Computational Electromagnetics

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Electronics and Computer Science


OWD 2003

Maxwell's equations:

$$\textit{curl}\ \mathbf{H} = \mathbf{J} + \partial\mathbf{D}/\partial t$$

$$\textit{curl}\ \mathbf{E} = -\ \partial\mathbf{B}/\partial t$$

$$\textit{div}\ \mathbf{D} = \rho$$

$$\textit{div}\ \mathbf{B} = 0$$

Aims:

- *analysis of performance*
- *design/optimisation*
- *virtual prototyping*

Electromagnetics is a challenge

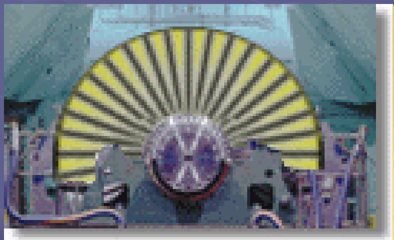
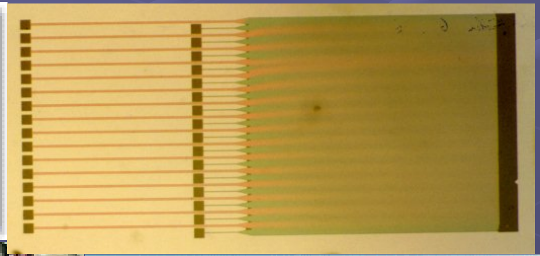
- *complex mathematics*
- *deals with invisible quantities (but measurable)*
- *fields exist everywhere (including vacuum)*
- *perceived as a difficult subject by students*

Electromagnetics

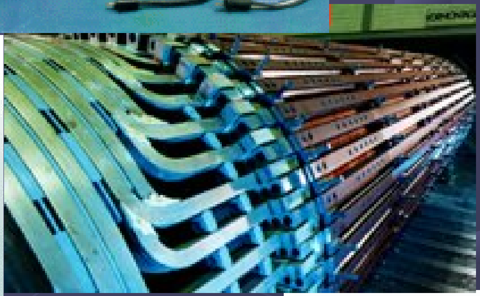
- **Dimensions: from nm to km**
- **Voltages: from μV to MV**
- **Power: from μW to GW**
- **Frequency: from DC to daylight**







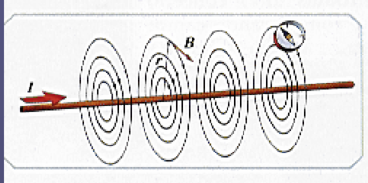

A little history

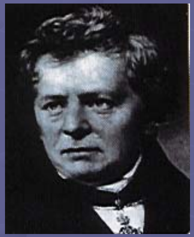
- First compasses were used for navigation in 6th Century BC
- By 800 – 1000 AD, Vikings using the compass for long navigation
- 1269 – Peter Peregrinus introduced the concept of “poles”
- 1600 – William Gilbert published ‘De Magnete’ which explained magnetism in terms of ‘magnetic power’ surrounding magnets
- In 1650, Otto van Guericke created an electric generator
- In 1785, electrostatic forces measured by Coulomb
- In 1799, Volta produced the first battery

A little history

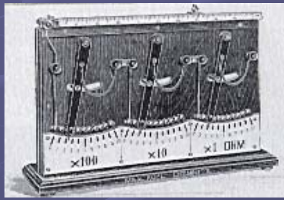
Andre Marie Ampere (1775-1836)

Jean Baptiste Biot (1774-1862)

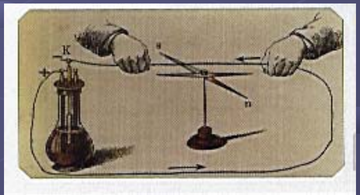




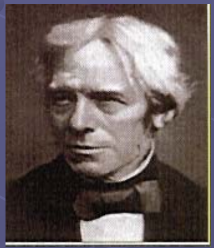
Georg Simon Ohm (1787-1854)



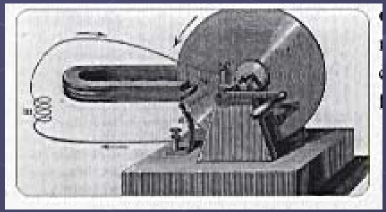
Hans Christian Oersted (1777-1851)



James Clerk Maxwell (1831-1879)



Michael Faraday (1791-1867)



Heinrich Rudolf Hertz (1857-1894)

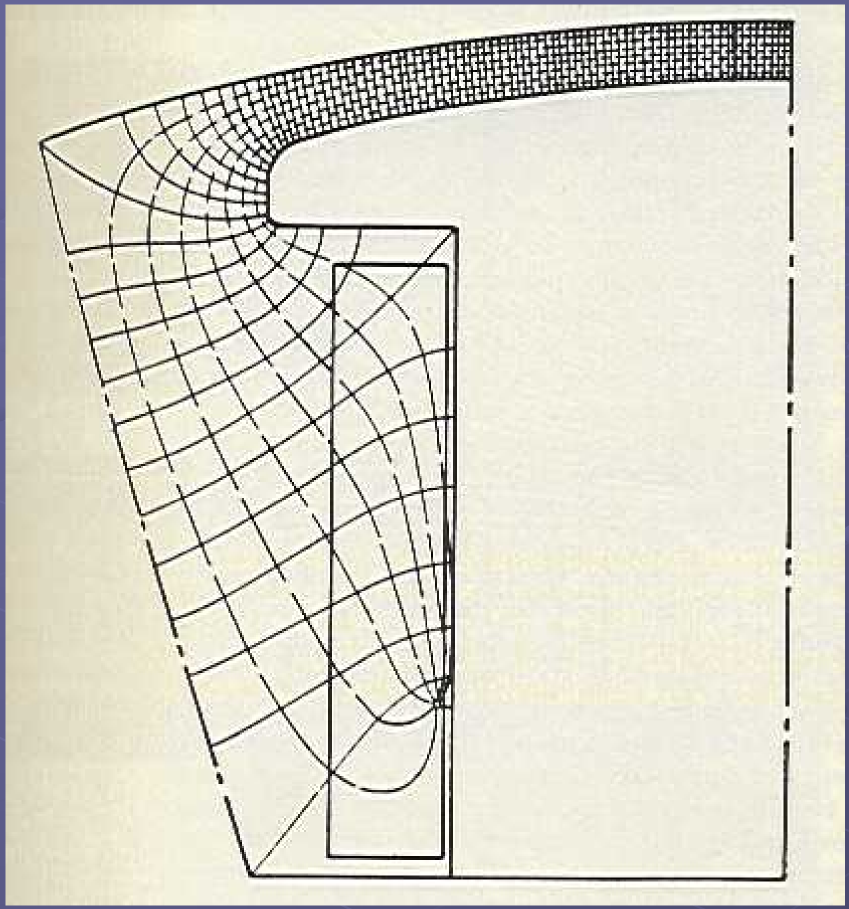
A little history

By 1864, the basic theory of electromagnetism and the underlying mathematical models were established.

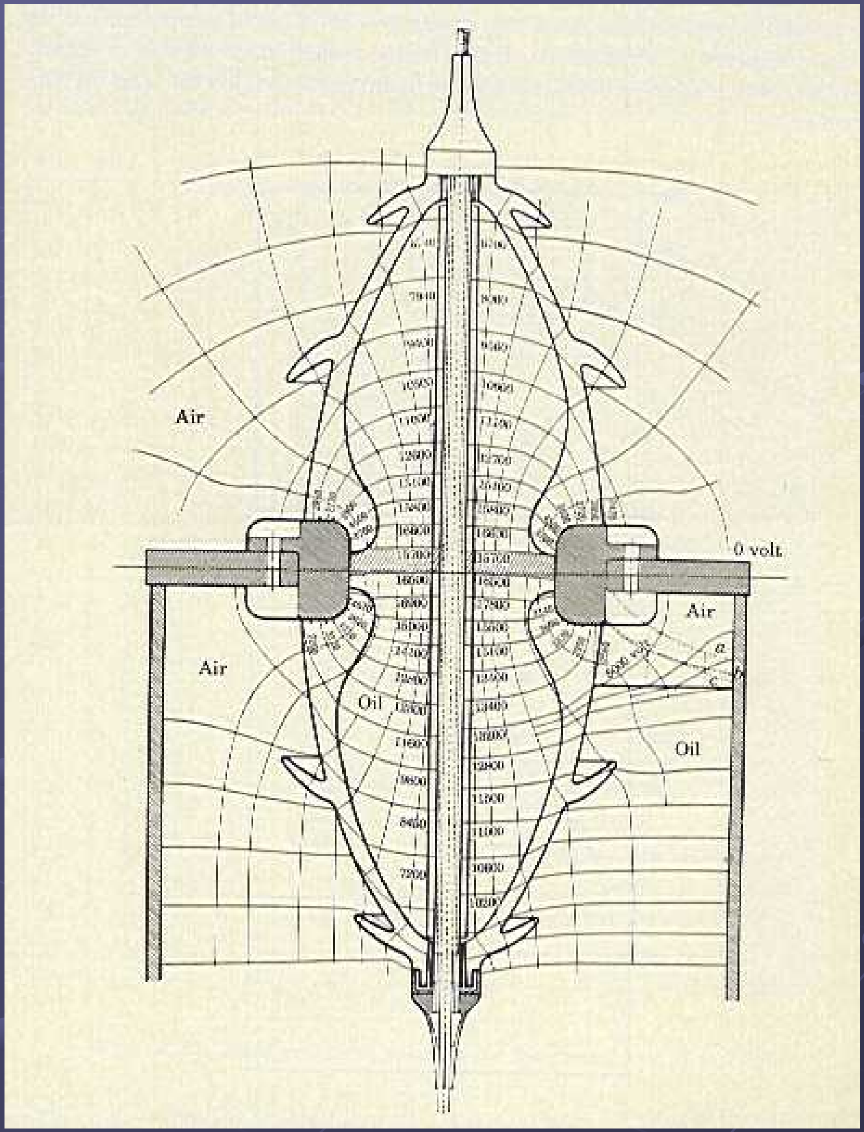
For the last 140 years the effort has been mainly directed to implementing the mathematical models and creating tools for design of new or better devices.

