to solve the 'inverse problem' and map particle precipitation from the magnetosphere into the ionosphere. We shall model Great Circle Path Trimpis with reference to the path from the 22.3kHz VLF transmitter NWC in NW Australia to Dunedin NZ, by using this frequency and path length [Dowden and Adams, 1989]. This is a mixed land/sea path of length 5.74 Mm. The modelling will use two distinct techniques. The first is the Finite Element Method (FEM) due to Baba and Hayakawa [1995, 1996], which is a 2D, rigorously non-Born approach (i.e. strong scattering). The second is due to Nunn [1997], and embodies a 3D linear scattering (Born) formalism. Our task is to compare the results from these two codes using identical parameters in each case. This will serve to validate both theoretical approaches.

2. Previous theoretical and numerical approaches to modelling

Progress towards accurate numerical modelling of the Trimpi phenomenon has been slow. Poulsen and coworkers [Poulsen et al., 1993] adopted a linear scattering (Born) approach and used an expression due to Wait [1964] which assumes that intermodal scattering is negligible and that the spatial scale of the patch is much greater than a wavelength, though neither condition is satisfied. Recently Baba and Hayakawa [1995, 1996] have used the finite element method (FEM) to attack the Trimpi problem [McDonald and Wexler, 1972]. In view of the complexity of the problem their treatment is 2D. The earth is assumed a perfect conductor, and the ionosphere isotropic (B_o=0) with a realistic electron density and collision frequency profile.

The approach of Nunn [1997] is 3 dimensional with an anisotropic ionosphere and earth curvature, but invokes the Born approximation. At each point in the LIE an effective source current $J_{eff}(r)$ may be defined by

$$J_{eff}(r) = \frac{jk^2}{\omega\mu_0}\chi'(r)E_o(r) = \sigma' E_o \qquad (1)$$

where $\chi'(r)$ is the perturbation in susceptibility tensor, and $E_o(r)$ is the zero order incident field [Nunn, 1997]. The scattered field $E'(r)$ is sourced upon the current distribution $J_{eff}(r)$ located within the LIE. It only remains to deal with the propagation problem. Nunn uses modal theory and the National Ocean Systems Centre, San Diego (NOSC) VLF propagation facility MODEFNDR [Morfitt and Shellman, 1976]. For short ranges less than 300 kms this is not valid and a full wave approach would be required.

3. The numerical modelling

(a) THE FEM CODE
The FEM code has two spatial coordinates, x along

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the direction of propagation and z vertically. The enhancement of electron density in the LIE, $\delta Ne(x,z)$ is assumed to be a Gaussian function as follows

$$\delta Ne(x,z) = \delta Ne^o \exp\{-(x-x_o)^2/S_x^2 - (z-z_o)^2/S_z^2\} \quad (2)$$

The average patch height is z_o=75 kms, which coincides with the level of maximum scattering [Nunn, 1997]. A Gaussian half width of S_z=10 kms was used. The horizontal scale of the LIE, S_x, is not well known and was varied from 10-100 kms. The FEM code employs an isotropic ionosphere with a night-time electron density profile given by

$$N_o(z) = (30.24)e^{0.24(z-h)} \qquad el/cc \quad (3)$$

derived from Reagan et al. [1982], where h=87 kms and z is in kms. The assumed collision frequency profile is

$$\nu(z) = (5.10^6)e^{-0.15(z-h)} \qquad s^{-1} \quad (4)$$

where h=70 kms and z is in kms. The ground is modelled as a conducting half space, and earth curvature is not modelled. The upper height of the simulation box is 100 kms. At the upper boundary the tangential field components are matched to those of an upgoing wave in a semi-infinite half space above z=100 km. At the downstream end of the box the boundary condition is one which can be obtained from the modal function expansions and is known as the finite element boundary integral method as proposed in McDonald and Wexler [1972]. The code analyses the field in terms of normalised TM modes in the waveguide. The incident field is assumed to consist of one TM mode only, either the dominant mode (no.2) or the semi-earth detached mode (no.1). In this paper we shall calculate Trimpis on the ground downstream from the LIE. The scattered field at the downstream end of the box will be expanded in TM modes, thus giving the modal scattering matrix M_{ij} for the strip perturbation.

(b) THE BORN 3D CODE.

