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# List of Symbols

## Chapter 1

$FWHM$	full width at half maximum
$N$	complex index of refraction

## Chapter 2

### Section 2.1

$\cdot$	scalar product of the <i>Euclidean</i> space
$\times$	cross product
$\nabla$	nabla operator, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)^T$
$\partial$	partial derivative symbol
$\partial S$	edge of surface $S$
$\partial V$	surface of volume $V$
$\alpha$	a scalar
$\varepsilon(\vec{r})$	dielectric function
$\varepsilon_0$	dielectric constant of vacuum, $\varepsilon_0 = 8.85 \cdot 10^{-12} As/Vm$
$\mu_0$	magnetic permeability of vacuum, $\mu_0 = 4\pi \cdot 10^{-7} Vs/Am$
$\omega$	angular frequency
$\phi, \phi(z)$	scalar function of the $z$ coordinate defining the amplitude of the magnetic field
$\rho(\vec{r})$	density of charges
$\Theta, \Theta(\bullet)$	operator on three-dimensional vector-field space
$a$	lattice constant
$b$	reciprocal lattice constant
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$\vec{c}_{k_x,q}$	<i>Fourier</i> expansion coefficients of the <i>Bloch</i> state
$da$	integration over a surface
$d\vec{l}$	integration along a curve
$dV$	integration over a volume

$\vec{j}(\vec{r})$	electric charge current
$k_x, k_y$	wave vector components in $x, y$ direction, respectively
$\hat{n}$	unit outward normal to the surface of integration
$p$	integer number
$q$	integer number
$\vec{r}$	position vector in the <i>Euclidean</i> space
$\vec{u}_{k_x}(x)$	periodic vector field function with one-dimensional domain, <i>Bloch</i> state
$t$	time
$x, y, z$	coordinates of the <i>Euclidean</i> space, $(x, y, z) = \vec{r}$
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes
$A$	a scalar constant
$\vec{B}(\vec{r})$	magnetic induction
$\vec{D}(\vec{r})$	dielectric displacement
$\vec{E}(\vec{r}), \vec{E}(\vec{r}, t)$ ,	electric field
$\vec{E}(\vec{r}, \omega)$	
$\vec{F}, \vec{F}(\vec{r})$	a vector field
$\vec{G}$	a vector field
$\vec{H}(\vec{r}), \vec{H}(\vec{r}, t)$ ,	magnetic field
$\vec{H}(\vec{r}, \omega)$	
$\vec{R}$	lattice vector in <i>Euclidean</i> space
$S$	surface
$T_{\vec{R}}$	discrete translation operator on vector-field space
$V$	volume

### Section 2.1.1

$\partial$	partial derivative symbol
$\varepsilon(\vec{r})$	dielectric function
$k_0$	wave number or magnitude of wave vector in vacuum
$x, y, z$	coordinates of the <i>Euclidean</i> space, $(x, y, z) = \vec{r}$
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes
$E(\vec{r})$	electric field magnitude
$H(\vec{r})$	magnetic field magnitude

### Section 2.1.2

$(\phi, \psi)$	one-dimensional scalar product $(\phi, \psi) = \int \phi \cdot \psi^* dx$
$\varepsilon(\vec{r})$	dielectric function
$\varepsilon_p$	<i>Fourier</i> coefficient of dielectric function
$\phi$	scalar function
$\psi$	scalar function
$b$	reciprocal lattice constant
$k_0$	wave number or magnitude of wave vector in vacuum
$p$	integer number
$q$	integer number
$x$	coordinate of one-dimensional space
$E(\vec{r})$	electric field magnitude
$E_p$	<i>Fourier</i> coefficient of electric field

### Section 2.1.3

$\angle(\bullet, \bullet)$	angle between two vectors
$\omega$	angular frequency
$a_x, a_z$	lattice constants in $x, z$ direction
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$\vec{k}$	wave vector
$k_x, k_z$	wave vector components in $x, z$ direction
$x, z$	coordinates of the <i>Euclidean</i> space
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes

### Section 2.1.4

$a_x, a_z$	lattice constants in $x, z$ direction
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$\vec{k}$	wave vector
$k_x, k_z$	wave vector components in $x, z$ direction
$x, z$	coordinates of the <i>Euclidean</i> space
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes

## Section 2.2

.	scalar product of the <i>Euclidean</i> space
$\nabla$	nabla operator, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)^T$
$\partial$	partial derivative symbol
$\partial\Omega$	surface of volume $\Omega$
$\epsilon_0$	dielectric constant of vacuum, $\epsilon_0 = 8.85 \cdot 10^{-12} As/Vm$
$\mu_0$	magnetic permeability of vacuum, $\mu_0 = 4\pi \cdot 10^{-7} Vs/Am$
$\mu(S)$	area of the surface $S$
$\omega$	angular frequency
$\theta_{inc}$	angle of incidence
$\theta_p$	angle of diffraction of $p^{\text{th}}$ order
$\Omega$	arbitrary volume
$b$	reciprocal lattice constant
$da$	integration over a surface
$i$	imaginary unit $i = \sqrt{-1}$
$\vec{k}_0$	wave vector of oscillating scalar field $U$ parallel to $\vec{r}_0$
$k_0$	magnitude of $\vec{k}_0$
$k_{0x}$	$x$ component of $\vec{k}_0$
$\hat{n}$	unit inward normal to the surface of integration
$p$	integer number
$\vec{r}$	position vector in the <i>Euclidean</i> space
$\vec{r}_0$	arbitrary point in space
$r_0$	magnitude of $\vec{r}_0$
$\hat{r}_0$	unit vector of $\vec{r}_0$
$\vec{r}_1$	mirror image of $\vec{r}_0$ with respect to a plane
$x$	coordinate of the <i>Euclidean</i> space
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$\vec{E}(\vec{r}), \vec{E}(x)$	electric field
$\vec{E}_p$	<i>Fourier</i> coefficient of scaled electric field
$\vec{H}(\vec{r}), \vec{H}(x)$	magnetic field
$\vec{H}_p$	<i>Fourier</i> coefficient of scaled magnetic field
$S$	plane surface generating the far-field effect
$U$	scalar function of the <i>Euclidean</i> space
$V$	scalar function of the <i>Euclidean</i> space
$Z_0$	impedance of vacuum, $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.73V/A$

### Section 2.3.1

$\cdot$	scalar product of the <i>Euclidean</i> space
$\times$	cross product
$\nabla$	nabla operator symbol, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)^T$
$\partial$	partial derivative symbol
$(x_i, z_j), (i, j)$	point of the discrete space, grid point
$(i, j, n)$	point of the discrete space/time
$\varepsilon, \varepsilon(\vec{r}), \varepsilon(i, j)$	dielectric function
$\varepsilon_0$	dielectric constant of vacuum, $\varepsilon_0 = 8.85 \cdot 10^{-12} \text{As/Vm}$
$\lambda$	vacuum wavelength of field oscillation
$\mu_0$	magnetic permeability of vacuum, $\mu_0 = 4\pi \cdot 10^{-7} \text{Vs/Am}$
$\omega$	angular frequency
$\sigma_{kl}$	electric conductivity tensor
$\sigma_{\perp}$	component of $\sigma_{kl}$ perpendicular to PML
$\sigma_0$	conductivity at the surface of a PML with geometric profile
$\tau_{kl}$	magnetic conductivity tensor
$\tau_{\perp}$	component of $\tau_{kl}$ perpendicular to PML
$\Delta\tau$	scalar parameter $\Delta\tau = c\Delta t$
$\Delta s$	space grid step
$\Delta t$	time step
$\Delta x$	scalar indicating displacement
$\Delta z$	scalar indicating displacement
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 \text{m/s}$
$d$	scalar, penetration depth into the PML
$g$	scalar, factor describing the geometric profile of the PML
$\vec{j}_e$	electric current
$\vec{j}_m$	magnetic current, artificial physical quantity
$\vec{k}$	wave vector of a plane wave
$\hat{\vec{k}}$	unit vector of $\vec{k}$
$k_0$	vacuum wave vector magnitude of $\vec{k}$
$n$	time-step index
$\vec{r}$	position vector in the <i>Euclidean</i> space

