Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

WeM11-4

A Non–singular Performance Comparison Between Two Robust Adaptive Control Designs

Ahmad Sanei Mark French
Department of Electronics and Computer Science, University of Southampton,
Southapmton, SO17 1BJ, U.K.
as99r|mcf @ecs.soton.ac.uk
http://www.isis.ecs.soton.ac.uk/control

Abstract

We consider standard robust adaptive control designs based on the dead-zone and projection modifications, and compare their performance w.r.t. a worst case transient cost functional penalizing the $\mathcal{L}^\infty$ norm of the state, control and control derivative. If a bound on the $\mathcal{L}^\infty$ norm of the disturbance is known, it is shown that the dead-zone controller outperforms the projection controller when the a-priori information on the unknown system parameter is sufficiently conservative. For simplicity the results are presented for a scalar system and generalizations are briefly discussed.

1 Introduction

It is well known that adaptive controllers are susceptible to phenomena such as parameter drift even when small disturbances are present. To overcome such problems, a number of standard techniques are widely utilized, such as dead-zones, σ modification, projection modification [5] etc.

Each of these designs have advantages and drawbacks. For example, dead-zone modifications require a-priori knowledge of the disturbance level, and only achieve convergence of the output to some pre-specified neighbourhood of the origin (whilst keeping all signals bounded). In particular if the disturbance vanishes, then the dead-zone controller does not typically achieve convergence to zero, the convergence remains to the pre-specified neighbourhood of the origin. On the other hand, projection modifications generally achieve boundedness of all signals, and furthermore have the desirable property that if no disturbances are present, then the output converges to zero, however, an arbitrarily small $\mathcal{L}^\infty$ disturbance can completely destroy any convergence of the state.

This illustrates that in the case of asymptotic performance, there are some known characterisations of 'good' and 'bad' behaviour. However, there are many situations in which we cannot definitively state whether a projection or dead-zone controller is superior even when only considering asymptotic performance. Furthermore, the known results, as with most results in adaptive control, are confined to non-singular performances, ie. without any consideration of the control signal.

The goal of this paper is to compare dead-zone and projection based adaptive controllers with respect to transient performance. Furthermore, the transient performance measure will be nonsingular (ie. penalise both the state (x) and the input (u) of the plant); specifically we will consider cost functionals of the form:

$$\mathcal{P} = \|x\|_{\mathcal{L}^\infty} + \|u\|_{\mathcal{L}^\infty} + \|\dot{u}\|_{\mathcal{L}^\infty}.$$

We will identify a circumstance in which the dead-zone controller is superior to the projection controller w.r.t. $\mathcal{P}$.

2 Statement of the Problem and Main Result

2.1 System and Basic Control Design

Consider the following class of SISO nonlinear systems and controller:

$$\begin{aligned}
&\Sigma_n(\theta, d(\cdot)):\\
&\qquad \dot{x}_i = x_{i+1}, && 1 \le i \le n-1\\
&\qquad \dot{x}_n = \theta^T\phi(x) + u + d(\cdot), && x(0) = x_0\\
&\Xi(\alpha):\\
&\qquad u = -\hat{\theta}^T\phi(x) - a^T x,\\
&\qquad \dot{\hat{\theta}} = \alpha\, x^T\phi(x), && \hat{\theta}(0) = 0
\end{aligned}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, θ and $\hat{\theta} \in \mathbb{R}^m$ are unknown constant parameter and its adaptive estimator respectively, $\phi \in \mathbb{R}^m$ is a known basis, which is taken to be locally Lipschitz, $d(\cdot)$ is a bounded disturbance, α is the adaptation gain, and

$a = [a_1, \ldots, a_n]^T$ is chosen such that the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -a_1 & -a_2 & -a_3 & \ldots & -a_n \end{bmatrix}$$

is Hurwitz. Letting $B = (0 \ldots 1)^T$, one can rewrite $(\Sigma_n(\theta, d(\cdot)), \Xi(\alpha)$ as follows:

$$\dot{x} = Ax + B(\,(\theta - \hat{\theta})^T \phi(x) + d(\cdot)\,), \qquad x(0) = x_0$$

$$\dot{\hat{\theta}} = \alpha\, x^T \phi(x), \qquad \hat{\theta}(0) = 0.$$

2.2 Robust Modifications to the Control Design

It has been shown that even a small $\mathcal{L}^\infty$ disturbance may cause a drift of the parameter estimates $\hat{\theta}$, see eg. [1]. The adaption law $\dot{\hat{\theta}}$ is often modified to avoid this problem. We briefly describe two popular methods, i.e. dead-zone and projection (see e.g. [5]).

