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Brief paper

Output feedback control of discrete linear repetitive processes[☆]

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Abstract

Repetitive processes are a distinct class of 2D systems (i.e. information propagation in two independent directions) of both systems theoretic and applications interest. They cannot be controlled by direct extension of existing techniques from either standard (termed 1D here) or 2D systems theory. Here, we give new results on the relatively open problem of the design of physically based control laws using an LMI setting. These results are for the sub-class of the so-called discrete linear repetitive processes which arise in applications areas such as iterative learning control.

Keywords: Repetitive dynamics; Stability; Stabilization; Output controller design; LMI

1. Introduction

Repetitive processes are a distinct class of 2D systems of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This in turn leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(p)$, $0 \leqslant p \leqslant \alpha - 1$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of next pass profile $y_{k+1}(p)$, $0 \leqslant p \leqslant \alpha-1$, $k \geqslant 0$.

Physical examples of repetitive processes include longwall coal cutting and metal rolling operations (see for example, Rogers & Owens, 1992). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes (Owens, Amann, Rogers, & French, 2000) and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle (Roberts, 2000).

One feature of repetitive processes in comparison to some other classes of 2D linear systems is that it is possible to define physically meaningful control laws for their dynamics. For example, in the ILC application, one such family of control laws is composed of state feedback control action on the current pass combined with information 'feedforward' from the previous pass (or trial in the ILC context) which,

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of course, has already been generated and is therefore available for use.

In the general case of repetitive processes it is clearly highly desirable to have an analysis setting where control laws can be designed for stability and/or performance. In which context, previous work has shown that an LMI reformulation of the stability conditions for discrete linear repetitive processes leads naturally to design algorithms to ensure closed-loop stability along the pass under control laws of the form referred to above—see, for example, Gałkowski, Rogers, Xu, Lam, and Owens (2002).

To implement a control law which uses the current pass state vector will, in general, require an observer to estimate the elements in this vector which are not directly measurable. As an alternative, this paper shows how to use the LMI setting to design control laws which only require pass profile information (which has already been generated and hence is available as a control signal) for implementation. Note here that LMI-based methods have also been investigated as a means of stability analysis and controller design for 2D discrete linear systems described by the well-known Roesser (Roesser, 1975) and Fornasini–Marchesini (Fornasini & Marchesini, 1978) state-space models, see, for example (Du & Xie, 2002). Discrete linear repetitive processes have strong structural links with such systems and some results can be exchanged between them. Other work (Du & Xie, 2002) has considered the use of a dynamic output feedback-based controller for other classes of 2D linear systems. Such a controller is obviously of a more complicated structure than a static alternative but is justified for these systems by the fact that static control is known to be of very limited use.

For discrete repetitive processes it is already known that static control laws can be highly effective (with attendant onward advantages in terms of eventual application). This has been established in previous work referred to above. The key novelty in this paper is the use of physically motivated control scheme which are actuated only by pass profile information and designed using LMI based methods. Then it is shown how to further strengthen such control laws by making enhanced use of already generated, and hence available for control purposes, current and previous pass profiles vector information.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively. Moreover, $M>0$ (<0) denotes a real symmetric positive (negative) definite matrix. We use $(*)$ to denote the transpose of matrix blocks in some of the LMIs employed (which are required to be symmetric).

2. Background

Following (Rogers & Owens, 1992), the state-space model of a discrete linear repetitive process has the following form over $0 \leqslant p \leqslant \alpha-1,\ k \geqslant 0$.

$$x_{k+1}(p+1) = Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p),$$

$$y_{k+1}(p) = Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p). \tag{1}$$

Here on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector and $u_k(p) \in \mathbb{R}^r$ is the vector of control inputs.

To complete the process description, it is necessary to specify the boundary conditions, i.e. the state initial vector on each pass and the initial pass profile. Here no loss of generality arises from assuming $x_{k+1}(0) = d_{k+1} \in \mathbb{R}^n,\ k \geqslant 0$, and $y_0(p) = f(p) \in \mathbb{R}^m$, where d_{k+1} is a vector with known constant entries and $f(p)$ is a vector whose entries are known functions of p. (For ease of presentation, we will make no further explicit reference to the boundary conditions in this paper.)