The ionosphere profile Ne, ν is as above for comparison purposes. In order to facilitate the comparison with the FEM code, for the runs in this paper the ambient magnetic field B_o will be switched off. A later paper will deal with the case of the anisotropic ionosphere. The ground is assumed to have conductivity and dielectric constant of sea water, although some of the path is over land. The transmitter and receiver are modelled as vertical electric dipoles (ved's). The LIE $\delta Ne(x,y,z)$ is modelled as a Gaussian in x and z as in equation (2), and the y dependence is also Gaussian with a scale length $S_y=2.75S_x$ i.e. the LIE is an ellipse with ellipticity $e=S_y/S_x$=2.75, with the long axis perpendicular to the direction of propagation in the y direction. In the case of S_x=10 the ellipticity e is increased to e=10. For comparison purposes we may restrict the incident zero order field to consist of one TM mode only and compute Trimpis at the ground as a function of range downstream from the LIE.

4. The results

We shall first investigate the behaviour of the modal scattering matrix M_{ij} in the case of the FEM code. This may be defined as follows. If the field incident

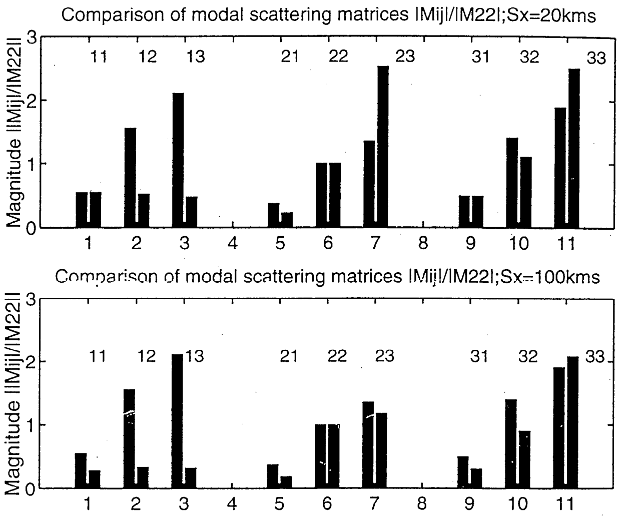

Figure 1. FEM 2D code. Plot of modal scattering matrix elements $\mid M_{ij} \mid$ as functions of electron density perturbation δNeo on a log-log scale. The quantity plotted is $Y_{ij}=[\mid M_{ij} \mid/\delta Neo/\mid M_{ij} \mid(1)]$. The line Y_{ij}=1 represents linear 'Born' behaviour.

on the strip perturbation consists of normalised mode i with unit amplitude, then the relative amplitude of mode j downstream of the patch will be $\mid M_{ij} \mid$. We select S_x=100 kms, and S_z=10 kms and z_o=75 kms. The phase of M_{ij} is important, but the magnitude gives us more intuitive information so we plot the $\mid M_{ij} \mid$ in Fig. 1 as a function of maximum electron density perturbation δNeo, over a range from 1-100 /cc. For clarity we plot the quantity $Y_{ij}=[\mid M_{ij} \mid /\delta Neo/ \mid M_{ij} \mid(1)]$. We only consider the leading 3 TM modes, nos 1, 2, 3, which correspond to nos 2, 3, 5 as returned by MODEFNDR. This gives us 9 coefficients in all. If the Born approximation holds then we expect the $\mid M_{ij} \mid$ to be proportional to δNeo, and Y_{ij} =1. Departure from this linear dependence indicates non-Born behaviour, and progressive exclusion of the incident field E_o from the patch due to the skin depth effect. We may ignore $\mid M_{11} \mid$, which has a very small value, since in the absence of earth curvature mode 1 has little penetration at the main scattering level at ~75 kms. We note that the approach to linearity is fairly asymptotic, and rigorously Born behaviour is only achieved at very small δNeo. Inspection shows that Born behaviour can be expected for δNeo less than 6 el/cc, and even then the Trimpis will be overestimated by some 20% in the Born code. Other computations showed that for smaller S_x=20 kms, δNeo less than 20 el/cc was close to the Born limit.

In Fig.2 we compare the $\mid M_{ij} \mid$ matrix from the FEM code with that calculated in the 3D Born code. We take the case δNeo=10 el/cc, z_o=75 kms, S_z=10 kms. In a 3D geometry the matrix $\mid M_{ij} \mid$ is defined for a single column of ionisation [Nunn, 1997]. We compare matrices normalised to $\mid M_{22} \mid$ for S_x=20, 100 kms. We see that the matrix structures are qualitatively similar, with the exception of $\mid M_{12} \mid$ and $\mid M_{13} \mid$, which the FEM code underestimates due to the absence of earth curvature in the model, and of $\mid M_{23} \mid$, which the FEM code calculates as being some 70% larger than the 3D Born code. The obvious features are (a) Di-