Let

$$\dot{\hat{\theta}} = \tau(x, \hat{\theta})$$

be the unmodified adaptation law. The idea of dead-zone is to modify the parameter estimator so that the adaptive mechanism is 'switched off' when system trajectory x lies inside a region Ω_0 where the disturbance has a destabilising effect on the dynamics. The modified adaptive law is taken to be $\dot{\hat{\theta}} = D_{\Omega_0}(x)\tau$, where

$$D_{\Omega_0}(x) = \begin{cases} 0, & x(t) \in \Omega_0 \\ 1, & x(t) \in \mathbb{R}^n \setminus \Omega_0 \end{cases}$$

The size of the disturbance is necessary a-priori knowledge in order to define the region Ω_0.

Projection is a alternative method to eliminate parameter drift by keeping the parameter estimates within some a priori defined bounds. Let us define the convex set

$$\Pi = \{\hat{\theta} \in \mathbb{R}^p \mid P(\hat{\theta}) \le 0\}$$

where P is a smooth convex function. Denote Π^o, $\partial\Pi$,the interior and the boundary of Π respectively and observe that $\nabla_{\hat{\theta}} P$ represents an outward normal vector at $\hat{\theta} \in \partial\Pi$. The idea behind this method is to project the adaptation law τ on the hyperplane tangent to $\partial\Pi$ at $\hat{\theta}$ when $\hat{\theta}$ is on the boundary $\partial\Pi$ and τ pointing outward i.e.

$$Proj(\tau) = \begin{cases} \tau, & \text{if } \hat{\theta} \in \Pi^o \text{ or } \nabla_{\hat{\theta}} P^T \tau \le 0 \\ \left(I - \dfrac{\nabla_{\hat{\theta}} P \nabla_{\hat{\theta}} P^T}{\nabla_{\hat{\theta}} P^T \nabla_{\hat{\theta}} P}\right)\tau & \text{if } \hat{\theta} \in \partial\Pi \text{ and } \nabla_{\hat{\theta}} P^T \tau > 0. \end{cases}$$

The modified adaptive law is taken to be $\dot{\hat{\theta}} = Proj(\tau)$.

2.3 Specific System and Controllers

The goal of this paper is to establish a comparison between dead-zone and projection methods on a scalar system with $\alpha = 1$ and $\phi(x) = x$, ie. consider the following system:

$$\Sigma_1(\theta, d(\cdot)): \; \dot{x}(t) = \theta x(t) + u(t) + d(\cdot) \qquad x(0) = x_0. \tag{1}$$

The unmodified controller $\Xi \equiv \Xi(\alpha = 1)$ is

$$\Xi: \; u(t) = -ax(t) - \hat{\theta}(t)x(t) \qquad \dot{\hat{\theta}}(t) = x(t)^2, \quad \hat{\theta}(0) = 0 \tag{2}$$

Consequently we define the dead-zone and projection controllers $\Xi_D(\eta)$ and $\Xi_P(\theta_{\max})$ as follows:

$$\Xi_D(\eta): \; u(t) = -ax(t) - \hat{\theta}(t)x(t) \qquad \dot{\hat{\theta}}(t) = D_{[-\eta,\eta]}(x)x(t)^2 \qquad \hat{\theta}(0) = 0 \tag{3}$$

$$\Xi_P(\theta_{\max}): \; u(t) = -ax(t) - \hat{\theta}(t)x(t) \qquad \dot{\hat{\theta}}(t) = Proj(x(t)^2) \qquad \hat{\theta}(0) = 0 \tag{4}$$

where $a > 0$, the convex set Π is defined as $\Pi = [-\theta_{\max}, \theta_{\max}]$ where $\theta_{\max}$ is an upper bound of $|\theta|$, and the dead-zone region Ω_0 is taken to be $\Omega_0 = [-\eta, \eta]$.

Finally let us denote the respective closed loops by $(\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta))$ and $(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max}))$.