$t$	time
$u, u(i, j)$	scalar function representing a field component
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes
$\vec{E}, \vec{E}(\vec{r}, t),$	electric field vector
$\vec{E}(i, j, n)$	
$E_x, E_y, E_z$	components of the electric field vector
$\vec{H}, \vec{H}(\vec{r}, t)$	magnetic field vector
$\vec{H}(i, j, n)$	
$H_x, H_y, H_z$	components of the magnetic field vector
$P(i, j)$	polynomial
$Z_0$	impedance of vacuum, $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.73V/A$

### Section 2.3.2

$\times$	cross product
$\partial$	partial derivative symbol
$(i, j)$	point of the discrete space, grid point
$(i, j, n)$	point of the discrete space/time
$\theta$	angle of scattering
$\Delta s$	space grid step
$\Delta t$	time step
$\Gamma$	line
$d\gamma$	line integral
$n$	time-step index
$n'$	retarded time-step index
$\hat{n}$	unit inward normal to the surface of integration
$p$	a point in space $p = (i_0, j_{p+\frac{1}{2}})$
$\vec{r}$	position vector in the <i>Euclidean</i> space
$\vec{r}_0$	arbitrary point in space
$r_0$	magnitude of $\vec{r}_0$
$\hat{r}_0$	unit vector of $\vec{r}_0$
$t$	time
$t'$	retarded time
$C$	scalar constant
$\vec{E}, \vec{E}(\vec{r}),$	electric field vector

$\vec{E}(\vec{r}, t), \vec{E}(n, \theta)$	
$\vec{E}(\omega, \theta)$	
$\vec{E}_1(\omega, \theta)$	contribution to electric field from first sampling window
$\vec{E}_2(\omega, \theta)$	contribution to electric field from second sampling window
$E_x, E_z$	components of the electric field vector
$\vec{H}, \vec{H}(\vec{r}, t),$	magnetic field vector
$H_y$	components of the magnetic field vector
$Z_0$	impedance of vacuum, $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.73V/A$

## Appendix A

### Section A.1

$\delta_{ij}$	<i>Kronecker</i> symbol
$\epsilon(\vec{r})$	dielectric function
$\mu(\Omega)$	volume of primitive cell
$\eta_{\vec{G}}$	<i>Fourier</i> coefficient of the inverse dielectric function
$\omega$	angular frequency
$\Omega$	primitive cell
$\vec{a}_1, \vec{a}_2, \vec{a}_3$	basis vectors of primitive lattice cell
$\vec{b}_1, \vec{b}_2, \vec{b}_3$	basis vectors of <i>Brillouin</i> zone
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$i$	imaginary unit $i = \sqrt{-1}$
$j$	integer index
$\vec{k}$	wave vector in the periodic structure
$l_1, l_2, l_3$	integers
$\vec{r}$	position vector in the <i>Euclidean</i> space
$\vec{u}_{\vec{k}}(\vec{r}, \omega)$	periodic vector field function with three-dimensional domain, <i>Bloch</i> state
$\vec{u}_{\vec{k}, \vec{G}'}$	<i>Fourier</i> coefficient of a <i>Bloch</i> state
$\vec{G}, \vec{G}', \vec{G}''$	reciprocal lattice vectors
$\vec{H}_{\vec{k}}(\vec{r}, \omega)$	magnetic field vector plane wave with wave vector $\vec{k}$
$\vec{R}$	lattice vector in <i>Euclidean</i> space

