AN OPTIMUM LINEAR FREQUENCY-SELECTIVE MIMO EQUALISER USING TIME-DOMAIN ANALYTIC INVERSION.

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Key words to describe this work: MIMO Equalisation, Frequency-Selective Channels.

Key results: An optimum linear MIMO equaliser.
How does the work advance the state-of-the-art?: Shows an optimum technique for equalising highly frequency-selective MIMO systems.
Motivation (Problems addressed): The majority of other work focuses of flat MIMO channels.

Introduction

In recent years, theoretical and practical investigations have shown that it is possible to realise enormous channel capacities, far in excess of the point-to-point capacity given by the Shannon-Hartley law [1], if the environment is sufficient multipath. The majority of work to date on this area has assumed flat sub-channels composing the MIMO channel. As the aim of MIMO systems is often to increase the data transmission rate of a communication system, a wideband and hence highly time-dispersive model would be more appropriate. To properly exploit this environment to realise these capacity increases, the MIMO channel must be equalised so that the performance of any system attempting to harness the multipath diversity can do so while maintaining a satisfactory BER performance. Assuming that the response of the MIMO channel is known at the receiver, a method to create a suitable equaliser is to analytically invert the frequency-selective, or time-dispersive, MIMO channel using a time-domain technique described in this paper. The technique calculates the optimum equaliser coefficients in the MMSE sense.

System Model

Our aim is to calculate a MIMO equaliser so that we can negate the cross-channel interference (CCI) inherent in MIMO systems, and also the ISI present in wideband transmission systems. We must first define a model to represent the transmission system $\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\nu}$ where $\mathbf{x}$ is a length $M \cdot N$ vector representing the input to the MIMO system and M is the number of transmitters and N is the length of the input signal. Further, $\mathbf{H}$ is the frequency selective MIMO matrix of dimensions $P\cdot(N-L_h+1)\times M\cdot N$, where P is the number of receivers and L_h is the impulse response length of the MIMO channel, $\boldsymbol{\nu}$ is a length $P\cdot(N-L_h+1)$ vector represent the noise and $\mathbf{y}$ is a length $P\cdot(N-L_h+1)$ vector representing its output. We have used a time-domain representation similar to that used in [2] to represent both the MIMO cross-channel transfer and the ISI. As such the output vector, $\mathbf{y}$, are given as follows; $\mathbf{y} = [\ \mathbf{y}_1^H \ \ \mathbf{y}_2^H \ \ \cdots \ \ \mathbf{y}_P^H\ ]^H$ where $\mathbf{y}_p = [\ y_1[0] \ \ y_1[1] \ \ \cdots \ \ y_1[N-L_h+1]\ ]^H$ and $y_p(n)$ is the received signal at receiver p at time n. The input vector $\mathbf{x}$ and AWGN vector $\boldsymbol{\nu}$ are defined similarly. Finally we have the MIMO channel *convolution* matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{21} & \cdots & \mathbf{H}_{M1} \\ \mathbf{H}_{12} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{1P} & \mathbf{H}_{2P} & \cdots & \mathbf{H}_{MP} \end{bmatrix} \quad (1)$$

where

$$\mathbf{H}_{mp} = \begin{bmatrix} \mathbf{h}_{mp} & \cdots & \cdots & 0 & 0 \\ 0 & \mathbf{h}_{mp} & \cdots & \vdots & 0 \\ 0 & 0 & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \mathbf{h}_{mp} & \cdots \end{bmatrix}, \quad (2)$$

where $\mathbf{h}_{mp} = [\ h_{mp}[0] \ \ \cdots \ \ h_{mp}[L_h-1]\ ]$. To find the MIMO equaliser we must obtain a matrix $\mathbf{G}$ so that after a signal is passed through the channel and equaliser, they should ideally only be a delay. The system is shown in Figure 1

Optimum MIMO Inverse

To solve this equalisation problem we may use the Wiener-Hopf solution [3] $\mathbf{g}_m = \mathbf{R}^{-1}\mathbf{p}_m$ where $\mathbf{g}_m = [\ \mathbf{g}_{m1}^H \ \ \mathbf{g}_{m2}^H \ \ \cdots \ \ \mathbf{g}_{mP}^H\ ]^H$ and $\mathbf{g}_{mp} = [\ g_{mp}[0] \ \ g_{mp}[1] \ \ \cdots \ \ g_{mp}[L_g-1]\ ]^H$, where L_g is the length of the MISO equaliser filters and $L_g = N - L_h + 1$. After some mathematical development we can calculate that $\mathbf{R} = \sigma_x^2\mathbf{H}\mathbf{H}^H + \sigma_\nu^2\mathbf{I}$, assuming that all the input variances and noise powers are the same, where σ_x^2 is the power of the input signal $x[n]$ and σ_ν^2 is