2.4 Statement of the Main Result

We will compare the performances of the controllers with respect to the following worst case non-singular transient cost functional $\mathcal{P}$, defined as follows:

$$\mathcal{P}(\,\Sigma_1(\theta, d(\cdot), \Xi\,) = \sup_{\|d\|_{\mathcal{L}^\infty} \le \epsilon} (\|x\|_{\mathcal{L}^\infty} + \|u\|_{\mathcal{L}^\infty} + \|\dot{u}\|_{\mathcal{L}^\infty})\,. \tag{5}$$

We are not concerned in this paper with the comparison of asymptotic performance, this has been studied previously, see eg. [5] and the references therein.

The main theorem in this paper is as follows:

Theorem 2.1 *Consider the scalar system* $\Sigma_1(\theta, d(\cdot))$ *defined by* (1), *and the controllers* $\Xi_D(\eta)$, $\Xi_P(\theta_{\max})$ *defined by equations* (3), (4), *where* $\epsilon, \theta_{\max} > 0$, *and*

$$\eta > \frac{\epsilon}{a}, \qquad \theta_{\max} \geq |\theta|.$$

Consider the transient performance cost functional (5). *Then there exists* $\theta^*_{\max} > 0$ *such that for all* $\theta_{\max} \geq \theta^*_{\max}$ *we have:*

$$\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max})) \;>\; \mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta)).$$

In fact, we will prove the stronger result that the ratio between the two costs can be made arbitrarily large, i.e

$$\frac{\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max}))}{\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta))} \to \infty \quad \text{as } \theta_{\max} \to \infty.$$

3 Proof of the Main Result

In order to prove theorem 2.1, we establish three propositions:

Proposition 3.1 *For the closed loop* $(\Sigma_1(\theta, d(\cdot)), \Xi)$ *defined by* (1), (2), *where*

$$d(\cdot) = \epsilon,$$

the following statements are true:

1. $x(t) \to 0$, $\hat{\theta}(t) \to \infty$ *as* $t \to \infty$,
2. *Suppose there exists* $L \in \mathbb{R}$ *such that*

$$\lim_{t\to\infty} \hat{\theta}(t)\, x(t) = L$$

then $L = \epsilon$.

Proof: For a proof of statement 1, see [3]. To establish 2, let f and f' be defined as follows:

$$f = \hat{\theta}x = \frac{x}{1/\hat{\theta}} \qquad \text{and} \qquad f' = \frac{\frac{d}{dt}x}{\frac{d}{dt}(1/\hat{\theta})}$$

then

$$f' = \frac{-\hat{\theta}^2(-ax + \theta x + \epsilon)}{x^2} + \frac{\hat{\theta}^2}{x^2} f$$

so,

$$f = \frac{-ax + \theta x + \epsilon}{1 - \frac{x^2}{\hat{\theta}^2}\frac{f'}{f}}.$$

Now since there exist L s.t. $\lim_{t\to\infty} f = L$ then by considering the Taylor series of f, f',[1] we have $\lim_{t\to\infty} f' = \lim_{t\to\infty} f = L$ i.e $\lim_{t\to\infty} f'/f = 1$.

$$\lim_{t\to\infty} f = \lim_{t\to\infty} \left(\frac{-ax + \theta x + \epsilon}{1 - \frac{x^2}{\hat{\theta}^2}\frac{f'}{f}} \right)$$

$$= \frac{\lim_{t\to\infty}(-ax + \theta x + \epsilon)}{\lim_{t\to\infty}(1 - \frac{x^2}{\hat{\theta}^2}\frac{f'}{f})} = \frac{\epsilon}{1 - \lim_{t\to\infty}\frac{x^2}{\hat{\theta}^2}}$$

so

$$L = \lim_{t\to\infty} \hat{\theta}x = \lim_{t\to\infty} f = \epsilon,$$

since $\lim_{t\to\infty}\left(\frac{x^2}{\hat{\theta}^2}\right) = 0$ by Proposition 3.1, (1).
■

Proposition 3.2 *Consider the closed loop* $(\Sigma_1(\theta, d(\cdot)), \Xi)$ *defined by equations* (1),(2) *and the transient performance cost functional* (5). *Then*

$$\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi) = \infty$$

Proof: We choose $d(\cdot) = \epsilon > 0$. Suppose for contradiction $\mathcal{P} < \infty$. Consider $\dot{x}$. There are two cases either 1. $\limsup_{t\to\infty} \dot{x} = \infty$ or 2. $\limsup_{t\to\infty} \dot{x} < \infty$:

1. $\limsup_{t\to\infty} \dot{x} = \infty$ i.e.

$$\limsup_{t\to\infty} (-ax + (\theta - \hat{\theta})x + \epsilon) = \infty.$$

Since $x \to 0$ by Proposition 3.1, therefore $\limsup_{t\to\infty} \hat{\theta}x = \infty$. It follows that

$$\sup_{\|d\|_{\mathcal{L}^\infty} < \epsilon} \|u\|_{\mathcal{L}^\infty} \geq \|\hat{\theta}x\|_{\mathcal{L}^\infty} = \infty, \qquad (6)$$

i.e $\mathcal{P} = \infty$, which is a contradiction.

2 $\limsup_{t\to\infty} \dot{x} < \infty$. Again there are two cases a) $\limsup_{t\to\infty} \hat{\theta}x = \infty$, b) $\limsup_{t\to\infty} \hat{\theta}x < \infty$:

2.a. $\limsup_{t\to\infty} \hat{\theta}x = \infty$. Then similarly to (6), $\sup_{\|d\|_{\mathcal{L}^\infty} < \epsilon} \|u\|_{\mathcal{L}^\infty} = \infty$, and that is a contradiction.

2.b. $\limsup_{t\to\infty} \hat{\theta}x < \infty$ i.e. $|\hat{\theta}x| \leq M$ for some $M > 0$. There are two possibilities, either i) $\lim_{t\to\infty} \hat{\theta}x$ does not exist, or ii) $\lim_{t\to\infty} \hat{\theta}x$ does exist:

[1] These exist since the r.h.s. of the equations (1),(2) are analytic, hence the solutions are analytic.

2b.i. $\lim_{t\to\infty} \hat{\theta}x$ does not exist. By considering $\limsup_{t\to\infty} \dot{u}$, we see

$$\limsup_{t\to\infty} \dot{u} = \limsup_{t\to\infty}(-a\dot{x} - x^3 - \hat{\theta}\dot{x})$$
$$= (2a-\theta)\hat{\theta}x - a\epsilon + \hat{\theta}(\hat{\theta}x - \epsilon) \to \infty,$$

since the first two terms are bounded, $\hat{\theta} \to \infty$ by Proposition 3.1, and $\hat{\theta}x$ cannot converge to ϵ since the limit does not exist. It follows that $\sup_{\|d\|_{\mathcal{L}^\infty}<\epsilon} \|\dot{u}\|_{\mathcal{L}^\infty} = \infty$; hence contradiction.

2b.ii Let L denote the limit, ie. $\lim_{t\to\infty} \hat{\theta}x = L$. Therefore $L = \epsilon$ by Proposition 3.1. So

$$\forall\delta > 0 \quad \exists T_1 > 0 \quad s.t. \ \forall t > T_1 \quad |\hat{\theta}x - \epsilon| < \delta$$

Since $\hat{\theta} \to \infty$ by Proposition 3.1, it follows that

$$\forall\hat{\theta}^* > 0 \quad \exists T_2 > 0 \quad s.t. \ \forall t > T_2 \quad \hat{\theta} > \hat{\theta}^* \qquad (7)$$

Taking $T_3 = \max(T_1, T_2)$ we have that

$$\forall t > T_3 \quad \hat{\theta} > \hat{\theta}^*; \quad \epsilon - \delta < \hat{\theta}x < \epsilon + \delta \qquad (8)$$

Now we choose $d(\cdot)$ as follows

$$d(t) = \begin{cases} \epsilon & t \le T_3 \\ -\epsilon & t > T_3 \end{cases} \qquad (9)$$

With this choice, by causality, (8) holds also, and it can be easily shown that the whole proof from the beginning is also hold for the case that $d = -\epsilon$; hence with the choice of d given by equation (9) we have $\hat{\theta}x \to -\epsilon$ as $t \to \infty$. Since $\hat{\theta}(T3)x(T_3) < \epsilon + \delta$, by intermediate value theorem,

$$\exists\, T_3 \le T < \infty \ s.t. \ \hat{\theta}(T)x(T) = -\frac{\epsilon}{2} \qquad (10)$$

So look at $\dot{u}(T)$, we see that

$$\dot{u}(T) = (a^2 - a\theta)x(T) - x(T)^3 + \theta\frac{\epsilon}{2} + \hat{\theta}(T)\frac{\epsilon}{2}$$

