FINITE-ELEMENT AIDED OPTIMISATION IN ELECTROMAGNETICS

Jan K. Sykulski

Electrical Power Engineering Research Group, School of Electronics and Computer Science, University of Southampton
Highfield, Southampton, SO17 1BJ, UK, e-mail: jks@soton.ac.uk

Abstract **- The paper reviews recent advances in methods of design and optimisation of electromechanical devices where intensive field simulation studies are required. Six techniques appear to be particularly promising and are summarised, including: Minimal Function Calls Method; combined Evolution Strategy, Differential Evolution and Multiquadrics Interpolation; Neuro-Fuzzy Modelling; Combined Finite Elements with Neural Network; Sensitivity Analysis; and finally Pareto Optimisation.**

I. INTRODUCTION

Optimal design often necessitates repetitive usage of finite-element or other numerically intensive field computation. Calling the FE package every time an objective function evaluation is needed is straightforward but very inefficient, as each set of selected design parameters leads to full field analysis. The number of FE runs escalates as more design variables are used; moreover, additional calls are normally required to evaluate gradients of the objective function. In the design office environment such an approach becomes impractical and thus more efficient schemes are sought. This contribution builds on a recently published review [1].

II. MINIMAL FUNCTION CALLS APPROACH

The *Minimum Function Calls* (MFC) approach relies on evaluating the objective function *a priori* for a number of pre-determined cases and fitting an interpolating function through the data points [2]. The optimiser then uses the interpolating function rather than calling the FE directly. In this *Response Surface Methodology* (RSM) it is usual to use polynomial interpolating functions. The minimum number of function evaluations needed for curve fitting is equal to the number of coefficients in the interpolating equation. For example, using a third order polynomial and five design variables requires 56 function calls, which will be quite acceptable in practical situations. The position of initial points is carefully selected to be optimal in a sense that the resulting algorithms have proven stable. Using the Response Surface Methodology reduces computing times dramatically, and accuracy is maintained by introducing *on-line learning* with *dynamic weighting*. As the optimisation process proceeds, more points become available for curve fitting and thus the estimate of the optimum position becomes more accurate. It is therefore appropriate to apply lower weighting to points far from the predicted optimum. To illustrate the process a brushless permanent magnet motor has been optimised for efficiency (with minimum torque constraint) in terms of magnet height, tooth width and stack length. Figure 1 shows a section through the response surface illustrating the nature of the optimisation problem. The efficiency is calculated by integrating input power and losses in a time-stepping model.

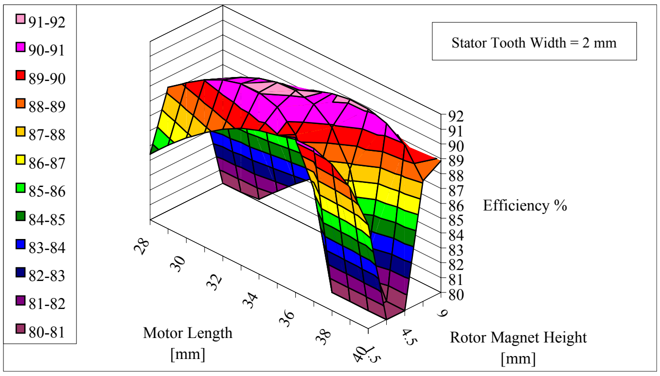

Fig.1. Brushless PM motor optimisation response surface [2].

III. EVOLUTION STRATEGIES

If local minima traps are identified as a potential problem, stochastic techniques may be preferred. Most such techniques are very expensive in terms of number of necessary function evaluations and thus impractical. Some more recent methods, however, look more promising and one such techniques, introduced originally in [3], is reported here. It uses a combination of *Evolution Strategy*, *Differential Evolution* and *Multiquadrics Interpolation* (ES/DE/MQ) as shown in Fig. 4.

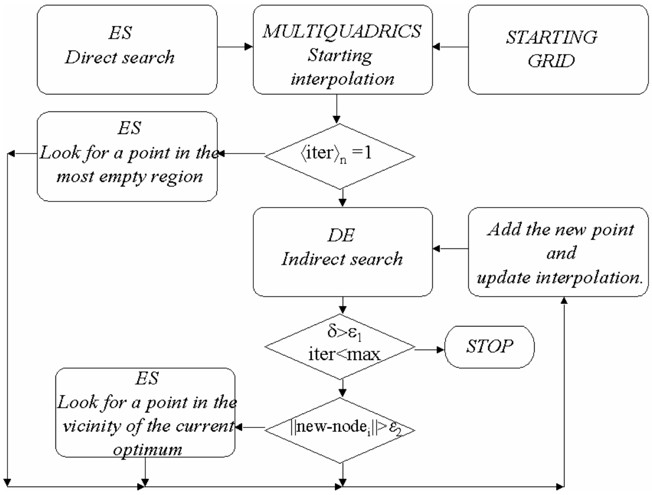

Fig.2. Flowchart of the ES/DE/MQ method [3].