The stability theory (Rogers & Owens, 1992) for linear repetitive processes consists of two distinct concepts, termed asymptotic stability and stability along the pass respectively. In effect, asymptotic stability is bounded-input bounded-output stability (defined in terms of the norm on the underlying function space) over the finite pass length, and for the processes considered here requires that all eigenvalues of the matrix D_0 have modulus strictly less than unity (written $r(D_0)<1$ where $r(\cdot)$ denotes the spectral radius). If this property holds, and the control input sequence applied $\{u_k\}_{k \geqslant 1}$ converges strongly to u_∞ as $k \to \infty$, then the resulting output pass profile sequence $\{y_k\}_{k \geqslant 1}$ converges strongly to y_∞—the so-called limit profile—which is described (with $D=0$ for simplicity) by a 1D discrete linear system with state matrix $A_{\mathrm{lp}} := A + B_0(I-D_0)^{-1}C$.

The fact that the pass length is finite means that the limit profile may not be stable as a 1D linear system, i.e. $r(A_{\mathrm{lp}})<1$, e.g. $A=-0.5,\ B=0, B_0=0.5+b_0,\ C=1, D=D_0=0$, and the real scalar b_0 is chosen such that $|b_0| \geqslant 1$. Stability along the pass prevents this from arising by demanding the bounded-input bounded-output property uniformly, i.e. independent of the pass length α. (Mathematically, this can be analyzed by letting $\alpha \to +\infty$.) Several equivalent sets of necessary and sufficient conditions for processes described by (1) to have this property are known, but here the essential starting point is based on the so-called 2D characteristic polynomial given next.

Define the shift operators z_1, z_2 in the along the pass (p) and pass-to-pass directions (k) acting e.g. on the state and pass profile vectors respectively as

$$x_k(p) := z_1 x_k(p+1), \quad y_k(p) := z_2 y_{k+1}(p). \tag{2}$$

Then the 2D characteristic polynomial for processes described by (1) is defined as

$$\mathscr{C}(z_1, z_2) = \det \begin{bmatrix} I - z_1 A & -z_1 B_0 \\ -z_2 C & I - z_2 D_0 \end{bmatrix} \tag{3}$$

and it can be shown (Rogers & Owens, 1992) that stability along the pass holds if, and only if,

$$\mathscr{C}(z_1, z_2) \neq 0 \quad \text{in } \overline{U}^2,$$

where $\overline{U}^2 = \{(z_1, z_2) : |z_1| \leqslant 1, |z_2| \leqslant 1\}$. Note that stability along the pass can also be expressed in the form

$$\mathscr{C}(z_1, z_2) = \det(I - z_1\widehat{A}_1 - z_2\widehat{A}_2) \neq 0 \quad \text{in } \overline{U}^2, \tag{4}$$

where

$$\widehat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}, \quad \widehat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix}. \tag{5}$$

In this work, we use the following LMI-based sufficient condition derived from (4) which, unlike all other existing stability tests, leads immediately (see below) to systematic methods for control law design. The proof of this result can be found in Gałkowski et al. (2002).

Theorem 1. *A discrete linear repetitive process described by* (1) *is stable along the pass if there exist matrices* $Y > 0$ and $Z > 0$ *such that the following LMI holds*:

$$\begin{bmatrix} Y - Z & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y & \widehat{A}_2 Y & -Y \end{bmatrix} < 0.$$

The control law considered in previous work has the following form over $0 \leqslant p \leqslant \alpha - 1,\ k \geqslant 0$:

$$u_{k+1}(p) := K \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix} = K_1 x_{k+1}(p) + K_2 y_k(p), \tag{6}$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed. In effect, this control law uses feedback of the current state vector (which is assumed to be available for use) and 'feedforward' of the previous pass profile vector. Note that in repetitive processes the term 'feedforward' is used to describe the case where state or pass profile information from the previous pass (or passes) is used as (part of) the input to a control law applied on the current pass, i.e. to information which is propagated in the pass-to-pass (k) direction. The basic result for the design of this control law for closed-loop stability along the pass is as follows. (For a similar approach in the case of 1D linear systems see Crusius & Trofino, 1999.)

