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Distance-Regularised Graphs Are Distance-Regular or Distance-Biregular

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One problem with the theory of distance-regular graphs is that it does not apply directly to the graphs of generalised polygons. In this paper we overcome this difficulty by introducing the class of distance-regularised graphs, a natural common generalisation. These graphs are shown to either be distance-regular or fall into a family of bipartite graphs called distance-biregular. This family includes the generalised polygons and other interesting graphs. Despite this increased generality we are also able to extend much of the basic theory of distance-regular graphs to our wider class of graphs. © 1987 Academic Press, Inc.

1. Introduction, Examples and Definitions

A graph G is *distance-regular* if, for any integer k and vertices x and y, the number of vertices at distance k from x and adjacent to y only depends on $d(x, y)$, the distance between x and y (note that such graphs must be regular). A *generalised polygon* or *generalised n-gon* is a bipartite graph of diameter n with vertices in the same colour class having the same degree and with pairs of vertices less than distance n apart joined by a unique shortest path.

Both distance-regular graphs and generalised polygons are important combinatorial objects. Moreover they are closely related. Any generalised polygon is by definition semiregular and determines, in a natural way, two

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distance-regular graphs [6]. Any regular generalised polygon is itself a distance-regular graph. Given the importance of these two classes of graphs, and their close connection, it seems worthwhile to look for a common framework.

Let us call a graph *distance-regularised* if, for any integer k and any vertices x and y, the number of vertices at distance k from x and adjacent to y only depends on $d(x, y)$ *and* the vertex x. Clearly any distance-regular graph is distance-regularised and also any generalised polygon is distance-regularised. However, it appears that, if we sought only to catch distance-regular graphs and generalised polygons, we have cast a rather wide net.

In this paper we show that a distance-regularised graph is either distance-regular or is a bipartite semiregular graph from which we can derive two distance-regular graphs. These bipartite graphs will be called distance-biregular (thus we caught almost exactly the fish we wanted). We then present a short study of the basic theory of distance-biregular graphs, roughly corresponding to that of distance-regular graphs (for which we refer the reader to, e.g., [1]). The theory of distance-biregular graphs is also discussed at length in [5].

The existence of generalised polygons implies, by our remarks above, that some distance-regular graphs come in pairs. One interesting consequence of our work is that this pairing is a more widespread phenomenon than was previously realised.

1.1. *Examples of Distance-Biregular Graphs*

We have already met the generalised polygons. The complete bipartite graphs form another, somewhat trivial, family. We now present some other nontrivial examples.

EXAMPLE 1.1.1. Consider the set $\{1, ..., n\}$. Let $A = \{k\text{-subsets}\}$ and $B = \{k+1\text{-subsets}\}$ where k is a number less than n. $VG = A \cup B$ and adjacency is defined in the natural way: u is adjacent to v, with $u \in A$ and $v \in B$ if $u \subseteq v$.

EXAMPLE 1.1.2. Consider an n-dimensional vector space over $\mathrm{GF}(q)$, where q is the power of a prime and $\mathrm{GF}(q)$ is the (unique) Galois field of order q. Let $A = \{k\text{-subspaces}\}$ and $B = \{k+1\text{-subspaces}\}$ and $VG = A \cup B$. For $u \in A$ and $v \in B$, u is adjacent to v if $u \subseteq v$.

EXAMPLE 1.1.3. Let D be a quasisymmetric 2-design with block intersection numbers i_1, i_2, with $i_2 = 0$. Then the incidence graph G of D is a distance-biregular graph of diameter 4. (Any 2-design with $\lambda = 1$ is an example of such a design.)

1.2. *Definitions and Notation*

Let G be a graph. By VG we denote the vertex set of G and by EG the edge set. For $u, v \in VG$ we say u is adjacent to v if $(u, v) \in EG$. With $d(u, v)$ we denote the usual distance in G between vertices u and v. For $v \in VG$ and $i \in N$, $G_i(v)$ denotes the set of vertices at distance i from v. For $u \in VG$ and $v \in G_i(u)$ we write $c(u, v) = |G_{i-1}(u) \cap G_1(v)|$, $b(u, v) = |G_{i+1}(u) \cap G_1(v)|$, $a(u, v) = |G_i(u) \cap G_1(v)|$ and $k_i(u) = |G_i(u)|$.

