Applying Continuum Design Sensitivity Analysis Combined With Standard EM Software to Shape Optimization in Magnetostatic Problems

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***Abstract*—This paper explores the physical meaning of pseudosources of an adjoint system in a continuum design sensitivity analysis (CDSA) when applied to shape optimization in magnetostatics. An efficient and practical way to compute the required gradient information with standard electromagnetic (EM) software packages is suggested. Based on this novel methodology, designers will be able to deal with practical design optimization of electromechanical devices using existing analysis tools without the need to access the complicated code of the EM software. The applicability of the proposed methodology is demonstrated by optimizing the salient pole face shape of a high-tempearature superconducting synchronous generator.**

***Index Terms*—Design optimization, design sensitivity analysis (DSA), electromagnetic (EM) analysis, superconducting generator.**

I. Introduction

IN RECENT years, general-purpose electromanetic (EM) software packages, such as OPERA, FLUX, MAGNET, MAXWELL, ANSYS, and others, have been widely used in industry as well as in academia to estimate and evaluate the performance of electromechanical devices. However, it is difficult to satisfy—using such commercial programs—the ultimate aim of the engineers of achieving an optimal design of a machine.

Some attempts to use general optimization methods, such as response surface methodology and stochastic techniques, with commercial EM software were reported in [1] and [2]. Even though such methods contain desirable modular programming, where the optimization algorithm and the EM analysis are separated, they still cause difficulties when practical devices are attempted due to the restriction on the number of design variables and excessive number of necessary iterations.

From the point of view of accuracy and time-efficiency in finding the optimum solution in design space, the design sensitivity analysis (DSA) appears to be very competitive compared with other optimization methods. Depending on the technique used to compute the derivative of an objective function, the DSA can be categorized as the discrete DSA (DDSA) or the continuum DSA (CDSA) [3]. The former obtains gradient information from direct differentiation of the discretized algebraic system matrix, whereas the latter uses an analytically derived sensitivity formula by exploiting the material derivative-adjoint variable (MDAV) method.

So far, most of the studies have been devoted to the DDSA and a few attempts have been made to combine it with existing EM software [4]. However, from the practical point of view, as a general-purpose approach, the DDSA has a critical drawback as it requires some amendments to the software source code to perform sensitivity calculation. In the meantime, Park *et al.* [5] suggested that CDSA could overcome this problem and provide an easy way of interfacing with existing EM software. In spite of this advantage, no in-depth work on the CDSA has been carried out to date. It is believed that the complication and ambiguity of the adjoint system may lead researchers to the wrong conclusion that the CDSA also needs to access the source code at low level so as to replace the load vector of the primary system matrix.

In this paper, the physical meanings of pseudosources related to the applied loads and boundary conditions of the adjoint system are thoroughly investigated. Moreover, an efficient and practical way to easily compute the gradient information using an analysis tool is also suggested. The proposed methodology allows the incorporation of the CDSA into existing EM software packages without the need to modify the source code. Thus, this method enables designers to deal with all foreseeable shape designs of magnetostatic problems by using their own EM packages with the API and command language. The validity of the approach is shown later by optimizing the salient pole face shape of a high-tempearature superconducting (HTS) synchronous generator [6].

II. Primary and Adjoint Systems

In order to understand the meaning of the adjoint system, a brief review of deriving the analytical sensitivity formula in the CDSA is first given. Then, the mathematical expression and the dimensions of units of pseudosources imposed on the adjoint system are compared with those of the primary system.

When dealing with the optimization of magnetostatic devices, the objectives can be classified into the following three categories [Fig. 1(a)].

1) Control of the global quantities (i. e. energy, force or inductance) connected with the magnetic vector potential, $\mathbf{A}$, inside the region of interest Ω_f.
2) Shaping of the local quantity distribution (i. e. magnetic flux density or field intensity) inside the region Ω_g.

Manuscript received July 1, 2003.

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Digital Object Identifier 10.1109/TMAG.2004.824604

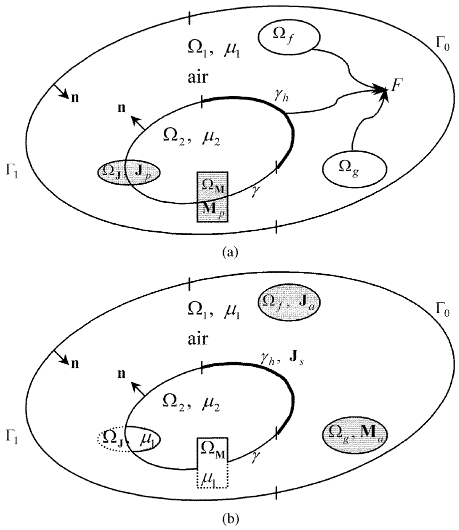

Fig. 1. Dual system of the CDSA. (a) Primary system. (b) Adjoint system.