Theorem 2 (*Gałkowski et al., 2002*). *Consider a discrete linear repetitive process of the form described by* (1) *subject to a control law of the form* (6). *Then the resulting closed-loop process is stable along the pass if there exist matrices* $Y > 0,\ Z > 0$, *and* N *such that the following LMI holds*:

$$\begin{bmatrix} Z - Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N & \widehat{A}_2 Y + \widehat{B}_2 N & -Y \end{bmatrix} < 0, \tag{7}$$

where $\widehat{A}_1$, $\widehat{A}_2$ *are given in* (5) *and*

$$\widehat{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \widehat{B}_2 = \begin{bmatrix} 0 \\ D \end{bmatrix}. \tag{8}$$

If (7) *holds, then a stabilizing* K *in the control law* (6) *is given by*

$$K = NY^{-1}. \tag{9}$$

3. Output feedback-based controller design

In many cases the state vector $x_{k+1}(p)$ may not be available or, at best, only some of its entries are. Hence, we now consider the use of output-based feedback-based control laws to achieve closed-loop stability along the pass. The first law considered has the following form over $0 \leqslant p \leqslant \alpha - 1,\ k \geqslant 0$:

$$u_{k+1}(p) = \widetilde{K}_1 y_{k+1}(p) + \widetilde{K}_2 y_k(p). \tag{10}$$

This control law is, in general, weaker than that of (6) and examples are easily given where stability along the pass can be achieved using (6) but not (10). It is important to note here that by definition the pass profile produced on each pass is available for control purposes before the start of each new pass. As such, this control law (and extensions) assumes storage of the required previous pass profiles and that they are not corrupted by noise etc.

To consider the effect of a controller of form (10) on the process dynamics, first substitute the pass profile (second) equation of (1) into (10) to obtain (assuming the required matrix inverse exists)

$$\begin{aligned} u_{k+1}(p) &= (I - \widetilde{K}_1 D)^{-1} \widetilde{K}_1 C x_{k+1}(p) \\ &\quad + (I - \widetilde{K}_1 D)^{-1} [\widetilde{K}_2 + \widetilde{K}_1 D_0] y_k(p) \end{aligned} \tag{11}$$

and hence (11) can be treated as a particular case of (6) with

$$K_1 = (I - \widetilde{K}_1 D)^{-1} \widetilde{K}_1 C,$$

$$K_2 = (I - \widetilde{K}_1 D)^{-1} (\widetilde{K}_2 + \widetilde{K}_1 D_0). \tag{12}$$

This route may, however, encounter serious numerical difficulties (arising from the fact that (12) is a set of matrix nonlinear algebraic equations) and hence we proceed by rewriting these last equations to obtain

$$(I - \widetilde{K}_1 D) K_1 = \widetilde{K}_1 C,$$

$$(I - \widetilde{K}_1 D) K_2 = \widetilde{K}_2 + \widetilde{K}_1 D_0 \tag{13}$$

and assume that

$$K_1 = L_1 C. \tag{14}$$

Note that this assumption imposes no restrictions on the results developed here but could be a source of difficulty in other cases, e.g. in uncertainty analysis where the resulting robust control problem may not be convex.

It now follows immediately that

$$\widetilde{K}_1 = L_1(I + DL_1)^{-1},$$

$$\widetilde{K}_2 = [I - L_1(I + DL_1)^{-1}D]K_2 - L_1(I + DL_1)^{-1}D_0 \tag{15}$$

for any L_1 such that $I + DL_1$ is nonsingular, and we have the following result.

Theorem 3. *Suppose that a discrete linear repetitive process of the form described by* (1) *is subject to a control law of form* (10) *and that* (14) *holds. Then the resulting closed-loop process is stable along the pass if there exist matrices $Y > 0$, $Z > 0$, $X > 0$ and N such that the following LMI holds*

$$\begin{bmatrix} Z - Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N\widetilde{C} & \widehat{A}_2 Y + \widehat{B}_2 N\widetilde{C} & -Y \end{bmatrix} < 0,$$

$$X\widetilde{C} = \widetilde{C}Y, \tag{16}$$

where $\widehat{B}_1, \widehat{B}_2, \widehat{A}_1, \widehat{A}_2, N$ are defined as in Theorem 2, *and $\widetilde{C} = diag(C, I)$. Also if this condition holds, the controller matrices $\widetilde{K}_1$ and $\widetilde{K}_2$ can be obtained from* (15), *where*

$$[L_1 \;\; K_2] = NX^{-1} \tag{17}$$

and it is required that $I + DL_1$ is nonsingular.