### Section A.2

$\epsilon(z)$	dielectric function
$\epsilon_1, \epsilon_2$	dielectric constants

$\eta_G, \eta_l$	<i>Fourier</i> coefficient of the inverse dielectric function
$\omega$	angular frequency
$a$	period of a one-dimensional lattice
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$f$	filling fraction
$g(z)$	scalar top-hat function
$i$	imaginary unit $i = \sqrt{-1}$
$\vec{k}$	wave vector in the periodic structure
$k_x, k_z$	components of wave vector $\vec{k}$
$l, l', l''$	integers
$\vec{u}_{\vec{k},l}$	<i>Fourier</i> coefficient of a <i>Bloch</i> state
$u_{\vec{k},l,x}, u_{\vec{k},l,y},$	components of $\vec{u}_{\vec{k},l}$
$u_{\vec{k},l,z}$	
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes
$z$	coordinate of the <i>Euclidean</i> space
$z_0$	arbitrary point on $\hat{z}$ axis
$G$	multiple of $\frac{2\pi}{a}$
$\vec{G}', \vec{G}''$	reciprocal lattice vectors
$M$	square matrix of the eigenvalue problem
$M_{l,l'}$	$2 \times 2$ block matrix components of $M$ in the <i>TE</i> case
$N$	integer, limit for the number of terms in the expansions
$R$	multiple of $a$

### Section A.3

$\eta_{l_x,l_z}$	<i>Fourier</i> coefficient of the inverse dielectric function
$\omega$	angular frequency
$a_x, a_z$	lattice constants
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 m/s$
$\vec{k}$	wave vector in the periodic structure
$k_x, k_z$	components of wave vector $\vec{k}$
$l_x, l'_x, l_z, l'_z$	integers
$\vec{u}_{\vec{k},l_x,l_z}$	<i>Fourier</i> coefficient of a <i>Bloch</i> state
$u_{\vec{k},l_x,l_z,x},$	components of $\vec{u}_{\vec{k},l_x,l_z}$
$u_{\vec{k},l_x,l_z,y},$	

$u_{\vec{k}, l_x, l_z, z}$	
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$\vec{G}_x, \vec{G}'_x, \vec{G}_z, \vec{G}'_z$	reciprocal lattice vectors
$M$	square matrix of the eigenvalue problem
$M_{l_x, l_z}$	$2 \times 2$ block matrix components of $M$ in the <i>TE</i> case
$N$	integer, limit for the number of terms in the expansions

### Section A.3.1

$\varepsilon(x, z)$	dielectric function
$\varepsilon_1, \varepsilon_2$	dielectric constants
$\eta_{l_x, l_z}$	<i>Fourier</i> coefficient of the inverse dielectric function
$a_x, a_z$	lattice constants
$f_x, f_z$	linear filling fractions
$g_x(x), g_z(z)$	scalar top-hat functions
$l_x, l_z$	integers
$x, z$	coordinates of the <i>Euclidean</i> space

### Section A.3.2

$\delta_{l_x, l_z}$	<i>Kronecker</i> 's symbol
$\varepsilon_2$	dielectric constant
$\eta_{l_x, l_z}, \eta'_{l_x, l_z}$	<i>Fourier</i> coefficient of the inverse dielectric function
$l_x, l_z$	integers

## Appendix C

$\bullet^*$	complex conjugate
$\delta_{b\gamma}$	<i>Kronecker</i> symbol
$\delta_j$	phase delay for single travel in the $j^{\text{th}}$ layer of the stack
$\gamma$	symbolic index, $\gamma = f, b$
$\kappa_j$	imaginary factor to $n_j$ in $N_j$
$\nu_0$	angle of incidence
$\nu_1, \nu_2$	<i>Euler</i> angles of propagation
$\nu_j$	<i>Euler</i> angle of propagation in $j^{\text{th}}$ material in the stack
$\omega$	angular frequency
$\zeta_{j \text{TE}}, \zeta_{j \text{TM}}$	scalar terms for the construction of the $j^{\text{th}}$ transfer matrix

