Highly anisotropic distributions of energetic electrons and triggered VLF emissions

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Abstract. Bell et al. [2000] have reported an exciting set of new observations, made near the equatorial plane of the plasmasphere on the $L = 3.4$ flux tube, of hiss emissions between 4.0 and 5.6 kHz, highly anisotropic energetic electrons, and unusual VLF triggering effects at 10.2 kHz. Here we show that these data as a whole can be considered as the first experimental evidence of the existence of a special type of distribution function of energetic electrons with a step-like feature in the velocity component parallel to the magnetic field. Such a shape of velocity distribution can be crucial for explanation of triggered VLF emissions and chorus. Assuming the distribution to be step-like, we can readily explain the spatial distribution of energetic electrons along the field line observed by Bell et al. [2000], and also strong wave amplification in the triggering process. We discuss how the step in the velocity distribution is maintained by the hiss emission. The frequency gap between the hiss band and the triggered signal is connected with the excitation of the quasi-electrostatic mode near the upper frequency edge of the hiss band.

1. Introduction

An interesting highly anisotropic distribution function of energetic electrons (energy from 1 to 20 keV) was recently revealed in POLAR particle data [Bell et al., 2000, hereafter referred to as I], with an average equatorial pitch angle exceeding $\sim 75^\circ$. The measurements made along the $L = 3.4$ magnetic flux tube on 13 January, 1997, three days after a much studied space weather event, found these energetic particles only within $7^\circ$ latitude of the magnetic equator, in accordance with a high value of the pitch angle anisotropy. Hiss band emissions between 4.0 and 5.6 kHz, and VLF (10.2 kHz) triggered effects, were observed simultaneously (see Fig. 1 of [I]). Basing their study on these data, Bell et al. [2000] estimated the gyroresonance amplification, which could be sufficient for VLF triggering. They considered a conventional anisotropic distribution function with a very high degree of transverse anisotropy and found that the necessary amplification could be achieved if the higher energy ($>20$ keV) electron fluxes are sufficiently intense.

On the other hand, strong amplification of an initial signal, which is necessary for triggering effects, can be supplied by a very different type of electron distribution function with a step-like deformation in the magnetic field aligned velocity component. Such a distribution can have a moderate integral anisotropy, but a very sharp local gradient in velocity space providing strong amplification of waves in a narrow frequency band centered on the frequency of cyclotron resonance with the step. Step-like deformations can appear in electron distribution functions in a natural way in the course of development of the cyclotron instability related to the generation of hiss. Such a deformation was actually obtained in calculations performed by Lyons et al. [1972], who considered the formation of the energetic electron distribution function under the action of plasmaspheric ELF hiss emissions in the quasilinear approach. They also revealed some evidence for the step-like feature in data from the OGO-5 satellite. Within the framework of the self-consistent quasilinear theory, a transient step-like deformation of the distribution function of energetic electrons was obtained by Trakhtengerts et al. [1996] in computer simulations of the cyclotron instability in the Earth’s magnetosphere. Distributions with a step are important in relation to another important problem, namely, chorus generation. As shown by Trakhtengerts [1995; 1999], a step-like deformation can give rise to an absolute instability (backward wave generation regime) that can be the basis for a quantitative theory of chorus formation (see, as well, Villalon and Burke [1997]).

The aim of this paper is to show that the results of [I] can actually serve as the first convincing, though indirect, experimental evidence of the existence of step-like distributions. We demonstrate that the detected properties of the energetic electron distribution and the strong amplification of the initial wave, responsible for triggering, can be explained by the step-like deformation without additional assumptions. We also discuss how the simultaneously observed VLF hiss can maintain this deformation in the distribution function.

2. Step-like distribution function and related effects

The step, which appears during the development of the cyclotron instability [Lyons et al., 1972; Trakhtengerts et al., 1996; Trakhtengerts and Rycroft, 2000], divides the velocity phase space into two parts, one (with lower values of $F$) corresponding to electrons which are at some point in cyclotron resonance with the hiss band generated, and the other corresponding to always nonresonant electrons (Fig. 1). In the case when the value of the velocity, $V_F$, corresponding to the step, is much less than the characteristic velocity $V_0$ of the energetic electrons, we have a highly anisotropic distribution. Fig. 2 shows the dependence of the energetic electron number density on the magnetic latitude from the equator, along the $L = 3.4$ magnetic flux tube, in the case of a step-like distribution. The case ($V_F/V_0) = 0.2$ corresponds to the observations made on POLAR. If we put the energy...
along the geomagnetic field $-\mathbf{B}$.