Let $d(u) = \max\{i \mid G_i(u) \neq \emptyset\}$. We are interested in vertices $u \in VG$ for which, for each i $(1 \leqslant i \leqslant d(u))$, the numbers $b(u, v)$, $a(u, v)$, and $c(u, v)$ are independent of the choice of $v \in G_i(u)$. In this case we say u is distance-regularised and we denote $b(u, v)$, $a(u, v)$, and $c(u, v)$ by $b_i(u)$, $a_i(u)$, and $c_i(u)$. Then the array

$$i(u) = \begin{bmatrix} * & c_1(u), \dots, c_{d(u)-1}(u), & c_{d(u)}(u) \\ 0 & a_1(u), \dots, a_{d(u)-1}(u), & a_{d(u)}(u) \\ b_0(u), & b_1(u), \dots, b_{d(u)-1}(u), & * \end{bmatrix}$$

is called the intersection array for u, and the matrix

$$I(u) = \begin{bmatrix} 0 & c_1(u) & 0 & \cdots & & & \\ b_0(u) & a_1(u) & c_2(u) & & & & \\ 0 & b_1(u) & a_2(u) & & & & \\ 0 & 0 & b_2(u) & & & & \\ & & & \ddots & & & \\ \vdots & & & & b_{d(u)-2}(u) & a_{d(u)-1}(u) & c_{d(u)}(u) \\ & & & & 0 & b_{d(u)-1}(u) & a_{d(u)}(u) \end{bmatrix}$$

is called the intersection matrix for u.

We will call a connected graph in which every vertex is distance-regularised a *distance-regularised* graph. The much studied *distance-regular* graphs are distance-regularised graphs in which all vertices have the same intersection array. Another special case of distance-regularised graphs are bipartite distance-regularised graphs in which vertices in the same colour class have the same intersection array. We call these graphs *distance-biregular.*

Unless explicitly stated, we use the following standardised notation for a distance-biregular graph. Sets A and B denote the colour partition of VG, $d_A = d(u)$ $(u \in A)$, u is a vertex in A and has intersection array

$$i(A) = \begin{bmatrix} * & 1 & c_2 & \cdots & c_{d_A} \\ 0 & 0 & 0 & & 0 \\ r & b_1 & b_2 & \cdots & * \end{bmatrix}$$

or just

$$\begin{bmatrix} * & 1 & c_2 & \cdots & c_{d_A} \\ r & b_1 & b_2 & \cdots & * \end{bmatrix},$$

$d_B = d(v)$ $(v \in B)$, v is a vertex in B and has intersection array

$$i(B) = \begin{bmatrix} * & 1 & f_2 & \cdots & f_{d_B} \\ s & e_1 & e_2 & \cdots & * \end{bmatrix}.$$

The corresponding intersection matrices are denoted $I(A)$ and $I(B)$, respectively. The diameter d of G is of course $\max\{d_A, d_B\}$. Note that $\deg(u) = r$ and $\deg(v) = s$. We denote with k_i the numbers $|G_i(u)|$ and with l_i the numbers $|G_i(v)|$, $i = 0,..., d$. Note that $l_{d-1} \neq 0$ and $k_{d-1} \neq 0$ though one of l_d and k_d may be zero.

2. Distance-Regularised Graphs

We first present a lemma, which though not in itself very interesting is proved in a similar way to the main theorems and will be useful later.

Lemma 2.1. *Let G be a distance-regularised graph. Then either G is regular or G is bipartite with vertices in the same colour class having the same degree.*

Proof. Let $v, v' \in VG$ with $d(v, v') = 2$. We can therefore find u in VG adjacent to both v and v'. Then $\deg(v) = b_1(u) + a_1(u) + c_1(u) = \deg(v')$. Let v and w be any vertices of G such that there exists a path $v = v_1, v_2,..., v_{2k+1} = w$ from v to w of even length. By the above $\deg(v_{2i-1}) = \deg(v_{2i+1})$ for $i = 1,..., k$, and so $\deg(v) = \deg(w)$. Assume now that G is not bipartite. In this case we can find a path of even length between any two vertices. Hence G is regular. If on the other hand G is bipartite, vertices in the same colour class are at even distance and so have the same degree. ∎

We are now ready to tackle the main theorem of this section which deals with the non bipartite case.