Note that by (7), we have $\hat{\theta}(T) > \hat{\theta}^*$ and therefore by (10)

$$|x(T)| < \frac{\epsilon}{(2\hat{\theta}^*)},$$

i.e. by choosing $\hat{\theta}^*$, $x(T)$ can be sufficiently small and $\hat{\theta}(T)$ can be made arbitrarily large. Hence also $\|\dot{u}\|_{\mathcal{L}^\infty}$ can be made arbitrarily large, i.e.

$$\sup_{\|d\|_{\mathcal{L}^\infty}<\epsilon} |\dot{u}| = \infty$$

and this is a contradiction.

Therefore either

$$\sup_{\|d\|_{\mathcal{L}^\infty}<\epsilon} \|u\|_{\mathcal{L}^\infty} = \infty$$

or

$$\sup_{\|d\|_{\mathcal{L}^\infty}<\epsilon} \|\dot{u}\|_{\mathcal{L}^\infty} = \infty.$$

Hence $\mathcal{P} = \infty$. ∎

Proposition 3.3 *Consider the closed loop* $(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max}))$ *defined by equations* (1), (4) *and the transient performance cost functional* (5). *Then*

$$\mathcal{P}\left((\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max}))\right) \to \infty \text{ as } \theta_{\max} \to \infty.$$

Proof: It is convenient to define

$$\mathcal{P}|_{[0,T]} = \left(\|x\|_{\mathcal{L}^\infty[0,T]} + \|u\|_{\mathcal{L}^\infty[0,T]} + \|\dot{u}\|_{\mathcal{L}^\infty[0,T]}\right).$$

Now let $M > 0$. By Proposition 3.2 there exists $d(\cdot)$, $\|d\|_{\mathcal{L}^\infty} \le \epsilon$ s.t.

$$\mathcal{P}|_{[0,\infty]}(\Sigma_1(\theta, d(\cdot)), \Xi) \ge 2M.$$

It follows that $\exists T > 0$ s.t.

$$\mathcal{P}|_{[0,T]}(\Sigma_1(\theta, d(\cdot)), \Xi) \ge M$$

and also $\hat{\theta}(T) \ge M$. Let $\theta_{\max} = 2\hat{\theta}(T) \ge 2M$. Then the unmodified and the projection design are identical on $[0, T]$, hence

$$\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max})) \ge \mathcal{P}|_{[0,T]}(\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max}))$$
$$\ge \mathcal{P}|_{[0,T]}(\Sigma_1(\theta, d(\cdot)), \Xi) \ge M.$$

Since this holds for all $M > 0$, this completes the proof. ∎

Proposition 3.4 *Consider the closed loop* $(\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta))$ *defined by equations* (1), (3) *and the transient performance cost functional* (5). *Suppose*

$$\eta > \frac{\epsilon}{a}.$$

Then

$$\mathcal{P}(\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta)) < \infty.$$

Proof: Due to switching nature of the dead-zone, all our differential equations have a discontinuous right hand sides, for which the classical definition of solution is not valid, we therefore consider solutions in a Fillipov sense. A complete proof of stability can be found in [2]. A brief sketch of the proof is as follows:

We define a Lyapunov function:

$$V(x,\hat{\theta}) = \frac{1}{2}x^2 + \frac{1}{2}(\theta - \hat{\theta})^2$$

and let

$$V_0 = \frac{1}{2}\max(\, x_0^2, \epsilon^2 \,) + \frac{1}{2}\theta^2.$$

It has been shown [2] that

$$V(x,\hat{\theta}) \le V_0 \qquad \forall t > 0.$$

Hence we can bound $x(t)$ and $\hat{\theta}(t)$ in terms of V_0 as follows:

$$|x(t)| \le \sqrt{2V_0}, \quad (11)$$
$$|\hat{\theta}(t)| \le |\theta| + \sqrt{2V_0}. \quad (12)$$