A little history

Graphical solutions

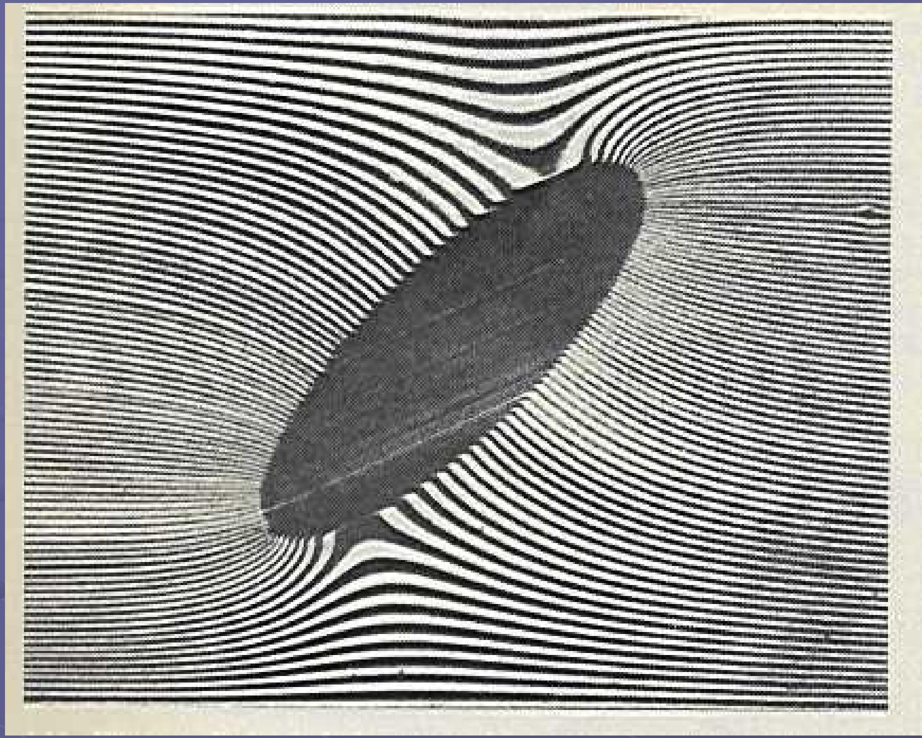


Magnetic field in an alternator (1927)

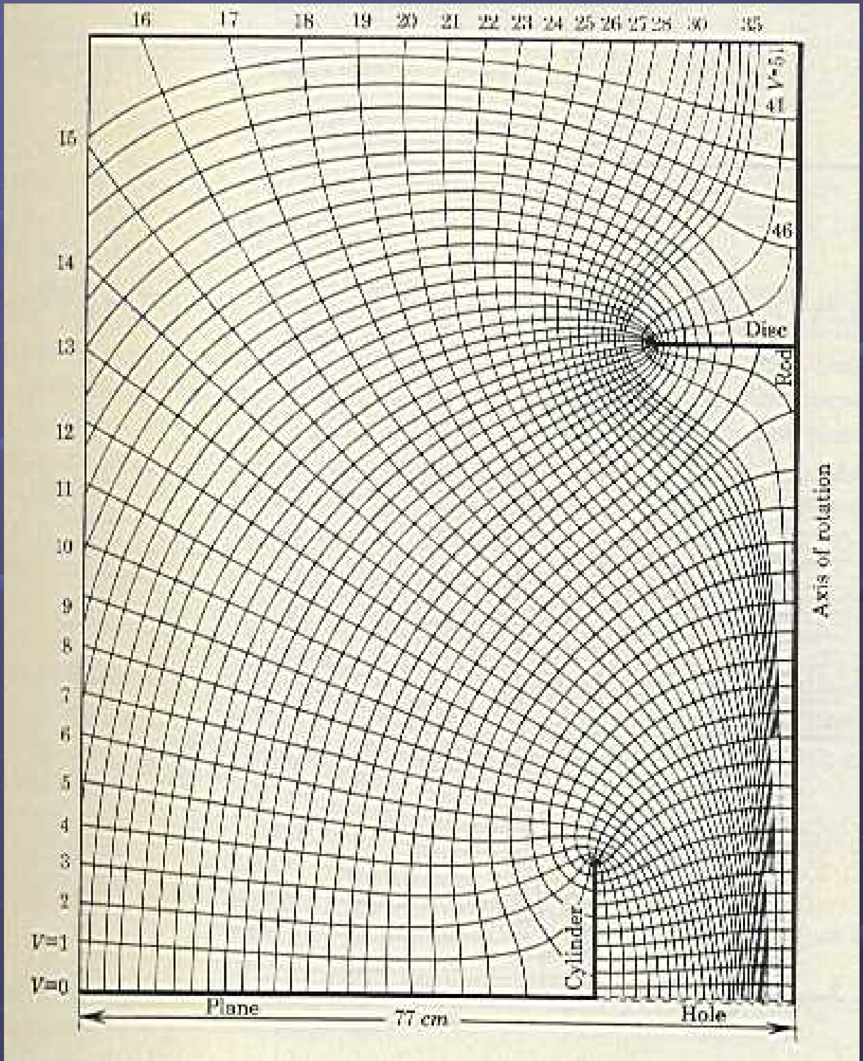


Transformer bushing (1914)

A little history Physical analogies



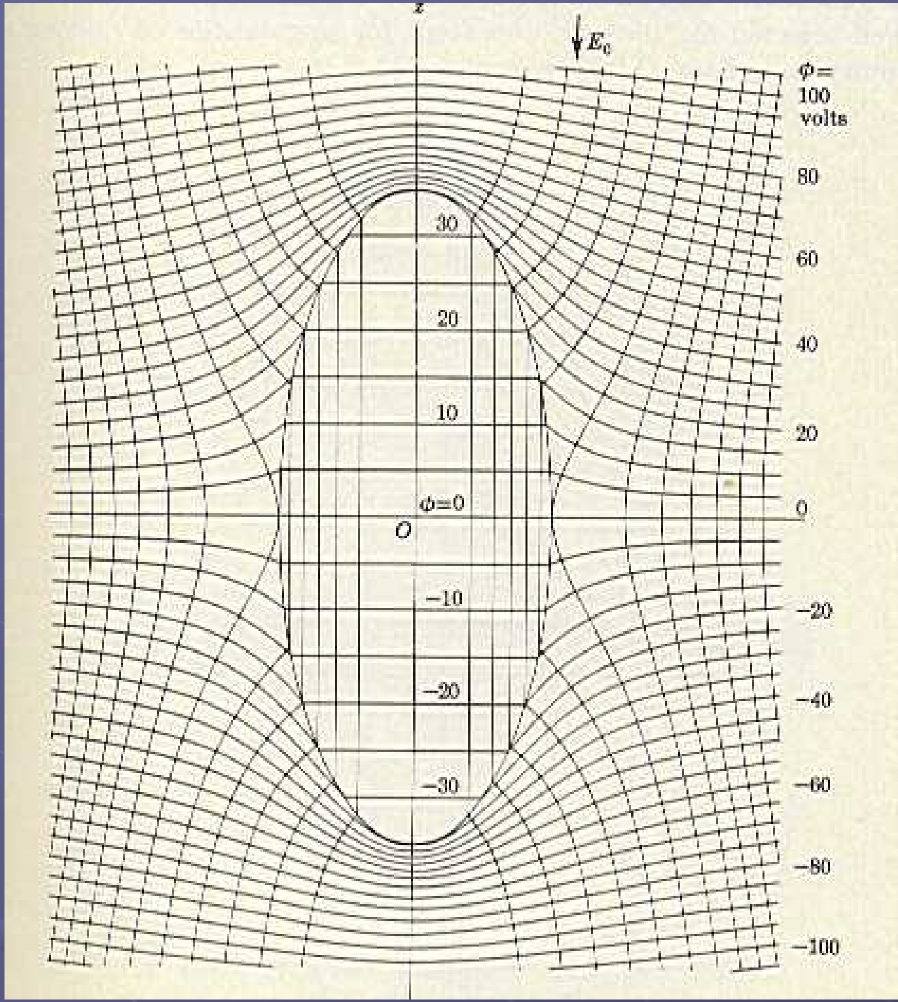
Elliptic cylinder immersed in a uniform magnetic field - map obtained experimentally by the liquid flow method (1901)