\begin{equation}
W_r = mV_r^2/2 \sim 1.2 \text{ keV}, \text{ this corresponds to the characteristic energy } W_0 \sim 30 \text{ keV}.
\end{equation}

For whistler-mode wave generation with wave vector \( \mathbf{k}_w \) along the geomagnetic field \( \mathbf{B} \), the modulus of the velocity \( V_r \) is related to the upper frequency \( \omega_h \) of the hiss emission by the cyclotron resonance condition

\begin{equation}
V_r = \frac{\omega_{BL} - \omega_h}{k_w}
\end{equation}

where \( k_w = \omega_{BL}/\omega_h \). \( \omega_{BL} \) and \( \omega_pL \) are the electron gyrofrequency and plasma frequency, respectively, on the equator at \( L = 3.4 \). For a dipolar geomagnetic field, at \( L = 3.4 \), \( \omega_{BL}/2\pi \approx 22 \text{ kHz} \).

According to the experimental data [1], hiss occurs with an upper frequency limit of \( \omega_h/2\pi \approx 5.6 \text{ kHz} \). The VLF triggered signal observed simultaneously, with a higher frequency \( f_{\text{trig}} = \omega_{\text{trig}}/2\pi = 10.2 \text{ kHz} \) can be explained by cyclotron amplification by a step in the distribution function (see below), if we take into account resonant cone waves with a frequency close to the resonant frequency \( \omega \approx \omega_{BL} \cos \theta \), where \( \theta \) is the wave normal angle between \( \mathbf{k} \) and \( \mathbf{B} \). Under real magnetospheric conditions the cyclotron instability generates whistler mode waves inside ducts of enhanced plasma density, which are formed just inside the plasmapause (or by filaments outside with much enhanced cold plasma density). These ducts permit multi-hop propagation and the amplification of whistler waves in limited frequency bands.

As the whistler-mode wave frequency \( \omega \) increases, the generation of oblique and resonant cone waves becomes important. These waves are detected in satellite data [Hayakawa et al., 1986; Hayakawa and Sazhin, 1992, Hayakawa, 1993], and their generation has been considered in many papers [e.g., Brinca, 1972; Cuperman and Sertl, 1974; Trakhtengerts, 1963]. Our estimations in the special case with a step-like distribution function show a more effective generation of resonance cone waves in comparison with whistler-mode waves with \( k || B \), for frequencies \( \omega > 0.3f_{\text{BL}} \). Then the wave frequency is not actually changing with a decrease of \( V_r \), and the cyclotron resonance condition is still kept due to the growth of \( k_z \) of the resonance cone waves. The wave number of these waves is given by

\begin{equation}
k_{\text{R}} = \frac{\omega_{pL} \omega^{1/2}}{c(\omega_{BL} \cos \theta - \omega)^{1/2}}
\end{equation}

with \( \cos \theta \sim \omega/\omega_{BL} \).

We can now estimate when the same electrons will be in cyclotron resonance both with the resonance cone wave at a frequency of \( \omega_h \) and with the whistler-mode wave at a frequency of \( \omega_{\text{trig}} \). For that condition we write

\begin{equation}
V_r = \frac{\omega_{BL} - \omega_h}{k_{\text{R}}(\omega_{\text{trig}})} = \frac{\omega_{BL} - \omega_{\text{trig}}}{k_w(\omega_{\text{trig}})}
\end{equation}

Putting \( \omega_h/2\pi = 5.6 \text{ kHz} \), \( \omega_{\text{trig}}/2\pi = 10.2 \text{ kHz} \) and \( L = 3.4 \), values which follow from [1], we find that equation (3) is fulfilled under the condition that \( \cos \theta \approx 0.1 \), (i.e. \( \theta \approx 70^\circ \)).

