

## Preface

### Special issue of DAM on the Vapnik–Chervonenkis dimension

The Vapnik–Chervonenkis (VC) is a combinatorial parameter of a class of binary functions or set system which has been shown to characterise the expressiveness of the system or class and in addition the learnability of the class from examples. The learnability follows from the fact that finite VC dimension implies an exponential bound on the probability of uniform relative deviation. The framework has also been formalised as the theory of  $\varepsilon$ -nets for range spaces in computational geometry, where the size of the net is used to characterise the complexity of a number of geometric problems. The concept also plays an important role in the study of logic, where it is seen as a tool to characterise the expressability of a function class.

Angus MacIntyre, Mark Jerrum and John Shawe-Taylor organised a *Workshop on the Vapnik–Chervonenkis dimension* at the International Centre for Mathematical Sciences in Edinburgh, Scotland which took place during the week of 9th–13th September 1996. The meeting received support from the UK funding body, the Engineering and Physical Sciences Research Council through a grant held by Mark Jerrum as well as from the European Commission through the ESPRIT Working Group, NeuroCOLT, coordinated by John Shawe-Taylor.

Arising out of the workshop a call for papers for a Special Issue of Discrete Applied Mathematics on the Vapnik–Chervonenkis dimension was announced, resulting in the current volume.

The aim of the workshop was to bring together theoreticians from all the disciplines that have reason to study the VC dimension, in order to exchange techniques, results and directions of research. It is, therefore, very pleasing to find the variety of papers that have been accepted for publication in the special issue.

Ben-David and Litman study the intriguing open question of the connection between VC classes and the existence of sample compression schemes. They show how half-spaces can be compressed and prove that they are universal with respect to families of algebraically defined classes. Using the approach they prove the existence of compression schemes for ‘geometric concept classes’.

Two of the papers consider generalisations of the concept and the implications for learning. Cesa-Bianchi and Haussler describe a generalisation to function classes whose outputs are in an arbitrary, totally bounded metric space. Horváth and Lugosi consider scale-sensitive dimensions as a way of improving bounds on the generalisation error based on improved learning algorithms.

In contrast, Koiran and Sontag consider the classical VC dimension but compute its value for various classes of recurrent neural networks. Maiorov and Ratsaby bring out the connection to approximation theory by computing degrees of approximation for sets when using sets with bounded VC dimension.

Sill considers the more powerful class of monotonic functions which has infinite VC dimension. Despite this fact, he is, however, able to show by estimating the metric entropy in specific distributions that in many natural cases learning is possible. In contrast, Steinsaltz considers classes with very low VC dimension and obtains tighter bounds for practically interesting applications of fluctuation bounds.

The VC dimension seems to have a knack of cropping up in very different contexts. The breadth of papers in this special issue bears witness to this fact and we hope that by bringing them together in one volume we will encourage a cross-fertilisation of both ideas and techniques.

John Shawe-Taylor

Guest Editor

*Department of Computer Science,  
Royal Holloway, University of London,  
Egham,  
Surrey TW20 0EX  
UK*