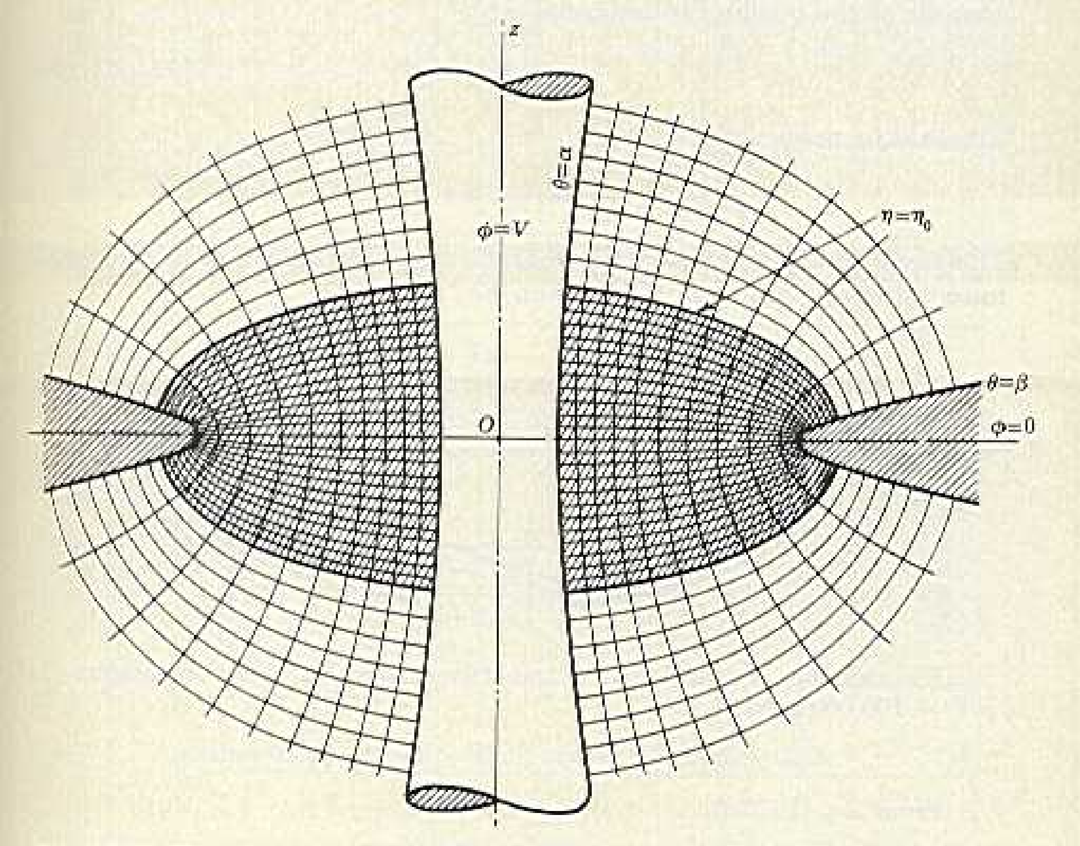
Electrostatic field about a high-tension lead through a transformer cover modelled using an electrolytic tank (1917)

A little history

Analytical solutions



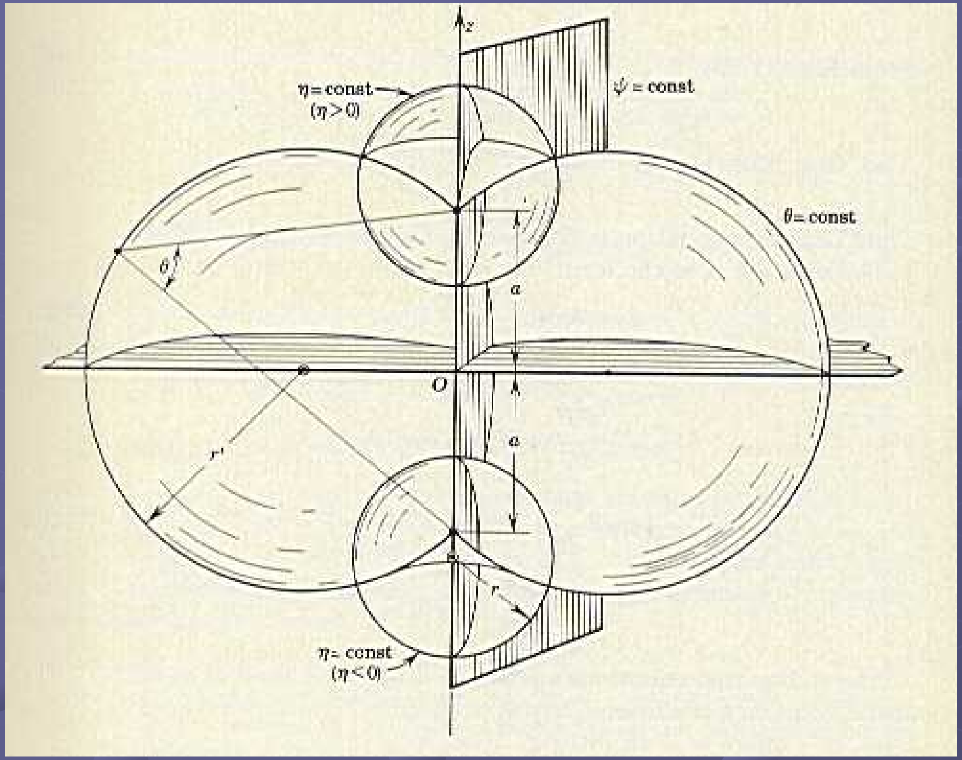
Electric field for a dielectric spheroid



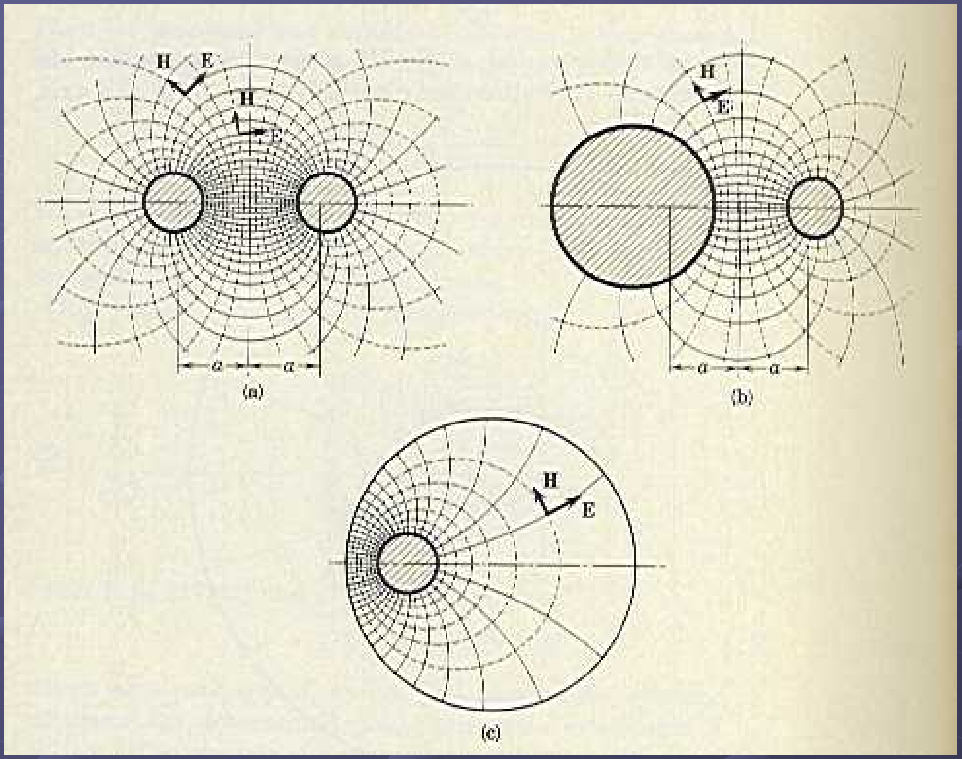
Transformer bushing of insulating material in the form of an oblate spheroid

A little history

Analytical solutions



Bispherical coordinates



TEM wave between parallel conductors

(Bicylindrical coordinates are used for all three cases)