Theorem 2.2. *Let G be a nonbipartite distance-regularised graph, then G is distance-regular.*

Proof. Let $u, v \in VG$ with u adjacent to v. We will prove by induction that these two vertices have the same intersection array. As G is connected the result will follow directly. Before beginning the inductive argument we calculate the number $|G_t(u) \cap G_t(v)|$. This is given by $k_t(u) - r_t(u, v) - s_t(u, v)$, where $r_t(u, v) = |G_t(u) \cap G_{t+1}(v)|$ and $s_t(u, v) = |G_t(u) \cap G_{t-1}(v)|$. Note that $s_1(u, v) = 1$ and $r_1(u, v) = b_1(v)$. By counting edges between $G_t(u) \cap G_{t-1}(v)$ and $G_{t-1}(u) \cap G_{t-2}(v)$ $(t-1 \leqslant d(v))$ we obtain

$$c_{t-1}(v)\, s_t(u, v) = s_{t-1}(u, v)\, b_{t-1}(u)$$

as each vertex in $G_t(u)$ adjacent to a vertex in $G_{t-1}(u) \cap G_{t-2}(v)$ must be in $G_t(u) \cap G_{t-1}(v)$, while each of the $c_{t-1}(v)$ neighbours nearer to v of a vertex in $G_t(u) \cap G_{t-1}(v)$ must lie in $G_{t-1}(u) \cap G_{t-2}(v)$. Hence

$$s_t(u, v) = \frac{b_{t-1}(u) \cdots b_1(u)}{c_{t-1}(v) \cdots c_1(v)}.$$

Similarly for $t \leqslant d(u)$

$$r_t(u, v) = \frac{b_t(v) \cdots b_1(v)}{c_t(u) \cdots c_1(u)}.$$

Note also that

$$k_t(u) = \frac{b_{t-1}(u) \cdots b_0(u)}{c_t(u) \cdots c_1(u)}.$$

We now start the induction on the columns of the intersection arrays. By Lemma 2.1 the first entry in each array is the same as G is regular. Now assume this is true for all entries up to and including the $(t-1)$-st column, for some t, $1 \leqslant t \leqslant d(u)$. In particular $b_{t-1}(u) = b_{t-1}(v) \neq 0$, so $d(v) \geqslant t$. The inductive assumption and the fact that $t \leqslant d(u)$ allows us to evaluate

$$|G_t(u) \cap G_t(v)| = k_t(u) - r_t(u, v) - s_t(u, v) \quad \text{as}$$

$$\frac{b_{t-1}(u) \cdots b_1(u)}{c_t(u) \cdots c_1(u)} \{b_0(u) - c_t(u) - b_t(v)\}. \qquad (*)$$

We consider two cases.

Case 1. $G_t(u) \cap G_t(v) = \varnothing$.

By the above formula and the fact that $t \leqslant \min\{d(u), d(v)\}$, $c_t(u) + b_t(v) = k$, the degree of G. By the symmetry of $|G_t(u) \cap G_t(v)|$ and $t \leqslant \min\{d(u), d(v)\}$, $c_t(v) + b_t(u) = k$ and so

$$c_t(u) + b_t(u) + b_t(v) + c_t(v) = 2k$$

and we must have $c_t(u) + b_t(u) = k = b_t(v) + c_t(v)$. In this case $a_t(u) =$

$a_t(v)=0$. Note also that $b_t(v)=k-c_t(u)=b_t(u)$, so that the arrays of u and v agree in the t-th column.

Case 2. $G_t(u)\cap G_t(v)\neq\varnothing$.

Hence $t\leqslant \min\{d(u), d(v)\}$. Let $w\in G_t(u)\cap G_t(v)$ and $q_i=|G_i(w)\cap G_{t-i}(u)|$. Clearly $q_1=c_t(u)$ and we can readily evaluate

$$q_i=\frac{c_t(u)\cdots c_{t-i+1}(u)}{c_1(w)\cdots c_i(w)}.$$

Using the induction hypothesis $q_{t-1}=c_t(u)$. But $q_{t-1}=c_t(w)$ by definition and so $c_t(u)=c_t(w)$. Similarly since $w\in G_t(u)\cap G_t(v)\neq\varnothing$, $c_t(w)=c_t(v)$ and so $c_t(u)=c_t(v)$. Finally calculating $|G_t(u)\cap G_t(v)|$ in two ways we have $c_t(u)+b_t(v)=c_t(v)+b_t(u)$, so $b_t(v)=b_t(u)$ and the tth column of the arrays of u and v agree.