3) Adjustment of the surface field distribution (i. e. magnetic force density or tensile stress) on γ_h, part of the interface γ.

The derivation of the sensitivity formula starts from the multiobjective function F of (1) including all the design goals mentioned above. The arguments of F must also satisfy the governing (2) of the primary system with the boundary conditions imposed on Γ_0, Γ_1 and γ as shown in (3). Thus, minimize

$$F = \int_{\Omega_1} f(\mathbf{A}_1) m_f d\Omega + \int_{\Omega_1} g(\mathbf{H}_1) m_g d\Omega + \int_{\gamma} h(\mathbf{A}_1) m_h d\Gamma \quad (1)$$

subject to

$$-\nabla \times (\upsilon \nabla \times \mathbf{A} - \mathbf{M}_p) + \mathbf{J}_p = 0, \qquad \text{in } \Omega = \Omega_1 \cup \Omega_2 \quad (2)$$

$$\mathbf{A} = c, \qquad \text{on } \Gamma_0 \quad \frac{\partial \mathbf{A}}{\partial n} = 0, \qquad \text{on } \Gamma_1$$
$$\mathbf{n} \cdot (\nabla \times \mathbf{A}_2 - \nabla \times \mathbf{A}_1) = 0$$
$$\mathbf{n} \times (\upsilon_2 \nabla \times \mathbf{A}_2 - \upsilon_1 \nabla \times \mathbf{A}_1) = 0, \qquad \text{on } \gamma \quad (3)$$

where f, g, and h are arbitrary scalar functions differentiable with respect to $\mathbf{A}$ or $\mathbf{H}$. The symbols m_f, m_g, and m_h denote the characteristic functions that represent parts of the analysis space Ω and γ, where F is defined. In (2), $\mathbf{M}_p$ is the permanent magnetization and $\mathbf{J}_p$ is the current density applied to the primary system.

To transfer the optimization problem under equality constraints of (2) and (3) into an unconstrained problem, the variational (4) of the primary system is added to (1) based on the augmented Lagrangian method

$$\int_{\Omega_1+\Omega_2} \upsilon \nabla \times \mathbf{A} \cdot \nabla \times \boldsymbol{\lambda} d\Omega + \int_{\Gamma_0+\Gamma_1+\gamma} \mathbf{n} \times (\upsilon \nabla \times \mathbf{A}) \cdot \boldsymbol{\lambda} d\Gamma$$
$$= \int_{\Omega_1+\Omega_2} [\mathbf{J}_p \cdot \boldsymbol{\lambda} + \mathbf{M}_p \cdot \nabla \times \boldsymbol{\lambda}] d\Omega, \qquad \text{for all } \boldsymbol{\lambda} \in \Phi \quad (4)$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector interpreted as the adjoint variable and Φ means admissible space of the state variable $\mathbf{A}$. The final sensitivity formula is obtained by exploiting

TABLE I
UNITS OF THE ADJOINT LOADS

Quantities used in F	Basic forms of F	Adjoint load vectors & their units	Relation
W_m [Wb A m^{-3}]	$f = (W_m - W_o)^2$	$\mathbf{J}_a \equiv \mathbf{f}_1 = 2\partial W_m / \partial \mathbf{A}$ [A m^{-2}]	Current source
F [Wb A m^{-2}]	$h = (F_i - F_o)^2$	$\mathbf{J}_s \equiv \mathbf{h}_1 = \partial h / \partial \mathbf{A}$ [A m^{-1}]	
B [T]	$g = (B_{xi} - B_{xo})^2$	$\mathbf{B}_{\mathbf{r}a} \equiv \mathbf{g}_1 = \partial g / \partial \mathbf{B}$ [T]	Magnet source
H [A m^{-1}]	$g = (H_{xi} - H_{xo})^2$	$\mathbf{M}_a \equiv \mathbf{g}_1 = \partial g / \partial \mathbf{H}$ [A m^{-1}]	