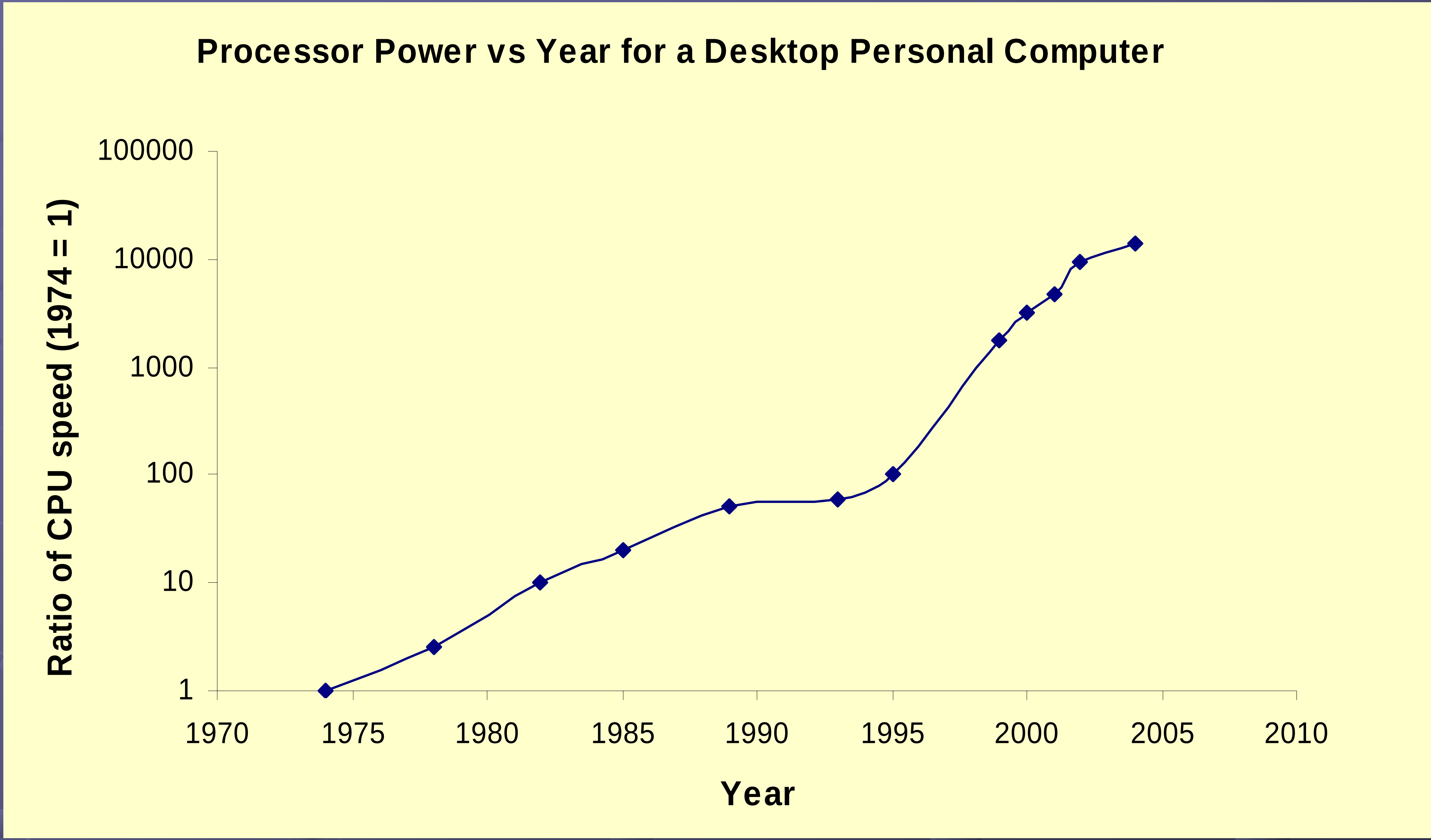
Classification of methods of field modelling:

- analogue methods
- analytical solutions
- numerical methods (algebraical computation)
- graphical computation

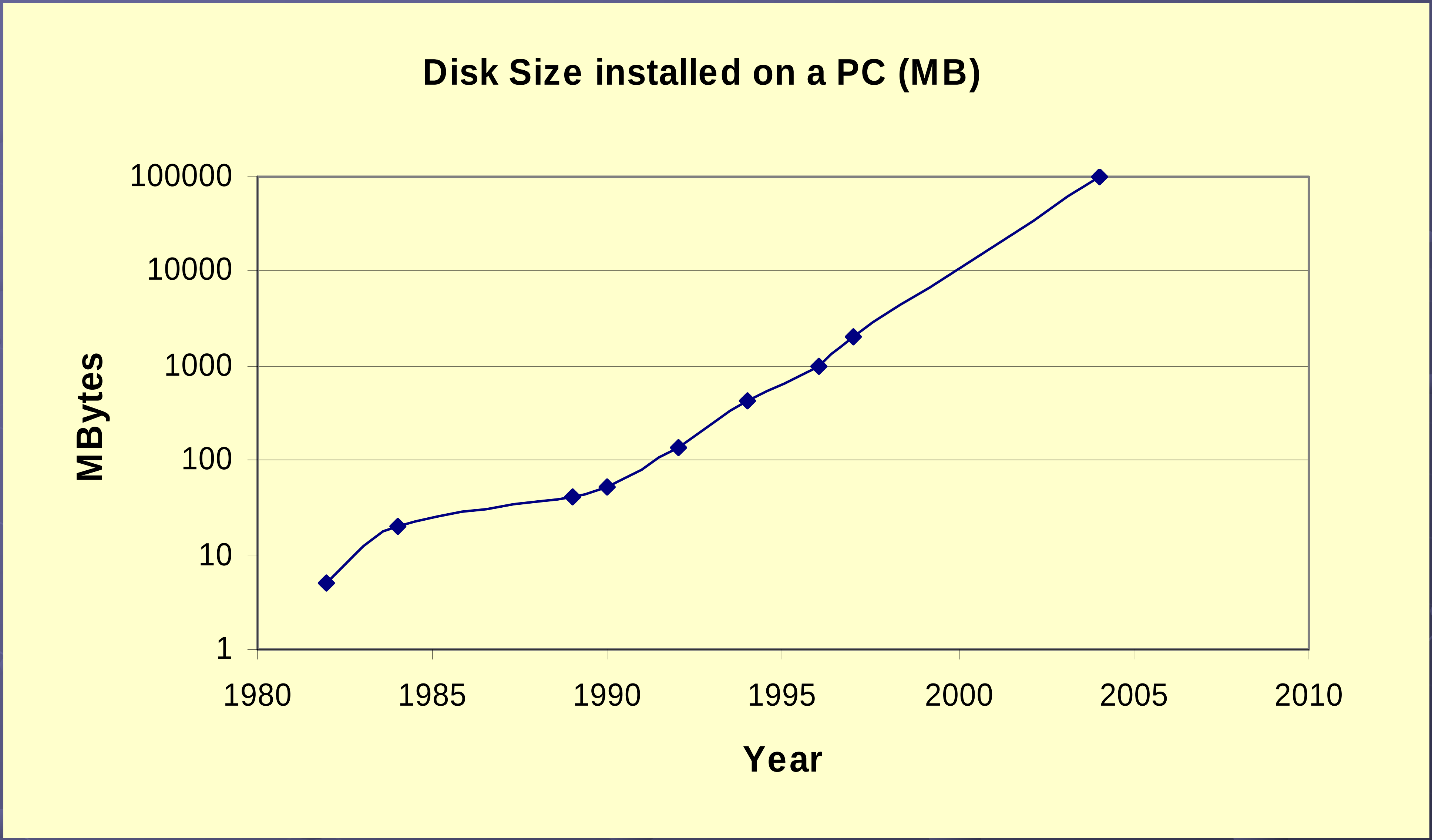
The most popular methods:

- separation of variables
- images
- analogue techniques
- conformal transformations
- Laplace transforms
- transmission-line modelling
- finite differences
- finite elements
- boundary elements
- integral formulations
- tubes and slices
- ...