Proof. From (17), we have that $N = LX$, $L := [L_1 \;\; K_2]$, and substitution into the LMI of (16) now gives with $X\tilde{C} = \tilde{C}Y$ applied

$$\begin{bmatrix} Z - Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 L\widetilde{C} Y & \widehat{A}_2 Y + \widehat{B}_2 L\widetilde{C} Y & -Y \end{bmatrix} < 0.$$

Finally, set $L\widetilde{C} = K$ to obtain the LMI stabilization condition (i.e Theorem 1 applied to the closed-loop process) which completes the proof. □

The design developed above is easily implemented using LMI toolboxes, such as Scilab or Matlab, but has the possible disadvantage that it is based on a sufficient but not necessary stability condition. (Also (14) can be a source of numerical difficulties when using the Matlab LMI toolbox but Scilab avoids such problems and hence is used in the numerical computations reported here.) This means that there could well be a not insignificant degree of conservativeness in the sense that in some cases it will fail to produce a stabilizing controller when one actually exists. To avoid, or lower, the level of conservativeness present, we next develop an extension of the control law considered in this section.

4. Extended output feedback-based controller design

The control law considered in this section has the following form and is, in effect, (10) augmented at point p by additive contributions from point $p-1$ on the previous pass profile and point p on the previous pass profile

$$u_{k+1}(p) = \widetilde{K}_1 y_{k+1}(p) + \widetilde{K}_2 y_k(p) + \widetilde{K}_3 y_k(p-1) + \widetilde{K}_4 y_{k-1}(p). \tag{18}$$

Substituting the second equation of (1) into the control law (18) now yields that this last control law is, in fact, a particular case of the so-called extended, mixed state, pass profile controller

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) + K_3 y_k(p-1) + K_4 y_{k-1}(p). \tag{19}$$

This last control law is, in effect, an extension of that of the previous section but here it is used as an intermediate step in the computation of the matrices $\widetilde{K}_i$, $i = 1, \ldots, 4$, through use of the following result.

Theorem 4. *Suppose that a discrete linear repetitive process of the form described by* (1) *is subject to a control law of form* (19) *and that* (14) *holds. Then the resulting closed loop process is stable along the pass if there exist matrices $Y > 0$, $X = diag(X_1, X_2, X_3, X_4) > 0$, $Z > 0$ and N such that*

$$\begin{bmatrix} Y - Z & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N\widehat{C} & \widehat{A}_2 Y + \widehat{B}_2 N\widehat{C} & -Y \end{bmatrix} < 0$$

$$X\widehat{C} = \widehat{C}Y, \tag{20}$$

where

$$\widehat{A}_1 = \begin{bmatrix} A & -I & 0 & B_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \widehat{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C & 0 & -I & D_0 \end{bmatrix},$$

$$\widehat{B}_1 = \begin{bmatrix} B & 0 & 0 & B \\ 0 & B & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & D & 0 \\ D & 0 & 0 & D \end{bmatrix}, \quad N = \begin{bmatrix} N_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -N_3 \\ 0 & 0 & 0 & -N_4 \\ 0 & 0 & 0 & N_2 \end{bmatrix},$$

$$\widehat{C} = \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{21}$$

with

$$\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_3 \\ 0 & 0 & 0 & K_4 \\ 0 & 0 & 0 & K_2 \end{bmatrix} = NX^{-1}. \tag{22}$$

Also if (20) *holds, the controller matrices* $\widetilde{K}_1$ *and* $\widetilde{K}_2$ *can be computed using* (15) *and then*

$$\widetilde{K}_3 = [I - L_1(I + DL_1)^{-1}D]K_3,$$
$$\widetilde{K}_4 = [I - L_1(I + DL_1)^{-1}D]K_4, \tag{23}$$

where it is assumed that $I + DL_1$ *is nonsingular*.