the material derivative and some mathematical manipulations as detailed in the previous paper [7]. Then, the variational equation of the adjoint system with its state variable $\boldsymbol{\lambda}$ is systematically deduced in the following form:

$$\int_{\Omega_1+\Omega_2} \upsilon \nabla \times \boldsymbol{\lambda} \cdot \nabla \times \dot{\mathbf{A}} \, d\Omega + \int_{\Gamma_0+\Gamma_1+\gamma} \mathbf{n} \times (\upsilon \nabla \times \boldsymbol{\lambda}) \cdot \dot{\mathbf{A}} \, d\Gamma$$
$$+ \int_{\gamma} \mathbf{h}_1 \cdot \dot{\mathbf{A}} \, d\Gamma = \int_{\Omega_1} [\mathbf{f}_1 \cdot \dot{\mathbf{A}} + \mathbf{g}_1 \cdot \nabla \times \dot{\mathbf{A}}] d\Omega, \qquad \text{for all } \dot{\mathbf{A}} \in \Phi \quad (5)$$

where $\mathbf{h}_1 \equiv \partial h / \partial \mathbf{A}$, $\mathbf{f}_1 \equiv \partial f / \partial \mathbf{A}$, $\mathbf{g}_1 \equiv \partial g / \partial \mathbf{H}$ and $\dot{\mathbf{A}}$ means the material derivative of the state variable $\mathbf{A}$.

From the analogy between (4) and (5), the meanings of the adjoint system can be clearly established as follows.

1) Geometric and material properties (except for the applied loads and the boundary conditions) are all the same as those of the primary system.
2) $\mathbf{f}_1$ and $\mathbf{g}_1$ play the role of magnetic sources such as electric current and permanent magnet, respectively.
3) $\mathbf{h}_1$ defined on γ_h results in discontinuity of the tangential component of the adjoint field.

Using these observations, a governing equation and its boundary conditions for the adjoint system can be found as:

$$-\nabla \times (\upsilon \nabla \times \boldsymbol{\lambda} - \mathbf{g}_1 m_g) + \mathbf{f}_1 m_f = 0, \qquad \text{in } \Omega = \Omega_1 \cup \Omega_2 \quad (6)$$

$$\boldsymbol{\lambda} = 0, \qquad \text{on } \Gamma_0, \frac{\partial \boldsymbol{\lambda}}{\partial n} = 0 \quad \text{on } \Gamma_1$$
$$\mathbf{n} \cdot (\nabla \times \boldsymbol{\lambda}_2 - \nabla \times \boldsymbol{\lambda}_1) = 0$$
$$\mathbf{n} \times (\upsilon_2 \nabla \times \boldsymbol{\lambda}_2 - \upsilon_1 \nabla \times \boldsymbol{\lambda}_1) = -\mathbf{h}_1 m_h, \qquad \text{on } \gamma. \quad (7)$$

The dimensions of each unit were investigated in order to have a clear understanding of the pseudosources, $\mathbf{f}_1$, $\mathbf{g}_1$ and $\mathbf{h}_1$. Table I represents all the possible pseudomagnetic sources induced by the objective functions that are related to the magnetic energy W_m, force density F, $\mathbf{B}$ and $\mathbf{H}$. The units of the pseudosources coincide precisely with those of the current density, surface current density, residual flux density and magnetization. In addition, Table I also includes one of the reasons why it is necessary to make the objective function in square form. The conversion relationship of the dual system, which consists of the primary and the adjoint systems, is illustrated in Fig. 1.

In Table I, the subscripts, i and o, mean the ith iterative design process and the initial design stage, respectively. $\mathbf{J}_a$ denotes the current density, $\mathbf{J}_s$ the surface current density, $\mathbf{B}_{\mathbf{r}a}$ the residual