Computational power



Computational power

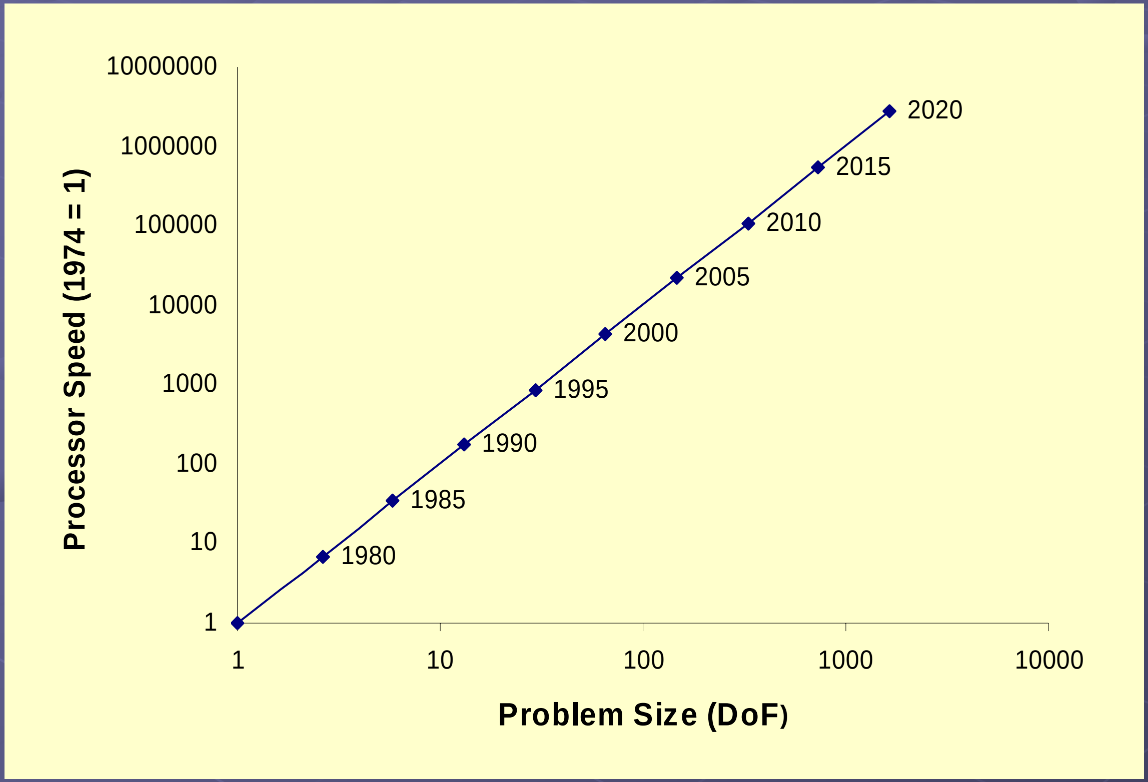


Computational power

- Processor speed
  - Growing at about 40% per year
- Installed memory sizes
  - Growing at about 40% per year
- Installed disk size
  - Growing at about 60% per year
- Data transfer rates on LANs
  - 10 Mbits/sec in 1990 to 1000 Mbits/sec in 2004

Computational power

How does the growth in computational power allow better representations of reality?



If I can solve a 1000 degree of freedom 2-D problem in t seconds in 1984, when will I be able to solve a 33,000 degree of freedom problem (the equivalent 3-D problem) in t seconds?

Answer: around 2005..

Thus the maximal size problem that can be solved on today's machine takes longer to solve than the maximal size problem on yesterday's … ***(*)***

Computational power not catching up !!!

Computational power

Available disk space is increasing faster than maximal problem size…

- Should we therefore rethink algorithms to use storage more than computation?

How can the processor capabilities be used more effectively?

- Solving M problems each of size D is much faster than solving one of size MD..
- Concentrate on those parts of the problem that need good representations?
- Use the virtual laboratory to develop heuristics?

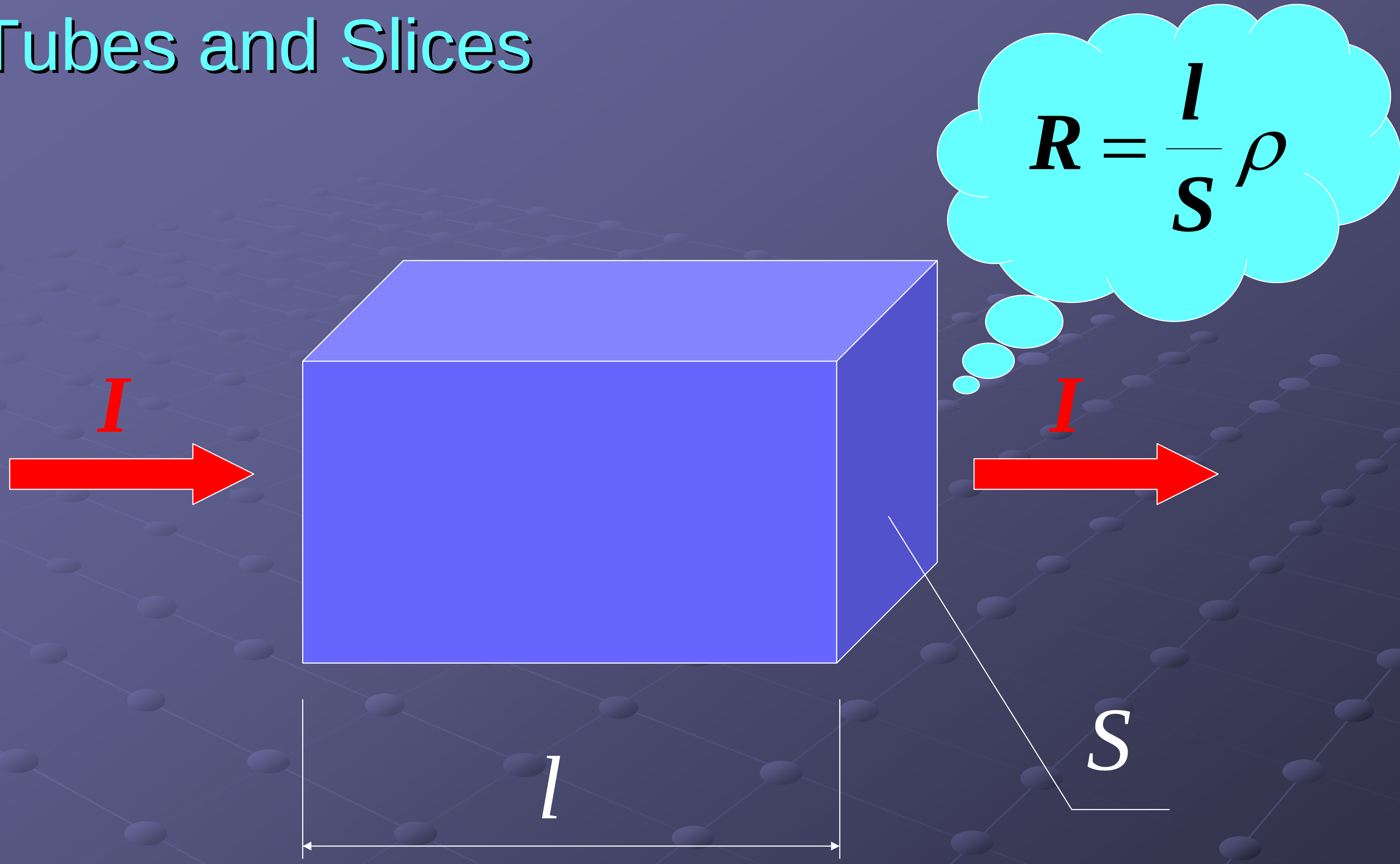
Digression:

Can we benefit by using approximate formulations but on-line (that is almost immediate) solutions?

Possible answer:

The method of Tubes and Slices.

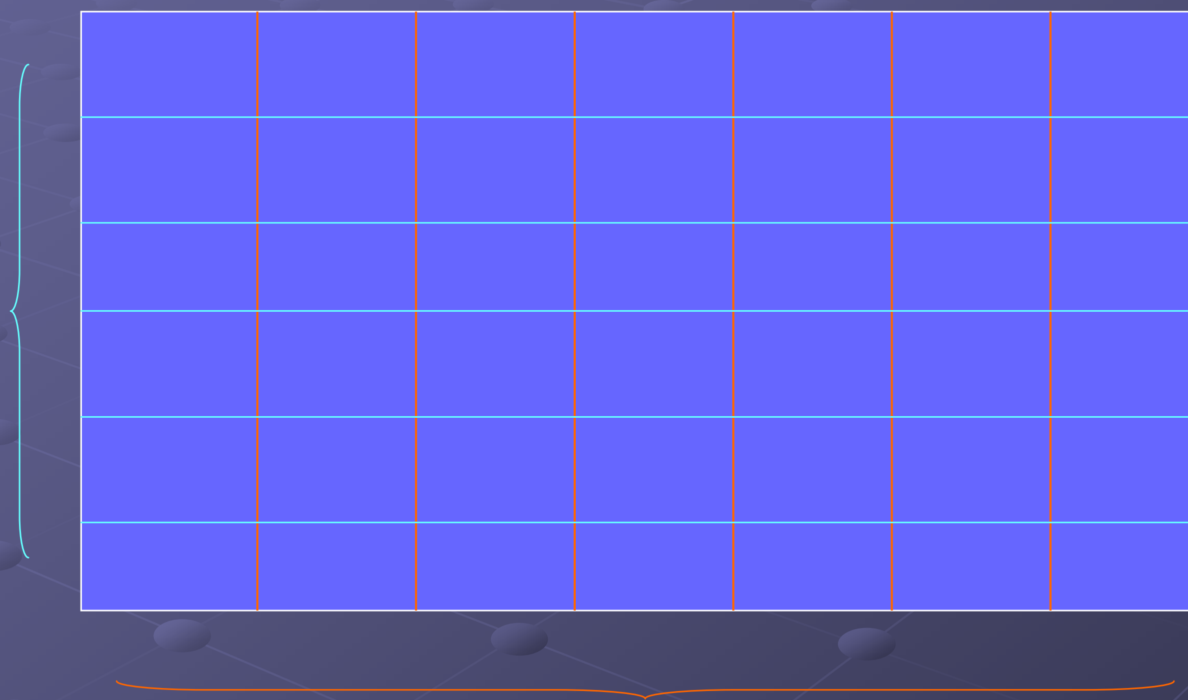
Tubes and Slices



Tubes and Slices

tubes

(thin insulating sheets)