In either case the intersection arrays for u and v agree up to the $d(u)$th column. But then $b_{d(u)}(v)=0$ and so $d(v)=d(u)$. Hence the arrays are identical. ∎

We have now dealt with the nonbipartite case. To cover the bipartite case we present

LEMMA 2.3. *Let G be a bipartite distance-regularised graph with $u, v\in VG$ and u adjacent to v. Then the intersection array for v can be computed from that of u.*

Proof. Note first that $|d(u)-d(v)|\leqslant 1$. We compute the intersection array for v. We have $G_i(v)\subseteq G_{i-1}(u)\cup G_{i+1}(u)$. Set $x_i=|G_i(u)\cap G_{i-1}(v)|$. Thus in the notation and by the derivation in the proof of Theorem 2.2 $x_i=s_i(u, v)$ and

$$x_i=\frac{b_1(u)\,b_2(u)\cdots b_{i-1}(u)}{c_1(v)\,c_2(v)\cdots c_{i-1}(v)}\qquad \text{for}\quad i=1,..., d(u).$$

Then $x_1=1$, $x_2=b_1(u)$. Note also that $k_0(v)=1$, $k_1(v)=b_1(u)+1$. Assume now that we know $b_j(v)$, $c_j(v)$, $k_j(v)$, $j<i$ for some i, $1<i\leqslant d(v)$. If $i=d(u)+1$, then $i=d(v)$ so $c_i(v)=b_0(u)$, if $d(v)$ is odd and $c_i(v)=b_1(u)+1$, otherwise. Clearly $b_i(v)=0$ and so we have computed the whole of $i(v)$. Hence we can assume that $i\leqslant d(u)$, enabling us to calculate x_i. This also means that $b_i(u)$ is defined (though possibly 0) and that $c_i(v)\neq 0$. So $k_i(v)=k_{i-1}(u)-x_{i-1}+x_{i+1}$, since

$$|G_i(v)\cap G_{i-1}(u)|=|G_{i-1}(u)|-x_{i-1}.$$

Note that $x_{i+1}=x_ib_i(u)/c_i(v)$, which correctly computes to 0 if $i=d(u)$.

We also have $k_i(v)=k_{i-1}(v)\,b_{i-1}(v)/c_i(v)$. If $k_{i-1}(u)=x_{i-1}$ then $G_{i-1}(u)\subseteq G_{i-2}(v)$ and so $G_i(v)=\emptyset$, as otherwise we could find a vertex in $G_i(v)\cap G_{i-1}(u)$. Hence $d(v)=i-1<i$, a contradiction. We conclude that $k_{i-1}(u)\neq x_{i-1}$, which enables us to eliminate $k_i(v)$ and obtain

$$c_i(v)=(k_{i-1}(v)\,b_{i-1}(v)-x_ib_i(u))/(k_{i-1}(u)-x_{i-1}).$$

We can then of course evaluate

$$b_i(v)=b_{i-1}(u)+c_{i-1}(u)-c_i(v),$$

while $k_i(v)=k_{i-1}(v)\,b_{i-1}(v)/c_i(v)$. This completes the calculations of another column of the array. We can thus inductively compute the array for v to the $d(v)$-th column, that is we can compute the whole of $i(v)$. ▌

COROLLARY 2.3.1. *A bipartite distance-regularised graph is distance-biregular.*

Proof. Let u, w be vertices of a bipartite distance-regularised graph G which lie in the same colour class. Then there exists a path of even length from u to w. Alternate vertices along this path have the same intersection array by the lemma. Hence u and w have the same array. ▌