Proof. Substitute (19) into (1) and using (14) we obtain the closed-loop state-space model

$$x_{k+1}(p+1) = (A + BL_1C)x_{k+1}(p) + (B_0 + BK_2)y_k(p) + BK_3y_k(p-1) + BK_4y_{k-1}(p),$$
$$y_{k+1}(p) = (C + DL_1C)x_{k+1}(p) + (D_0 + DK_2)y_k(p) + DK_3y_k(p-1) + DK_4y_{k-1}(p). \tag{24}$$

This last description is not in the form to which Theorem 1 can be applied but it is possible to obtain an equivalent state-space model for which this is the case. Here the route is by using the delay operators of (2) and the 2D characteristic polynomial. To begin, apply (2) to (24) to obtain (after some routine manipulations)

$$\mathscr{C}_c(z_1, z_2) := \det \begin{bmatrix} I - z_1\widetilde{A} & -z_1\widetilde{B}_0 - z_1^2F_1 - z_1z_2F_3 \\ -z_2\widetilde{C} & I - z_2\widetilde{D}_0 - z_1z_2F_2 - z_2^2F_4 \end{bmatrix},$$

where

$$\widetilde{A} = A + BL_1C, \quad \widetilde{B}_0 = B_0 + BK_2,$$
$$F_1 = BK_3, \quad F_2 = DK_3,$$
$$\widetilde{C} = C + DL_1C, \quad \widetilde{D}_0 = D_0 + DK_2,$$
$$F_3 = BK_4, \quad F_4 = DK_4.$$

Application of appropriate elementary operations (which leave the determinant invariant) to the right-hand side of this last expression now yields that it can be replaced by

$$\det \begin{bmatrix} I - z_1\widetilde{A} & z_1I & 0 & -z_1\widetilde{B}_0 \\ 0 & I & 0 & z_1F_1 + z_2F_3 \\ 0 & 0 & I & z_1F_2 + z_2F_4 \\ -z_2\widetilde{C} & 0 & z_2I & I - z_2\widetilde{D}_0 \end{bmatrix}. \tag{25}$$

At this stage, the closed-loop state-space model has a 2D characteristic polynomial which is of the form required for use in (4) (and therefore Theorem 1 can be directly applied).

Application of Theorem 1 together with some obvious algebraic operations now yield directly the LMI of (20) as a sufficient condition for closed-loop stability along the pass. Finally, by an identical argument to that of the previous section we have that $\widetilde{K}_1$ and $\widetilde{K}_2$ can be computed using (15) and $\widetilde{K}_3$ and $\widetilde{K}_4$ using (23), provided $I + DL_1$ nonsingular, and the proof is complete. □

Note that the LMI of (20) is of dimension $6(n+m) \times 6(n+m)$ but, since we are dealing with stability along the pass, this does not depend on the number of samples along the pass, i.e. on the pass length α. The existing LMI toolboxes also allow us to solve relatively large problems. For example, we have successfully completed this design for the case when in the basic process state-space model $n = 15$, $m = 3$, which gives an LMI of dimension 108×108.

5. Numerical example

As a numerical example, consider the following process, with $x_{k+1}(0) = 1$, $k \geqslant 0$, $y_0(p) = 1$, $1 \leqslant p \leqslant 19$, which is unstable along the pass since $r(D_0) > 1$.

$$A = \begin{bmatrix} -1.36 & -1.29 & -0.8 \\ 0.15 & 0.34 & 0 \\ -0.19 & 0 & -1.36 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0.44 & 0.51 \\ 0.93 & 0.14 \\ 0.65 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.18 & -2.35 & 0.8 \\ 1.07 & -2.5 & 0.5 \\ -0.43 & 0.8 & 2.82 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.38 & 0 & -0.37 \\ 0 & 0 & -0.98 \end{bmatrix},$$

$$D = \begin{bmatrix} -2.85 & -0.65 & -2.5 \\ -0.28 & -2.98 & 1.96 \end{bmatrix}, \quad D_0 = \begin{bmatrix} -1.15 & 0 \\ -0.42 & 1.13 \end{bmatrix}.$$

In this case

$$\widetilde{K}_1 = \begin{bmatrix} 49.5 & -40.8 \\ 14.27 & -11.77 \\ -44.49 & 36.46 \end{bmatrix}, \quad \widetilde{K}_2 = \begin{bmatrix} -1.77 & 0.96 \\ -0.18 & 0.31 \\ 1.26 & -0.97 \end{bmatrix},$$

$$\widetilde{K}_3 = 10^{-12} \times \begin{bmatrix} 0.37 & -0.33 \\ 0.11 & -0.1 \\ -0.33 & 0.29 \end{bmatrix},$$

$$\widetilde{K}_4 = 10^{-13} \times \begin{bmatrix} 0.68 & -0.6 \\ 0.2 & -0.17 \\ -0.61 & 0.53 \end{bmatrix}.$$

Fig. 1 shows the response of the first entry in the pass profile vector for this example and demonstrates that it is unstable along the pass. Fig. 2 shows the corresponding stable along the pass response with the control law of Section 4 applied.