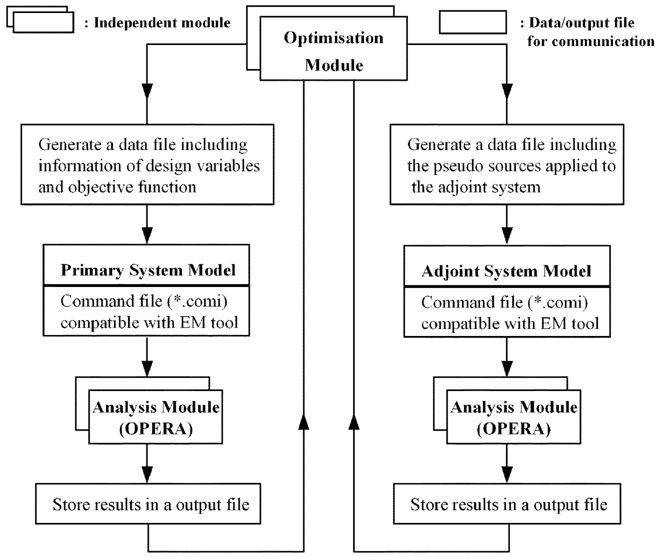

Fig. 2. Flowchart of design optimization using the CDSA.

flux density, and $\mathbf{M}_a$ the permanent magnetization in the adjoint system.

Based on the above discussion, it can be concluded that the adjoint system by itself satisfies all the necessary conditions in order to be solved with a standard EM package. Therefore, the pseudosources can be easily incorporated into the FE model without any amendments in the software source code.

III. Computation of Design Sensitivity Using Standard EM Software

The program architecture consists of two independent modules as shown in Fig. 2. The optimization module controls the overall design procedure and evaluates crucial quantities such as objective function, adjoint load term, and design sensitivity. This module generates two important data files, which store updated information about the changes of the design variables and the adjoint load. The function of the analysis module is to estimate the performance of the dual system and to execute the command files that include the complete specification of the design model. When changes to the design variables and adjoint loads are uploaded into the two data files at each iterative design process, the command file reads the improved design information from the data files using the user input/output commands offered by the software packages. The analysis module can contain any commercial EM software as long as the commands used are compatible with the software. It should be noted that the two modules are constantly communicating with each other and exchanging information about design variables, regions of interest and state variables through the data/output files.

The sensitivity coefficients are evaluated from the analytical formula [7] using the two post-processing output files of the dual system. However, in cases where the objective function is associated with the system energy, the Adjoint System Model does not need to be calculated (refer to Table I and [5]). The whole design process repeats itself until the objective function converges to the optimum solution.

There are some precautions that need to be taken prior to executing the main optimization program as explained in the following.

A. Definition of Design Variables and Objective Region Where F is Calculated

The design variables can be constructed by establishing a set of point coordinates. In the case of a parameterized model, a typical set will include angles, lengths, displacements, radii, or similar constraints. To facilitate the conformity of the FE mesh with the continuing shape changes of the design during the optimization process, it is recommended that each design variable has geometrical constraints, for example a direction and upper/lower limits of its acceptable range.

Dealing with the objective regions Ω_f, Ω_g, and γ_h (see Fig. 1), due to the presence of the objective functions and the adjoint system, requires additional work when compared to the conventional FE modeling. The objective regions have to be subdivided into multiple individual regions so that magnetic sources of the adjoint system model can be imposed on each region defined prior to FE mesh generation. Moreover, relevant information concerning the design variables and objective function are initially assigned to parameter values in the command files.

B. Construction of the Two Command Files for Dual System

The two command files for the FE analysis of the dual system play an important role in the interface between the optimization module and the analysis module. The command file of the primary system model can be constructed using the log file produced during FE analysis. The command file of the adjoint system model can be created following a similar procedure except that the sources are now applied to the subdivided objective region. an alternative approach to make the command file of the adjoint system model is to use the "restart" command embedded in EM software. This method allows the solver to easily append the variation of material or source distribution especially when material nonlinearity is taken into account. The two command files also include a command script to evaluate and store the characteristics of the two models mentioned, which is transferable to the optimization module.

IV. Application

The proposed methodology has been successfully applied to shape optimization of a salient pole rotor of a 100-kVA HTS synchronous generator [6]. The stator of the generator has 48 slots and a balanced two-pole, three-phase, star-connected stator winding. Due to symmetry and under no-load condition, a quarter of the generator needs to be analyzed using a nonlinear static OPERA-2D solver. In order to minimize the effect of the undesirable odd harmonics of the air-gap flux density, an optimal design of the pole face shape is required. To achieve this goal, the following objective function was evaluated over a 90° arc at a 160-mm radius:

$$F = \int_{\Omega_f} (B_{ri} - B_{rio})^2 d\Omega, \qquad B_{rio} = 0.7\sin(\theta) \qquad (8)$$