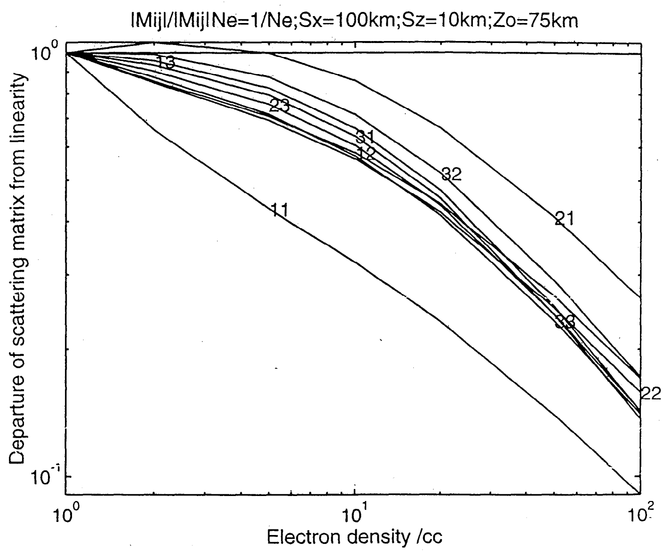

Figure 2. Comparison of the structure of the modal scattering matrices $| M_{ij} |$ for the FEM and 3D Born codes, with B_o=0 in both cases. Elements are normalised to $| M_{22} |$ for comparison purposes.

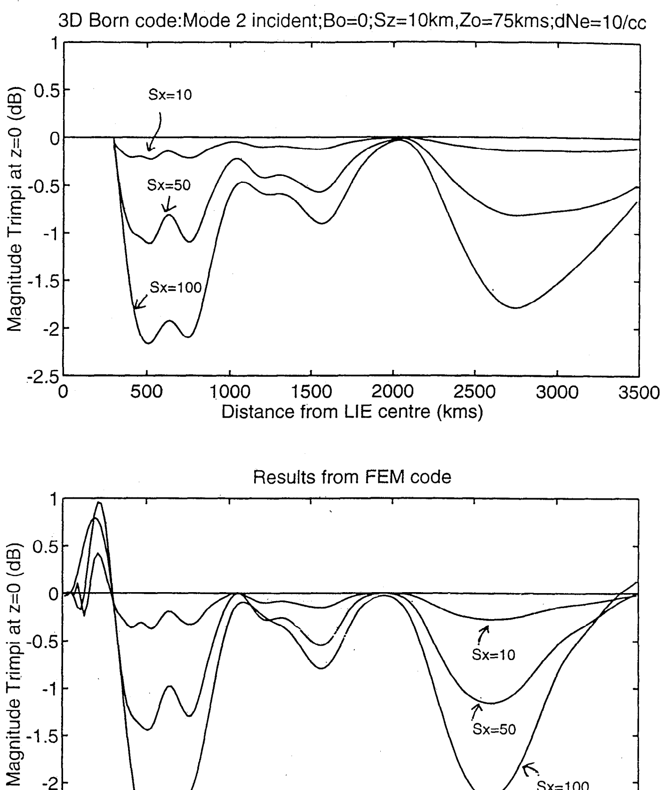

Figure 3. Top panel: 3D Born code. Plot of ground amplitude Trimpis as a function of distance downstream from the LIE centre, assuming B_o=0. Curves are for 3 values of S_x=10, 50, 100 kms. Only the dominant TM mode 2 is assumed incident. Bottom panel: Same case for the FEM code. The close similarity between the two results is apparent.

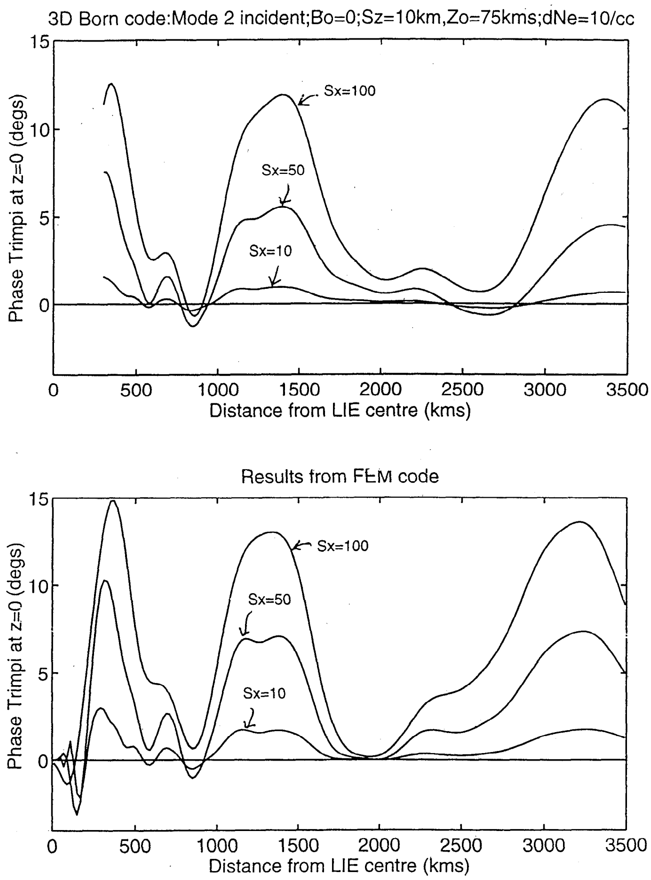

Figure 4. Top panel: 3D Born code. As figure 3 but comparing phase Trimpis for the FEM and 3D Born codes. Again close agreement will be noted.