$\Delta\phi$	phase delay between waves reflected at interfaces shifted by one period in a periodic stack
$c$	speed of light in vacuum, $c = 3 \cdot 10^8 \text{ m/s}$
$\hat{e}_0$	electric field orientation (unit vector) of incident plane wave
$j$	integer index
$k_0$	wave vector magnitude of incident plane wave
$\hat{k}_0$	propagation direction (unit wave vector) of incident plane wave
$\vec{k}_0$	wave vector of incident plane wave
$k_1, k_2$	wave vector magnitudes
$k_j$	magnitude of wave vector in $j^{\text{th}}$ material of the stack
$\hat{k}_{j\gamma}$	direction of propagation of plane wave $\vec{E}_{j\gamma}(\vec{r}, t)$
$\vec{k}_{j\gamma}$	wave vector of plane wave $\vec{E}_{j\gamma}(\vec{r}, t)$
$n_0$	real index of refraction of the incident medium
$n_1, n_2$	real indexes of refraction
$n_j$	real index of refraction of $j^{\text{th}}$ material in the stack
$p$	symbolic index, $p = TM, TE$
$\vec{r}$	position vector in the <i>Euclidean</i> space
$t_1, t_2$	layer thicknesses
$\hat{x}, \hat{y}, \hat{z}$	normalised (unit) vectors defining an orthogonal system of coordinates in the <i>Euclidean</i> space
$x, y, z$	coordinates of the <i>Euclidean</i> space, $(x, y, z) = \vec{r}$
$xz$	plane defined by the $\hat{x}$ and $\hat{z}$ axes of coordinates
$AB, AF, BC$	geometrical segments
$E_0$	electric field amplitude of incident plane wave
$E_{0bp}$	oscillation amplitude of the backward travelling plane wave component of the electric field in the incident medium for $p$ polarisation
$E_{0fp}$	oscillation amplitude of the forward travelling plane wave component of the electric field in the incident medium for $p$ polarisation
$E_{Nbp}$	oscillation amplitude of the backward travelling plane wave component of the electric field in the transmission medium for $p$ polarisation
$E_{Nfp}$	oscillation amplitude of the forward travelling plane wave component of the electric field in the transmission medium for $p$ polarisation

$\vec{E}_{j\gamma}, \vec{E}_{j\gamma}(\vec{r}, t)$	either forward or backward travelling electric field vector in the $j^{\text{th}}$ material in the stack
$E_{j\gamma x}, E_{j\gamma y}, E_{j\gamma z}$	components of $\vec{E}_{j\gamma}$
$\vec{H}_{j\gamma}$	either forward or backward travelling magnetic field vector in $j^{\text{th}}$ material of the stack
$M_{jp}$	characteristic matrix of the $j^{\text{th}}$ layer in the stack for $p$ polarisation
$N_j$	complex index of refraction of $j^{\text{th}}$ material in the stack
$P_j$	propagation matrix of the $j^{\text{th}}$ layer in the stack
$R_p$	reflected fraction of the power (reflectance) incident on a multilayer stack
$T_p$	transmitted fraction of the power (transmittance) incident on a multilayer stack
$Z_0$	impedance of vacuum, $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.73V/A$

# List of Acronyms

AR, ARC	antireflective coating
BBO	beta barium borate
BS-SEM	backscattered electrons scanning electron microscopy
ECR	electron cyclotron resonance
FD	finite difference
FDTD	finite-difference time-domain
FEM	finite element method
FWHM	full width at half maximum
GT	<i>Glan-Taylor</i> polariser
IPA	isopropyl alcohol
IR	infrared
KKR	<i>Korringa-Kohn-Rostocker</i>
KTP	potassium titanyl phosphate
MST	multiple-scattering theory
NFFT	near-field to far-field transformation
PC, PCs	photonic crystal, photonic crystals
PCF, PCFs	photonic crystal fibre, photonic crystal fibres
PECVD	plasma-enhanced chemical vapour deposition
PML	perfectly matched layer
PMMA	Polymethyl methacrylate
PWM	plane wave method
RCWT	rigorous coupled wave technique
RF	radio frequency
RIE	reactive ion etching
RMM	rigorous modal method
SC	supercontinuum
SEM	scanning electron microscopy

ST	scalar thory
TE	transverse electric
TEM	transmission electron microscopy
TM	transverse magnetic
TMM	transfer matrix method
TTV	total thickness variation