It follows that $\|x\|_{\mathcal{L}^\infty} < \infty$. Since

$$u = -ax - \hat{\theta}x,$$

and

$$\begin{aligned}\dot{u}(t) &= -a\dot{x} - \hat{\theta}\dot{x} - x^3 \\ &= -(a+\theta)(-ax + (\theta - \hat{\theta})x + d) - x^3,\end{aligned}$$

inequalities (11), (12) imply $\|u\|_{\mathcal{L}^\infty}$, and $\|\dot{u}\|_{\mathcal{L}^\infty}$ are bounded in terms of V_0, hence

$$(\|x\|_{\mathcal{L}^\infty} + \|u\|_{\mathcal{L}^\infty} + \|\dot{u}\|_{\mathcal{L}^\infty}) < \infty$$

thus completing the proof. ∎

We can now prove the main result, which we repeat for convenience of the reader:

Theorem 2.1 *Consider the scalar system $\Sigma_1(\theta, d(\cdot))$ defined by (1), and the controllers $\Xi_D(\eta)$, $\Xi_P(\theta_{\max})$ defined by equations (3), (4), where $\epsilon, \theta_{\max} > 0$, and*

$$\eta > \frac{\epsilon}{a}, \qquad \theta_{\max} \ge |\theta|.$$

Consider the transient performance cost functional (5). Then there exists $\theta^*_{\max} > 0$ such that for all $\theta_{\max} \ge \theta^*_{\max}$ we have:

$$\mathcal{P}(\,\Sigma_1(\theta, d(\cdot)), \Xi_P(\theta_{\max})\,) \; > \; \mathcal{P}(\,\Sigma_1(\theta, d(\cdot)), \Xi_D(\eta)\,).$$

Proof: This is a simple consequence of Proposition 3.3 and Proposition 3.4. ∎

4 Conclusion

In this paper we have established a rigorous result demonstrating a situation in which the transient performance of a projection based controller is worse than that of a dead-zone based controller. There are a number of directions in which the result can be fruitfully generalised, for example:

1. Relaxation of the assumption that $\phi(x) = x$. The first problem is that we need some conditions under which, the closed loop system has a drift in parameter estimator. For the simple case $\phi(x) = x$, it has been shown that $\hat{\theta} \to \infty$ and $x \to 0$ [3]. For a similar general argument, we would impose the conditions:

   $$x = 0 \Longleftrightarrow \phi(x) = 0,$$

   and

   $$x\phi(x) \ge 0,$$

   to ensure $\dot{\hat{\theta}} \ge 0$. In particular, similar results have been obtained for $\phi(x) = x^p$, p odd.

2. Generalisation of the result to the chain of integrators $\Sigma_n(\theta, d(\cdot))$ for $n > 1$. Some preliminary results has been obtained.

3. Generalisation of the result to strict feedback systems, via backstepping controllers [4].

4. Establishing whether the same results can be given for the alternative costs, for example, $\mathcal{P} = \|x\|_{\mathcal{L}^\infty} + \|u\|_{\mathcal{L}^\infty}$.

Similarly we are developing results to demonstrate the contrary relationship between the controllers, ie. establishing results which show when the projection controllers outperform the dead-zone controllers. We let d_{max} play a similar role to θ_{max} in theorem 2.1 by letting d_{max} be the upper bound on the $\mathcal{L}^\infty$ norm of the disturbance $d(\cdot)$. Then we define a worst case transient performance $\mathcal{P}$ and show that for the dead-zone controller, $\mathcal{P} \to \infty$ as $d_{max} \to \infty$, while the performance $\mathcal{P}$ of the projection controller remains bounded.

The aim is to establish good characterisations of the classes of problem in which one controller should be used in preference to another. The result of this paper represents a step towards these more general results.

References

[1] B. Egardt. *Stability of adaptive Controllers (Lecture notes in Control and Information Sciences)*. Springer-Verlag, New York, 1979.

[2] M. French, E. Rogers, and Cs. Szepesvári. Uncertainty, performance, and model dependency in approximate adaptive nonlinear control. *IEEE Trans. on Automatic Control*, 45(2):353–358, Feb. 2000.

[3] T. T. Georgiou and M. C. Smith. Remarks on 'robustness analysis of nonlinear feedback systems: An input–output approach'. *IEEE Trans. on Automatic Control*, 46(1):171–172, Jan. 2001.

[4] M. Kristic, I. Kanellakopoulos, and P. Kokotovic. *Nonlinear and Adaptive Control Design*. John Wiley & Sons, NewYork, 1995.

[5] K. S. Narendra and A. M. Annaswamy. *Stable Adaptive Systems*. Prentice-Hall, 1989.