Note that in the example here the elements of $\widetilde{K}_3$ and $\widetilde{K}_4$ are significantly smaller in magnitude than those in the other controller matrices. Also if these matrices are deleted from the control law then it can be verified that the closed-loop process is still stable along the pass and there is very little difference in the controlled response. Note also that direct use of the design method of Theorem 3 fails to give a stable design. Hence, it can be conjectured that this last design method can be exploited to reduce the degree of conservativeness due to the use of a sufficient but not necessary stability condition.

6. Conclusions

This paper reports substantial new results on the control of discrete linear repetitive processes. The major conclusion is that the LMI-based approach which was previously developed for control laws which included a current pass state

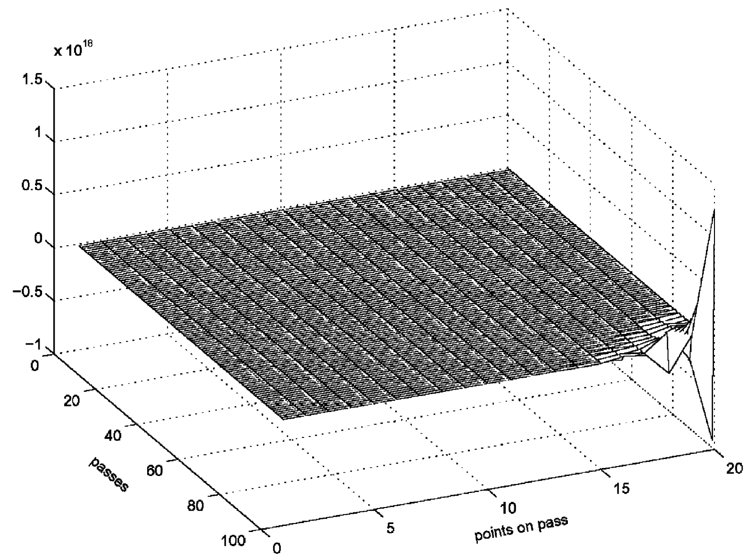

Fig. 1. Open-loop response.

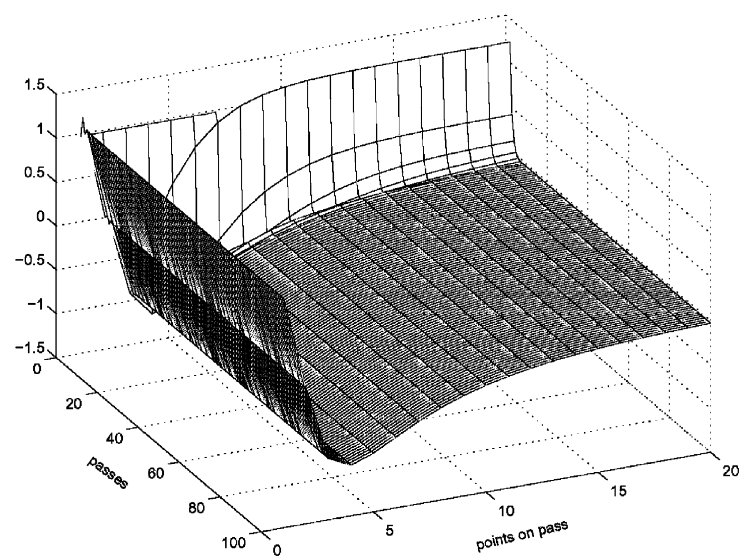

Fig. 2. Controlled response.

feedback component also extends to the more practically relevant case when state component is replaced by pass profile information from the current and the previous two pass profiles. In actual fact, there are a great number of other possibilities for partially (or completely) activating control laws with information from previous pass profiles and further work will include a detailed investigation of the relative merits of such laws. The results in Section 4 here provide a starting point for such work.

It is important to place these results in context, especially with respect to eventual practical applicability. In particular, the basic model is practically relevant as it does arise in approximating the dynamics of physical examples within the ILC area (see, for example, Hatonen, Harte, Owens, Ratcliffe, Lewin, & Rogers, 2003). It is eventually aimed to apply the control laws of this work to this problem area, for which the results in this paper provide part of the basic foundation.

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