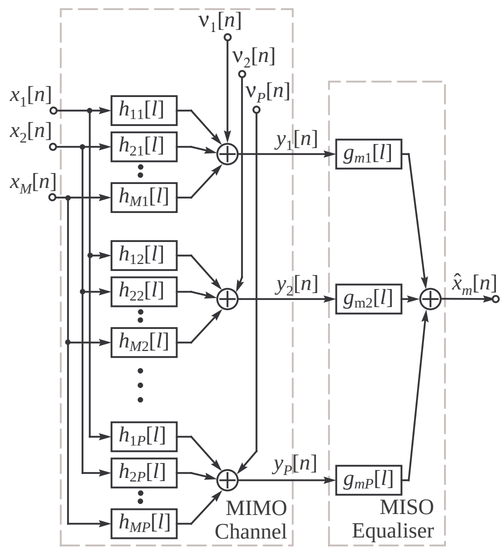

Figure 1: A MIMO channel and MISO equaliser applied to input m.

the power of the noise $\nu[n]$. Also, we can shown that $\mathbf{p}_m = \mathbf{H}\mathbf{d}_m$, where $\mathbf{d}_m$ is delay vector. We may now find $\mathbf{g}_m = \left(\sigma_x^2 \mathbf{H}\mathbf{H}^H + \sigma_\nu^2 \mathbf{I}\right)^{-1} \mathbf{H}\mathbf{d}_m$.

After some algebraic development the complexity, C, involved in calculating $\mathbf{g_m} \forall m \in 1:M$ can be shown to be $\mathcal{O}(L_g^3)$. From this we see that it is beneficial for computational simplicity to keep L_g as low as possible while still achieving satisfactory performance.

Simulations

For the simulations we shall employ real measured microwave channel impulse responses located at the Signal Processing Information Base at Rice University [4]. We modify these to create a range of highly frequency-selective channels with varying response lengths, L_h, and after calculating the MIMO equaliser with $L_g = 100$ using the method described in this paper, we pass random white Gaussian input signals through the system, which uses a BPSK constellation modulation pattern, to obtain SNR vs BER plots, shown in Figure 2. From this we see that the method performs well, achieving BER$> 10^{-3}$ for SNR$>\approx$ 15 dB for short channels ($L_h <\approx 6$), while approaching BER$= 10^{-2}$ by SNR 20 dB for $L_h = 50$. Finally, we calculate the complexity order of the $L_g = 100$ length equaliser relative to $L_g = 1$ to be $1,000,000$.

Conclusions

We have shown a time-domain analytic method of calculating the optimum MMSE MIMO

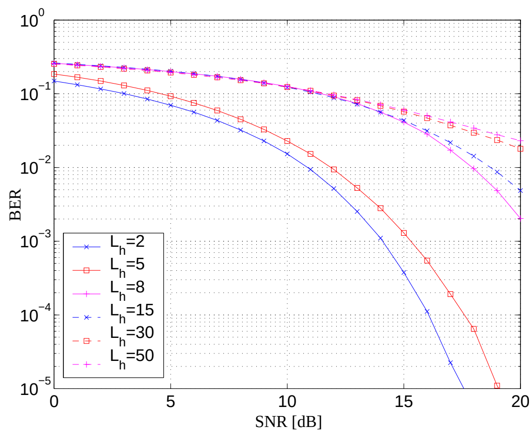

Figure 2: SNR vs BER curves BPSK AWGN systems with MIMO channels of varying impulse response length L_h when an optimum MMSE MIMO equaliser ($L_g = 100$) is used.

equaliser coefficients for highly frequency-selective MIMO channels. Whilst the BER performance of the resulting system is good, the computational complexity may be prohibitively high for current-day devices especially mobile devices requiring low power consumption. We may solve this problem by using lower order adaptive techniques such as the LMS algorithm; however as the convergence speed of the algorithm is low for highly coloured adaptive equaliser inputs, the adaptation time may be long. Subband techniques may provide an interesting solution to this problem [5]. Clearly, further research is required.

References

[1] E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, November-Decemeber 1999.

[2] O. Kirkeby, P. A. Nelson, F. Orduna-Bustamante, and H. Hamada, "Local sound field reproduction using digital signal processing," *Journal of the Acoustical Society of America*, vol. Vol.100, no. No.3, pp. 1584–1593, March 1996.

[3] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Englewood Cliffs, 3rd edition, 1996.

[4] "Signal processing information base: Microwave channel data," Rice University, http://spib.rice.edu.

[5] S. Weiss and R. W. Stewart, *On Adaptive Filtering in Oversampled Subbands*, Shaker Verlag, Aachen, Germany, 1998.