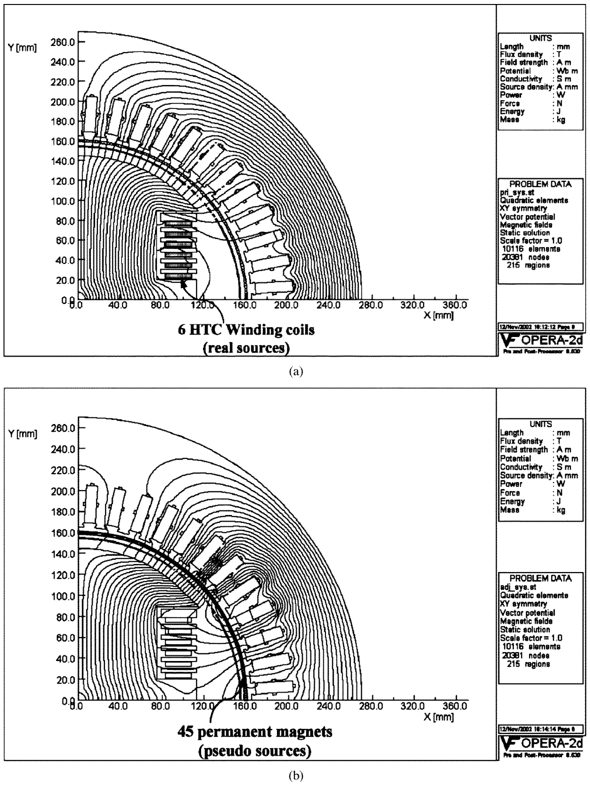

Fig. 3. Comparison of flux lines and source distributions between the dual system before starting the iterative design procedure. (a) Primary system. (b) Adjoint system.

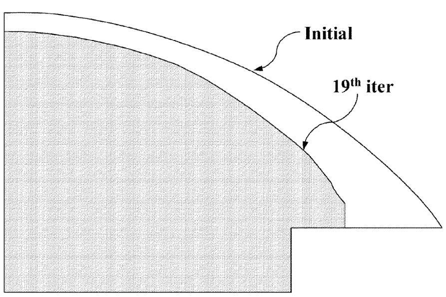

Fig. 4. Pole face shape optimization of a HTS generator [6].

where B_{ri} is the radial component of the air-gap flux density, in this case assumed to comprise of odd harmonics of order up to 19th, and θ is the angle between 0° and 90°. The objective region Ω_f was subdivided into 45 individual quadrilateral regions 2-mm wide, each, 2° along the 90° arc. A total of 53 grid points forming the outline of the rotor pole have been selected as design variables and allowed to move in the radial direction with the base point located at (0,89). In order to take manufacturing constraints into account, five grid points around the pole tip of the initial rotor shape are bound to move together in the x-direction.

The flux lines and source distribution of the primary and the adjoint systems are shown in Fig. 3. As expected, the flux distribution for the dual system is quite different due to the different sources applied at different locations as explained in the previous section. After only 19 iterations, the optimal pole shape of Fig. 4 was obtained. The magnitudes of the air-gap flux density, before and after optimization, are shown in Fig. 5.

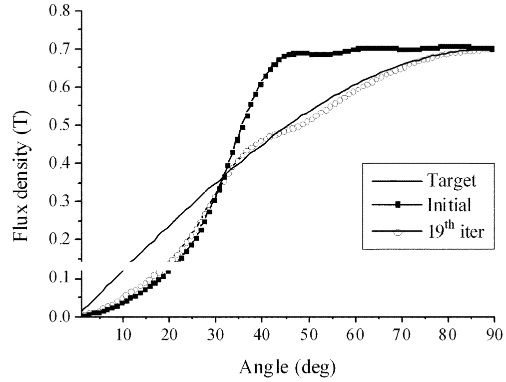

Fig. 5. Comparison of the air-gap flux density distributions.

The significant practical benefit of this approach, compared with other available optimization techniques, is a small number of time-expensive FE runs needed, despite a relatively large number of degrees of freedom (design variables). The final field shape, although not perfect, offers considerable improvement over the distributions obtained previously.

V. Conclusion

By exploring the physical meaning of the pseudosources arising in the adjoint system, a novel methodology of combining CDSA with standard EM software has been investigated. This approach avoids the need to access the often unavailable, or difficult to master, source codes of commercial programs. Moreover, the computing times required to find an optimal solution are not affected by the number of design variables. The modular programming between the optimization algorithm and the analysis tool has also been accomplished.

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