COROLLARY 2.3.2. *Let G be a distance-biregular graph with the standard notation. Then the intersection array $i(B)$ can be computed from the array $i(A)$ using the following method: set $s=b_1+c_1$, $e_0=s$, $f_1=1$, $e_1=b_0-1$,*

$$l_0=1,\qquad l_1=s,\qquad x_1=1,\qquad x_2=b_1.$$

Then for $i=2,\ldots,\min\{d_A,d_B\}$ we have

$$\begin{aligned}f_i&=(l_{i-1}e_{i-1}-x_ib_i)/(k_{i-1}-x_{i-1}),\\ e_i&=b_{i-1}+c_{i-1}-f_i,\\ x_{i+1}&=x_ib_i/f_i,\\ l_i&=l_{i-1}e_{i-1}/f_i,\end{aligned}$$

where d_B is the first i for which $e_i=0$. If $d_B>d_A$ then

$$f_{d_B}=\begin{cases}s & \text{if } d \text{ is even}\\ r & \text{if } d \text{ is odd,}\end{cases}$$

while $e_{d_B}=0$.

Proof. A distance-biregular graph is a bipartite distance-regularised graph. Hence we can compute the second intersection array using the method of the lemma. The equations obtained in the lemma are those listed. ∎

3. Feasible Arrays for a Distance-Biregular Graph

We begin by stating the main result of this section.

THEOREM 3.1. *Let G be a distance-biregular graph. Then the eigenvalues of G and their multiplicities can be determined from either of its two intersection arrays.*

Proof. We begin a proof of this theorem by introducing some notation. For any square matrix A we define

$$W(A, x) = \sum_{k=0}^{\infty} x^k A^k = (I - xA)^{-1}$$

and $\phi(A, x) = \det(xI - A)$. With a slight abuse of notation we write $W(G, x)$ for $W(A, x)$ and $\phi(G, x)$ for $\phi(A, x)$, where G is a digraph with adjacency matrix A. $W(G, x)$ is called the walk generating function for G, while $\phi(G, x)$ is the characteristic polynomial of G. The basic results for walk generating functions which we will require are the following:

(i) for $v \in VG$, $W_{vv}(G, x) = (1/x) \cdot \phi(G - v, 1/x)/\phi(G, 1/x)$,

(ii) $\operatorname{trace}(W(G, x)) = -x \cdot \phi'(G, 1/x)/\phi(G, 1/x)$.

A proof of (i) is given in [2], while (ii) is an immediate consequence of (i).

Consider a distance-biregular graph with the standard notation. Let P be the intersection matrix $I(A)$ for each vertex u in A and Q the matrix $I(B)$ for each vertex v in B. It can be readily verified by induction that for $u \in A$, the number of walks of length k in G which start at a specified vertex in $G_i(u)$ and finish anywhere in $G_j(u)$ is $(P^k)_{ji}$. A similar result holds for Q. This in turn means that $W_{uu}(G, x) = W_{00}(P, x)$ for $u \in A$ and $W_{vv}(G, x) = W_{00}(Q, x)$ for $v \in B$. Hence we can perform the following calculation:

$$\begin{aligned} -x\phi'(G, 1/x)/\phi(G, 1/x) &= \operatorname{trace}(W(G, x)) \\ &= \sum_{u \in A} W_{uu}(G, x) + \sum_{v \in B} W_{vv}(G, x) \\ &= nW_{00}(P, x) + mW_{00}(Q, x) \\ &= n(1/x)\, \phi(P - 0, 1/x)/\phi(P, 1/x) \\ &\quad + m(1/x)\, \phi(Q - 0, 1/x)/\phi(Q, 1/x), \end{aligned}$$

where $P-0$ is the matrix obtained from P by deleting the first row and column. Similarly for $Q-0$. Replacing x by $1/x$ yields:

$$\phi'(G, x)/\phi(G, x) = n\phi(P-0, x)/\phi(P, x) + m\phi(Q-0, x)/\phi(Q, x).$$