agonal components $| M_{jj} |$ increase with mode order j, (b) Strong two way coupling between the dominant TM mode (j=2) and the next TM mode (j=3), (c) Weak coupling from modes 2, 3 to the earth detached mode (j=1), (d) Strong coupling from the earth detached mode (j=1) to TM modes (j=2, 3), absent without earth curvature. Mode coupling by the scattering inhomogeneity is very important, in full agreement with results obtained theoretically in Wait [1995].

We will now compare Trimpis on the ground, computed as a function of x, the distance downstream from the LIE centre. The upper panels of Figs. 3 and 4 plot Trimpis for the 3D Born code, using δNeo=10 /cc, z_o=75 kms, S_z=10 km, and for 3 values of S_x=10, 50, 100 kms. The transmitter is placed 3Mm from the patch, and only the dominant TM mode no. 2 is incident on the LIE. The patch has an ellipticity $e=S_y/S_x$=2.75 for S_x=100, 50 kms and a value e=10 for S_x=10 km, and is placed with the long axis at right angles to the x axis. The lower panels are the plots for the FEM code using the same data. The FEM code computes the Trimpis due to an infinite slab perpendicular to the direction of propagation. In the Born 3D case an elongated ellipse oriented perpendicular to is expected to produce similar Trimpi values, since it is well known that LIE's more than 200 kms from the Great Circle Path produce very small Trimpis [Poulsen et al., 1993], the scattering being predominantly in the forward direction. In numerical experimentation it was found that the resultant Trimpis were almost independent of patch ellipticity provided this was greater than about two.

We see from Fig. 3 that the amplitude Trimpis are very similar with a double negative peak of 2.5 dB at x=600 km and a negative peak of ~-2 dB at x=2600 km. A third peak of ~-0.8 dB at x=-1500 km is apparent in both cases. The ratios of the Trimpis for the three horizontal scales are about the same.

The phase Trimpi plots (Fig. 4) show an excellent correlation. Positive peaks of about +12 degs. will be seen at 400, 1400 and 3300 kms. In view of the radically different nature of the two codes the closeness of these sets of results are remarkable. However, there are fundamental differences between the two models, so exact correspondence was not to be expected. The 3D Born code has earth curvature and calculates the Trimpis due to an elongated ellipse in the y direction. The Born 3D code will deviate from the FEM because it does not take into account non-Born shielding inside the LIE of the incident wavefield. In this paper neither code has an anisotropic ionosphere. Code comparisons with anisotropic ionosphere will be covered in a subsequent paper.

5. Conclusion

This paper has set out to investigate the scattering of subionospheric VLF radiation by nightime D region ionospheric inhomogeneities or LIEs. Results from the two codes (FEM and Born) have been compared, and in view of some fundamental differences the results are surprisingly close. This tends to confirm the validity of both modelling techniques.

The modal scattering matrices in both cases have very similar structures, and both models point to strong intermodal scattering beween the dominant TM modes. Analysis of the scattering matrix from the FEM code has shown the extent of validity of the Born approximation. Roughly speaking a LIE of radius 100kms will become non Born at a peak electron density perturbation of about 8/cc.

The path for future research on this problem is clear. For the FEM approach work is underway on inclusion of earth curvature and ionospheric B_o field and use of an appropriate mix of incident TM modes. A 3D model is being developed, and it will be ideally suited to compute scattering from small intense structures such as sprite columns. The 3D Born code has already been upgraded by incorporation of an attenuation function for the incident zero order field, calculated from the skin depth appropriate to the LIE. For short range problems use of a full wave propagation code is called for.

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K. Baba, Department of Electronic Engineering, Chubu University, 1200 Matsumoto-cho, Kasugai, Aichi 487-8501, Japan. (e-mail: baba@solan.chubu.ac.jp)

D. Nunn, Department of Electronics and Computer Science, University of Southampton, Southampton, Hants. SO17 1BJ, UK. (e-mail: dn@ecs.soton.ac.uk)

M. Hayakawa, Department of Electronic Engeneering, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182-8585, Japan. (e-mail: hayakawa@aurora.ee.uec.ac.jp)

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