The comparison of the ES/DE/MQ method with standard strategies (one *Evolution Strategy* ES, two versions of *Differential Evolution* DE1 and DE2 and a *Gradient Based Algorithm* GBA) for a popular *C*-core problem where the pole faces are shaped to achieve homogeneous field in a region in the centre of the air gap, is shown in Table I. The number of objective function calls is greatly reduced, whereas the value of the objective function is similar to ES and DE2 results and better than those obtained with DE1 and GBA.

TABLE I
COMPARATIVE OPTIMISATION RESULTS FOR A C-CORE

	Starting	Optimum	n
DE1	9 random	0.0803	720
DE2	13 random	0.0704	881
ES	0.7532 / 0.4344 / 0.6411	0.0642	450
GBA	0.7532	0.0855	188
ES/DE/MQ	0.7532	0.0718	118

IV. OTHER METHODS

The *Neuro-Fuzzy Modelling* (NFM) uses optimisation based on *Genetic Algorithm* (GA) and *Sequential Quadratic Programming* (SQP). In this NF/GA/SQP approach, an *n*-dimensional hyper-space is sampled initially using a grid structure or a suitable *Design of Experiment* (DoE) orthogonal array. The model data is subsequently employed to create a neuro-fuzzy model which approximates a real function [4]. Results for unconstrained optimisation using a magnetizer problem with six design parameters [5] are summarised in Table II and compared with the ES/DE/MQ method, as well as with standard evolutionary strategies and MATLAB's *gradient based algorithm*. On average the DE/ES/MQ method finds a better solution at the cost of slightly greater number of function evaluations. Both methods, however, require significantly fewer function calls than conventional stochastic techniques. The success of both methods lies in their ability to search unexplored regions of space whilst exploiting available knowledge to identify more accurately regions of minima.

TABLE II
UNCONSTRAINED OPTIMISATION RESULTS FOR MAGNETISER

	Starting	Optimum	n
DE1	11 Random	1.235E-5	987
DE2	11 Random	5.423E-5	1035
ES	1.457E-3	1.187E-5	433
ES	9.486E-2	1.318E-4	351
GBA	1.457E-3	1.238E-4	41
GBA	9.486E-2	2.433E-4	281
ES/DE/MQ	1.457E-3	1.961E-5	234
ES/DE/MQ	9.486E-2	2.125E-5	206
NF/GA/SQP		6.570E-5	189

There is growing interest in the ways in which the performance of a specific device could be modelled using a neural network. Such a network learns the shape of the hyper-surface and provides a fast evaluation of any point in it. Typically, the neural network is trained in a batch mode, prior to the optimisation process – essentially "off-line". A recent attempt has been made to construct a system which can provide "on-line" training, i.e. a network which is capable of learning and modifying its behaviour as it is used [6]. Such a network has major benefits over a static system in that it can handle a large number of variations of a device and track developments in design related to material changes and manufacturing processes.

Research into *Sensitivity Analysis* as an optimisation tool is also gaining momentum. Some successful implementations have already been reported. For example in [7] a generalized continuum sensitivity formula is applied to electrostatic problems. By exploiting the material derivative concept and the augmented Lagrangian method, the analytical sensitivity formula is derived from a multiobjective function and the variational equation describing the system, and can be expressed in terms of the fields of the primary system and the corresponding adjoint one. The formula is adaptable to all analysis methods (finite elements, boundary elements, finite differences) and the optimisation is not affected – in terms of overall computing times – by the number of design variables.

V. PARETO OPTIMISATION

Finally, *multi-objective optimisation* is becoming important as practical designs usually involve conflicting requirements. Problems are often converted into *single-objective* tasks with *a priori* application of some knowledge or imposition of a decision (for example *weighting factors*), but information can easily be lost in the process and some existing 'optimal' solutions may even be mathematically impossible to achieve. Instead the application of *Pareto Optimal Front* (POF) is advocated. The theory of Pareto multi-objective optimisation is somewhat complicated but some basic definitions and properties are easily explained using a special case of two objective functions being minimised as shown in Fig 3.

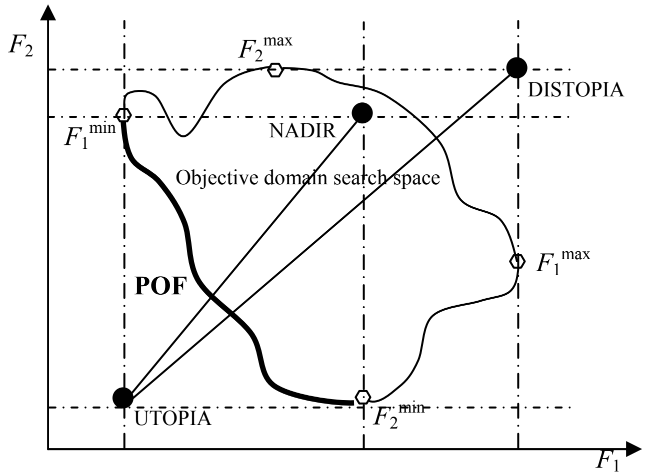

Fig. 3. Example of objective domain search space showing the Pareto Optimal Front (POF) and UTOPIA, DISTOPIA and NADIR points.

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