The matrix P is the adjacency matrix of a quotient multigraph of G of diameter d_A. Hence the eigenvalues of P are eigenvalues of G and P has at least d_A+1 distinct eigenvalues [1], so all its eigenvalues must be simple. A similar argument holds for Q. This means we can write:

$$1/\phi(P, x) = \sum_{\theta\in\lambda(P)} 1/((x-\theta)\,\phi'(P, \theta))$$

and similarly for Q. For the l.h.s. we have

$$\phi'(G, x)/\phi(G, x) = \sum_{\theta\in\lambda(G)} m(\theta)/(x-\theta)$$

where $m(\theta)$ is the multiplicity of θ in G. Hence

$$\sum_{\theta\in\lambda(G)} m(\theta)/(x-\theta) = n\sum_{\theta\in\lambda(P)} \phi(P-0, x)/(\phi'(P, \theta)(x-\theta)) + m\sum_{\theta\in\lambda(Q)} \phi(Q-0, x)/(\phi'(Q, \theta)(x-\theta))$$

equating residuals we obtain for each $\theta\in\lambda(G)$,

$$m(\theta) = n\phi(P-0, \theta)\,\chi_{\lambda(P)}(\theta)/\phi'(P, \theta) + m\phi(Q-0, \theta)\,\chi_{\lambda(Q)}(\theta)/\phi'(Q, \theta). \qquad (**)$$

Equation (**) enables us to calculate the multiplicities for each eigenvalue of $\lambda(G)$ and also tells us we have all the eigenvalues of G present on the r.h.s.:

$$\lambda(G) = \lambda(P)\cup\lambda(Q). \quad \blacksquare$$

In the theory of distance-regular graphs the multiplicities of the eigenvalues are normally expressed in terms of components of the eigenvectors of the intersection array. A similar formula can be obtained in our case. To be more precise, if t is an eigenvalue of P and y (x) a left (right) eigenvector corresponding to t, normalised with $x_0 = y_0 = 1$. Then

$$\phi'(P, t)/\phi(P-0, t) = y^T x.$$

Of course the same holds for Q. For details see [5].

Theorem 3.1 makes it reasonable to define a pair of feasible arrays for a distance-biregular graph in an analogous way to feasible arrays for distance-regular graphs [1]. We give here a definition by outlining a list of conditions which the two arrays must satisfy. Any statements that have not already been proved are elementary (Proofs are given in [5]).

DEFINITION 3.2. Two intersection arrays are said to be a pair of feasible arrays for a distance-biregular graph if

(i) they satisfy the following numerical conditions:

$$k_0 = 1, \qquad k_{i+1} = k_i \cdot b_i / c_{i+1},$$
$$l_0 = 1, \qquad l_{i+1} = l_i \cdot e_i / f_{i+1}$$

and the k_i and l_i are whole numbers.

Alternate (nonzero) columns in the intersection arrays sum to r and s,

$$c_i + b_i = \begin{cases} r & \text{if } i \text{ is even} \\ s & \text{if } i \text{ is odd,} \end{cases}$$

$$e_j + f_j = \begin{cases} r & \text{if } j \text{ is odd} \\ s & \text{if } j \text{ is even,} \end{cases}$$

$$e_{i-1} \geqslant b_i, \qquad i = 1, \ldots, d_A - 1,$$
$$b_{i-1} \geqslant e_i, \qquad i = 1, \ldots, d_B - 1$$
$$f_i \geqslant c_{i-1}, \qquad i = 2, \ldots, d_B,$$
$$c_i \geqslant f_{i-1}, \qquad i = 2, \ldots, d_A,$$
$$1 + k_2 + k_4 + \cdots + k_{d'} = l_1 + l_3 + \cdots + l_{d''} =: n$$

and

$$k_1 + k_3 + \cdots + k_{d''} = 1 + l_2 + l_4 + \cdots + l_{d'} =: m,$$

where d' is the largest even integer less than or equal to d and d'' is the largest such odd integer. Also $nr = ms$.

(ii) Each array can be computed from the other using the formulas of Corollary 2.3.2.

(iii) The values determined as multiplicities using Theorem 3.1 are positive integers. ∎

4. Further Directions

There is a clear need to determine whether the classes of distance-biregular graphs mentioned in this paper exhaust, in any sense, the possibilities. One approach to finding examples would be to attempt a thorough classification of the distance-biregular graphs with small minimum valency. The results of Section 3 should be useful for this. The distance-biregular graphs with vertices of degree two are completely characterised in [3].

We have not considered any group theoretic questions in the present paper. A reasonably complete development of the theory of distance-bitransitive graphs (the distance-biregular analogue of distance-transitive graphs) is presented in [4].

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