Now we estimate the one-hop cyclotron amplification \( \Gamma \) of an external whistler-mode signal, which is in resonance with the step (see equation (3)) for a certain \( L \)-shell. For the modified Maxwellian distribution function \( F \sim V^{2j+1} \cos(\theta - \omega_{BL}/2\pi) \exp(-V^2/V_0^2) \Theta(V_r - V_0) \), where \( j \) is the anisotropy index, \( \Theta \) is the Heaviside unit function and \( V_0 \) is electron parallel velocity at the equator, \( \Gamma \) can be evaluated as

\begin{equation}
\Gamma_{\text{step}} \approx 2(ka)^{1/3} \Gamma_0, \quad \Gamma_0 \approx 0.3 \frac{n_{hL}}{n_{eL}} \frac{a}{V_0} \omega_{BL}
\end{equation}

[Trakhtengerts et al., 1996; Hobara et al., 1998], where \( \Gamma_0 \) is the amplification for the standard distribution function (i.e., with \( \Theta(V_r - V_0) \) replaced by unity, the numerical factor in \( \Gamma_0 \) being weakly dependent on the anisotropy index \( j \)), \( n_{hL} \) and \( n_{eL} \) are the hot and cold electron densities, respectively, and \( a = LR_0/\sqrt{3} \) is the characteristic length scale of the Earth’s magnetic field near the equator. The amplification \( \Gamma \) for the smooth anisotropic distribution function depends only weakly on the anisotropy index for high anisotropy [Bell et al., 2000] and is close to \( \Gamma_0 \). Putting, in accordance with [1], \( n_b(W < 20 \text{ keV}) \sim 10^{-2} \text{ cm}^{-3} \), \( V_0 \sim 8 \times 10^9 \text{ cm s}^{-1} \), \( n_{eL} \sim 2.5 \times 10^2 \text{ cm}^{-3} \), \( k \sim 3 \text{ km}^{-1} \), and \( a \sim 1.28 \times 10^4 \text{ km} \) (for \( L = 3.4 \)), we find that \( \Gamma_0 \sim 0.24 \) and \( \Gamma_{\text{step}} \sim 15 \) or 60 dB. It is enough to explain the observed triggered effect.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Figure 1. Qualitative three-dimensional representation of the equatorial distribution function \( F \), with a step separating resonant and nonresonant electrons.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Figure 2. The dependence of the energetic (hot) electron density as a function of latitude (in degrees) along the magnetic flux tube in the case of a step-like distribution for: 1 – \( V_r/V_0 = 0.01 \), case 2 – \( V_r/V_0 = 0.05 \), case 3 – \( V_r/V_0 = 0.2 \). Here \( n_L \) is the energetic (hot) electron density in the equatorial plane.}
\end{figure}
3. Discussion

The above analysis demonstrates an important feature of wave-particle interactions, namely, their strong dependence on the fine structure of the distribution function $F$ of the energetic particles. In particular, a step-like deformation of $F$ in the velocity component $V_{\parallel}$ yields the cyclotron amplification $\Gamma_{\text{step}}$ which is much larger than the amplification $\Gamma_0$ by a smooth anisotropic distribution function (according to the above estimates, $\Gamma_{\text{step}}/\Gamma_0 \sim 60$). The step-like deformation can play a crucial role for the generation of discrete ELF/VLF emissions [Trakhtengerts, 1995, 1999]. Experimental detection of such a feature is rather difficult because it requires an instrument with a very fine resolution in $V_{\parallel}$ ($\sim 1'$). Moreover, as a rule this is a transient phenomenon, with a duration of less than one second. Electron data with such velocity and temporal resolution are not yet possible. Nevertheless, some indications of a step in the distribution function were revealed in OGO-5 satellite data [Lyons et al., 1972]. Unfortunately, the energy and pitch angle resolution of these data ($\sim 10\%$ and $\sim 5 \times 10^6$, respectively) were not enough to reconstruct the exact width and dynamic features of the step. From this point of view, the complex data set on energetic-electron distributions and whistler waves, presented in [I], can be considered to be the first actual evidence for the existence of step-like distribution functions, while the observed triggering effects yield some information about very fast processes. Simultaneous observations of the hiss, responsible for the formation of the step-like feature provide additional support for this suggestion.

As an additional experimental check of the model discussed, a comparison of the electric and magnetic field components of the hiss band could be made. We predict that the electric component should become stronger towards the upper frequency limit of the hiss band. It would also be important to determine the direction of the wave vector $k$ as a function of frequency. According to the above model, two particular directions of $k$ must be present. It is known that two wave-normal directions can be present in satellite VLF data [Hayakawa et al., 1986; Hayakawa and Sazhin, 1992; Hayakawa, 1993], but it is necessary to undertake a more sophisticated data analysis, including an investigation of the dependence of the $k$ direction on frequency.

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References