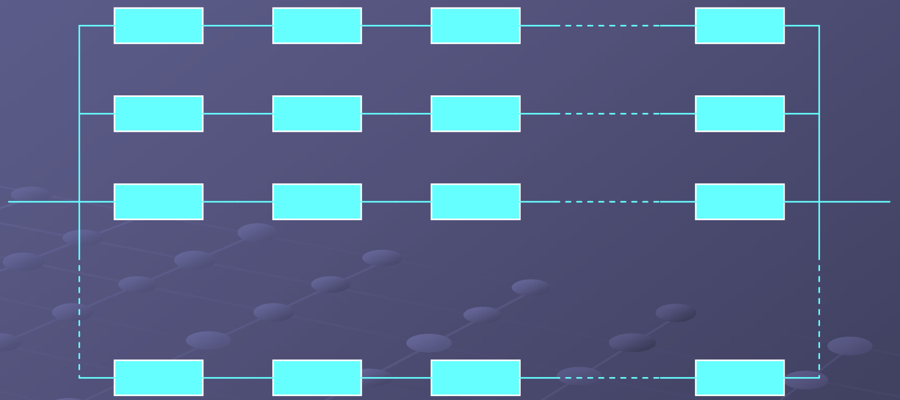
slices

(thin ‘superconducting’ sheets)

Tubes and Slices

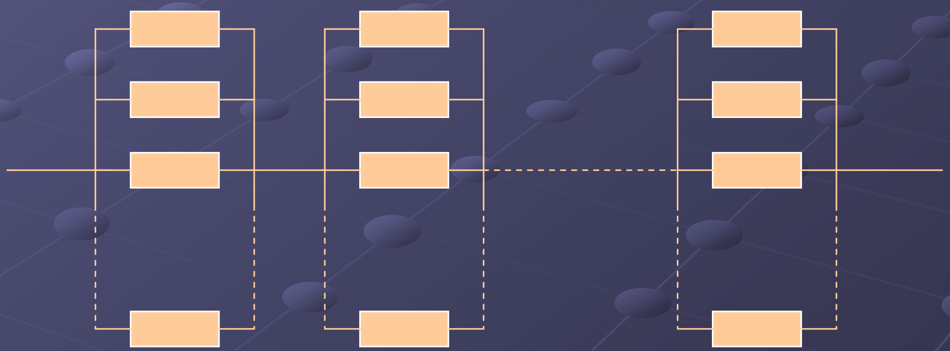
Tubes:

$$\boldsymbol{Power} = \boldsymbol{I}^2 \boldsymbol{R}^+ \quad \textit{where} \quad \boldsymbol{R}^+ = \frac{1}{\displaystyle\sum_{i=1}^{t} \frac{1}{\displaystyle\sum_{j=1}^{s} \boldsymbol{r}_{ij}}}$$

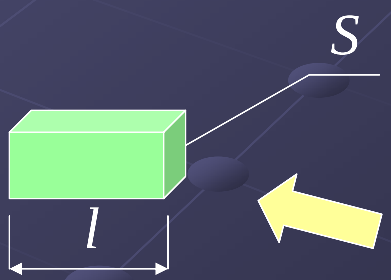


Slices:

$$\boldsymbol{Power} = \frac{\boldsymbol{V}^2}{\boldsymbol{R}^-} \quad \textit{where} \quad \boldsymbol{R}^- = \sum_{k=1}^{s} \frac{1}{\displaystyle\sum_{l=1}^{t} \frac{1}{\boldsymbol{r}_{ls}}}$$



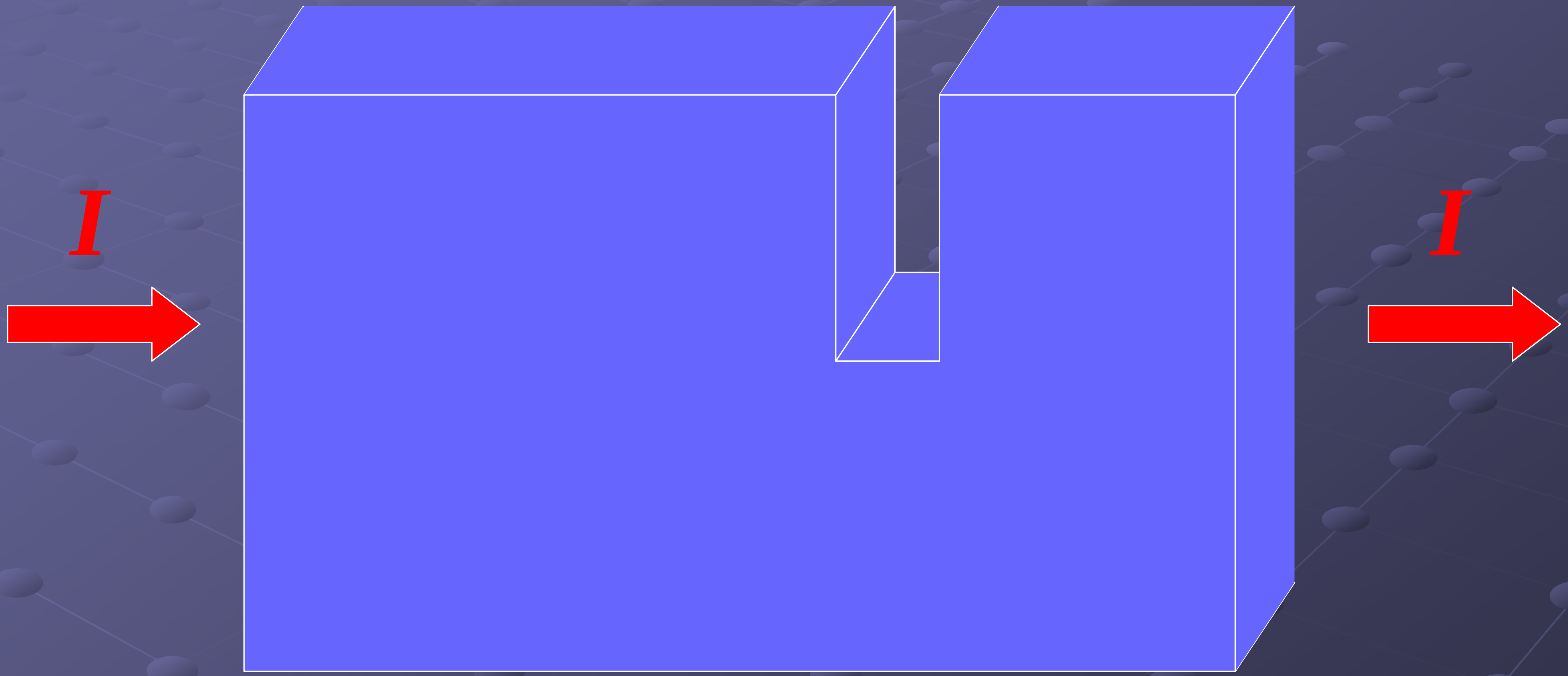
where for each r:



$$\boldsymbol{r} = \frac{\boldsymbol{l}}{\boldsymbol{S}} \rho$$

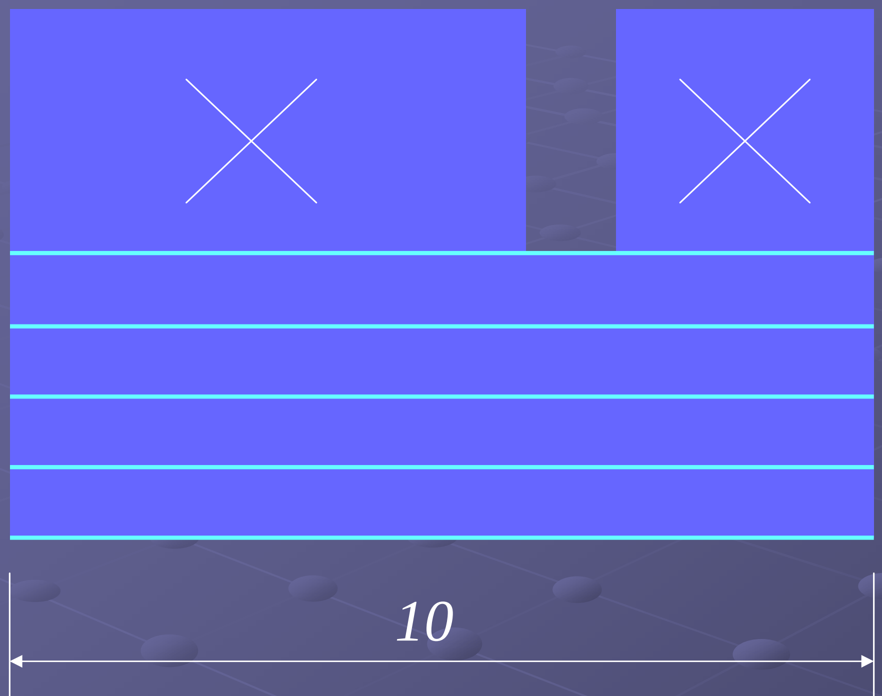
Tubes and Slices

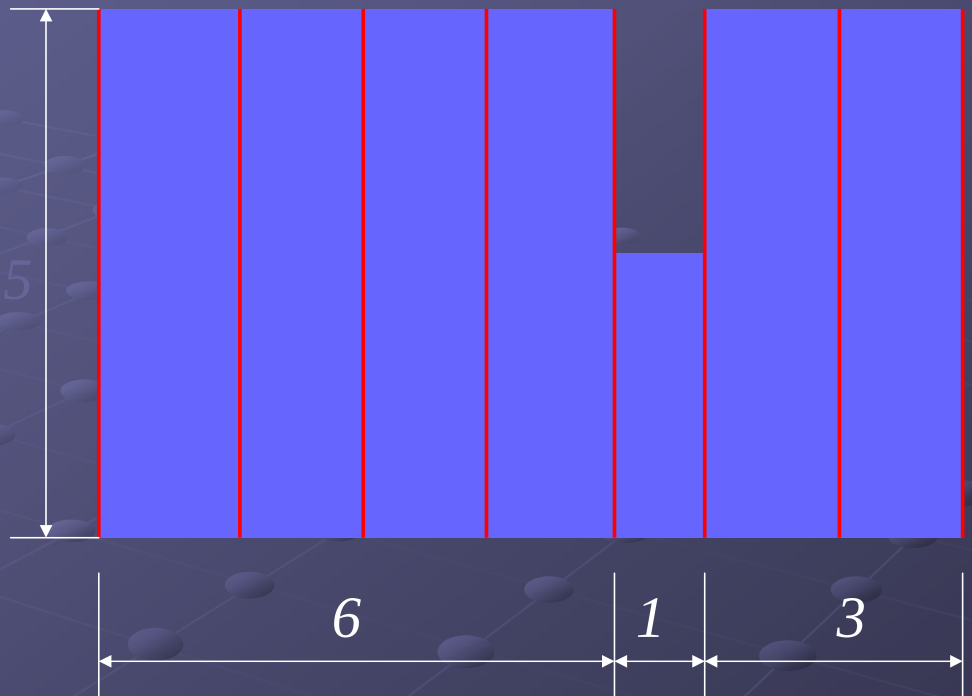
$$R_{ave} = \frac{R^{+} + R^{-}}{2}$$



Tubes and Slices

Simple hand calculation





Tubes:

$R^{+} = 10/3 = 3.33$

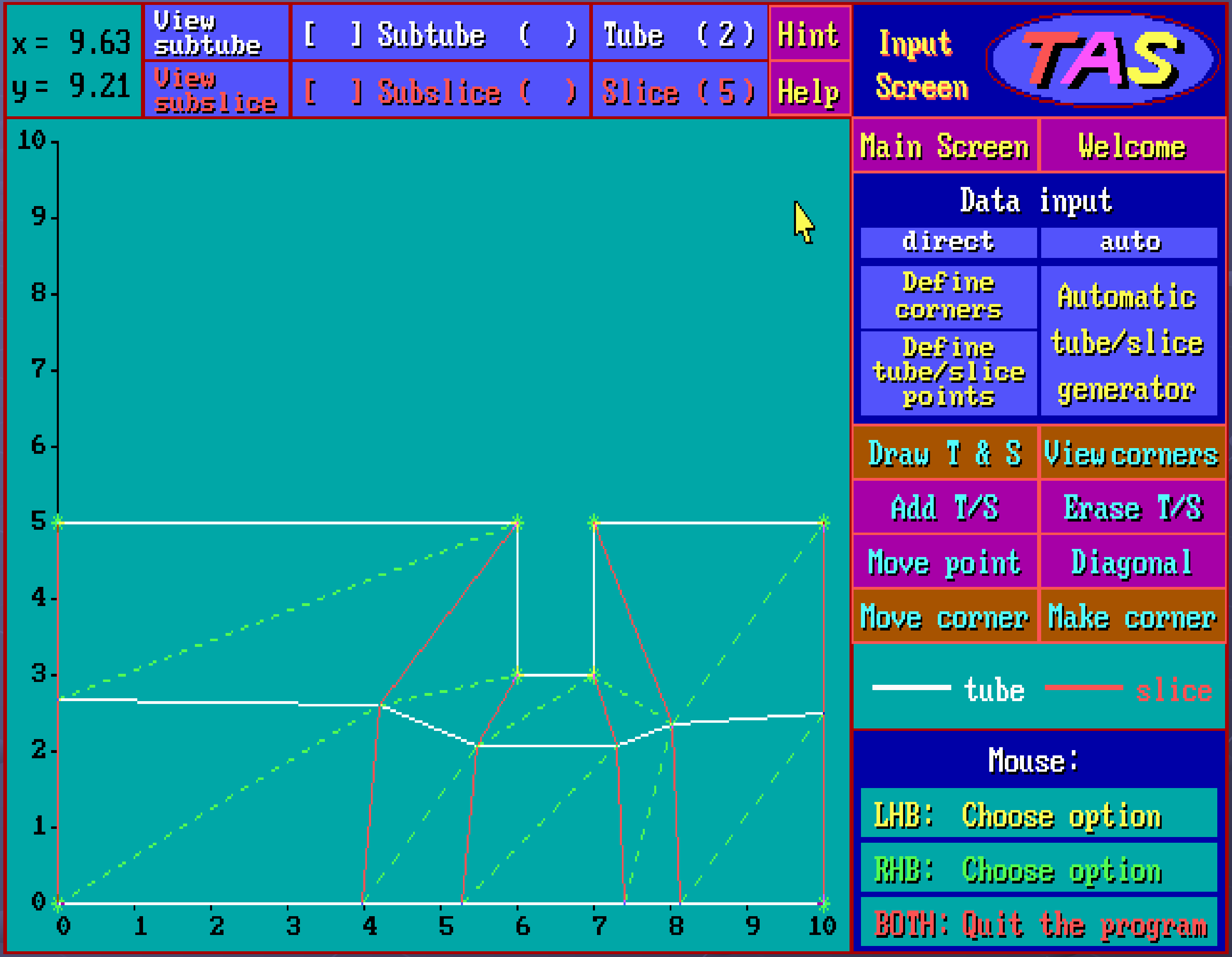
Slices:

$R^{-} = 6/5 + 1/3 + 3/5 = 2.13$

$R_{average} = 2.73 \times \rho$ *(per unit depth)*

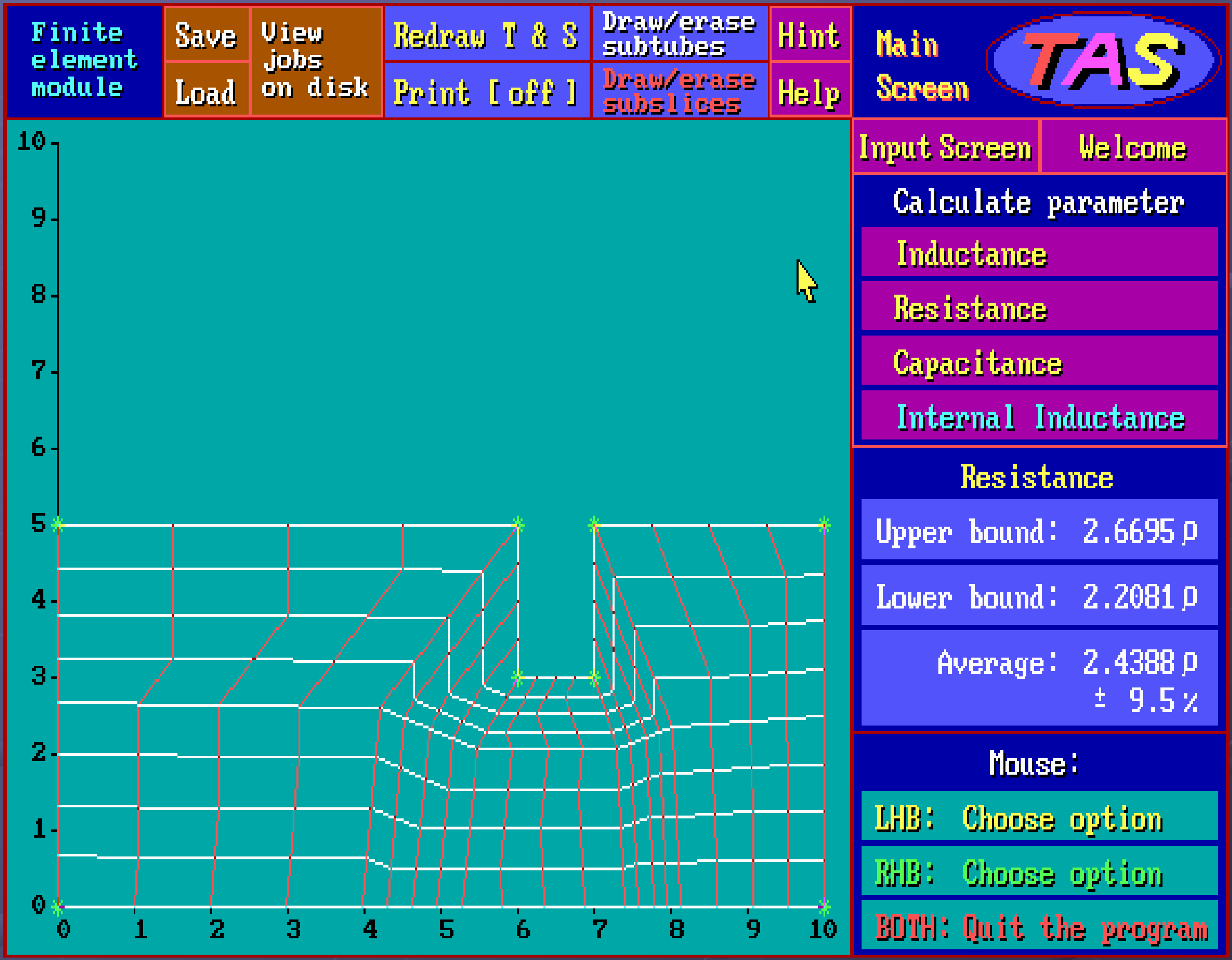
Tubes and Slices

Construction lines



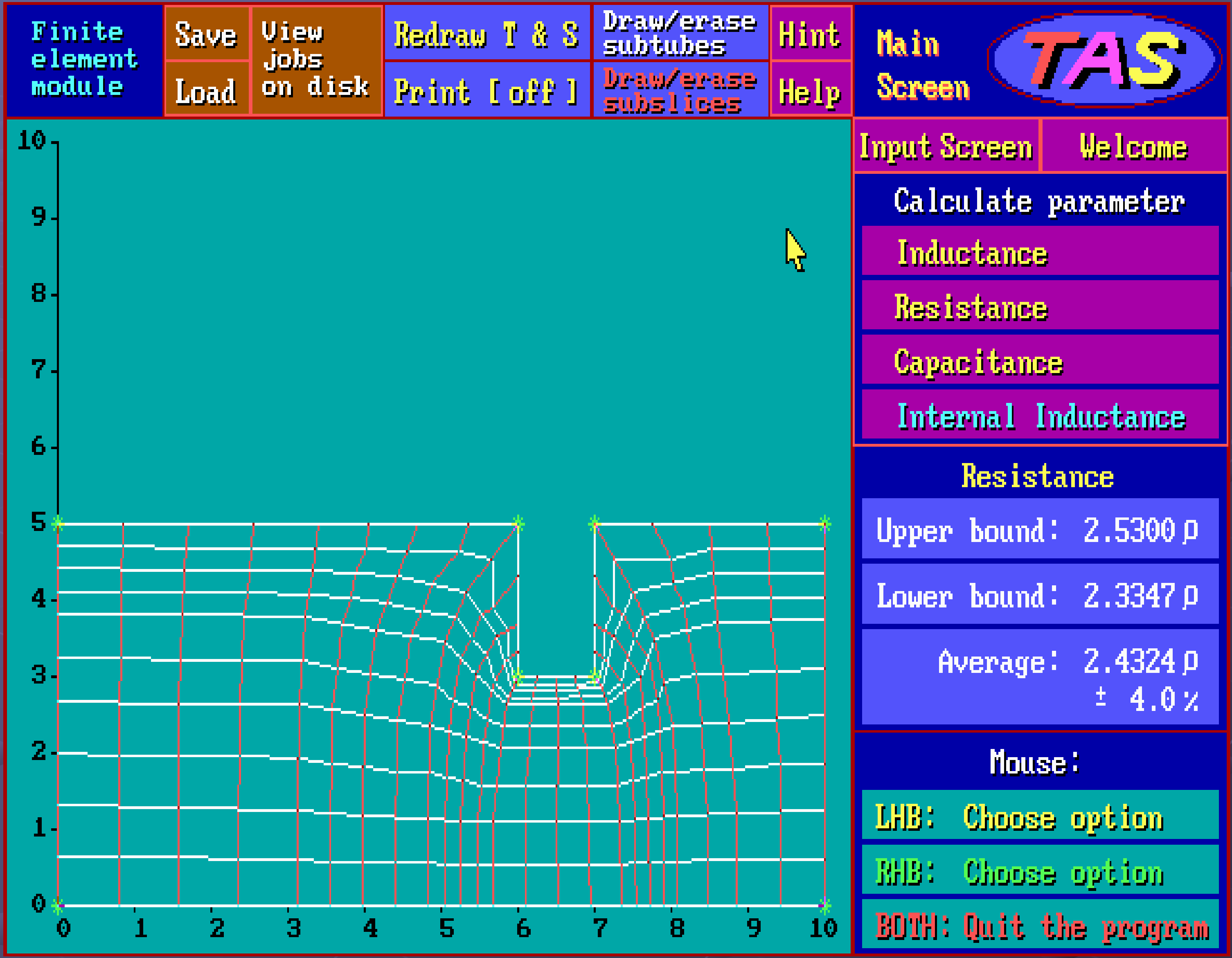
Tubes and Slices

Coarse tube/slice lines



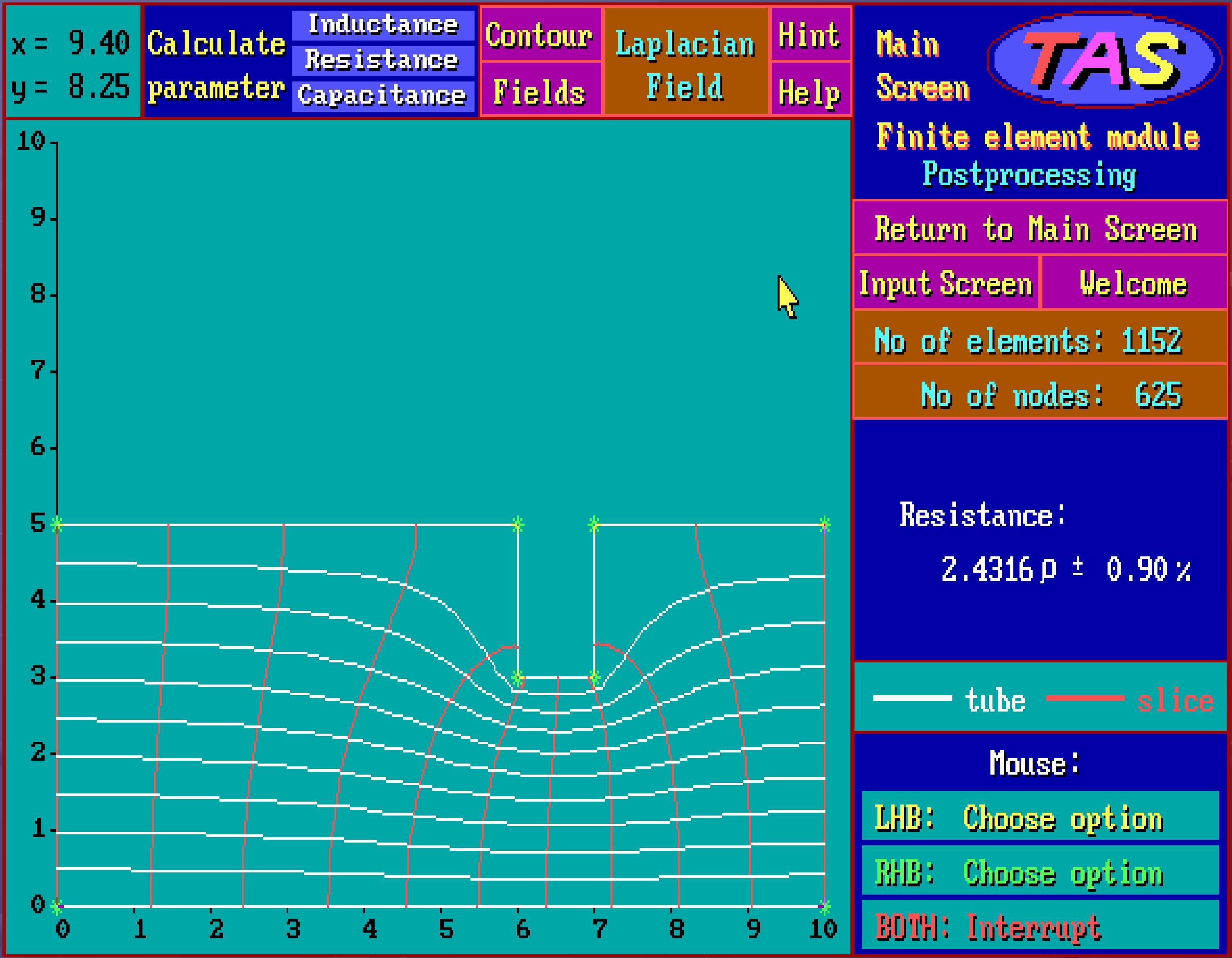
Tubes and Slices

Improved tube/slice lines



Tubes and Slices

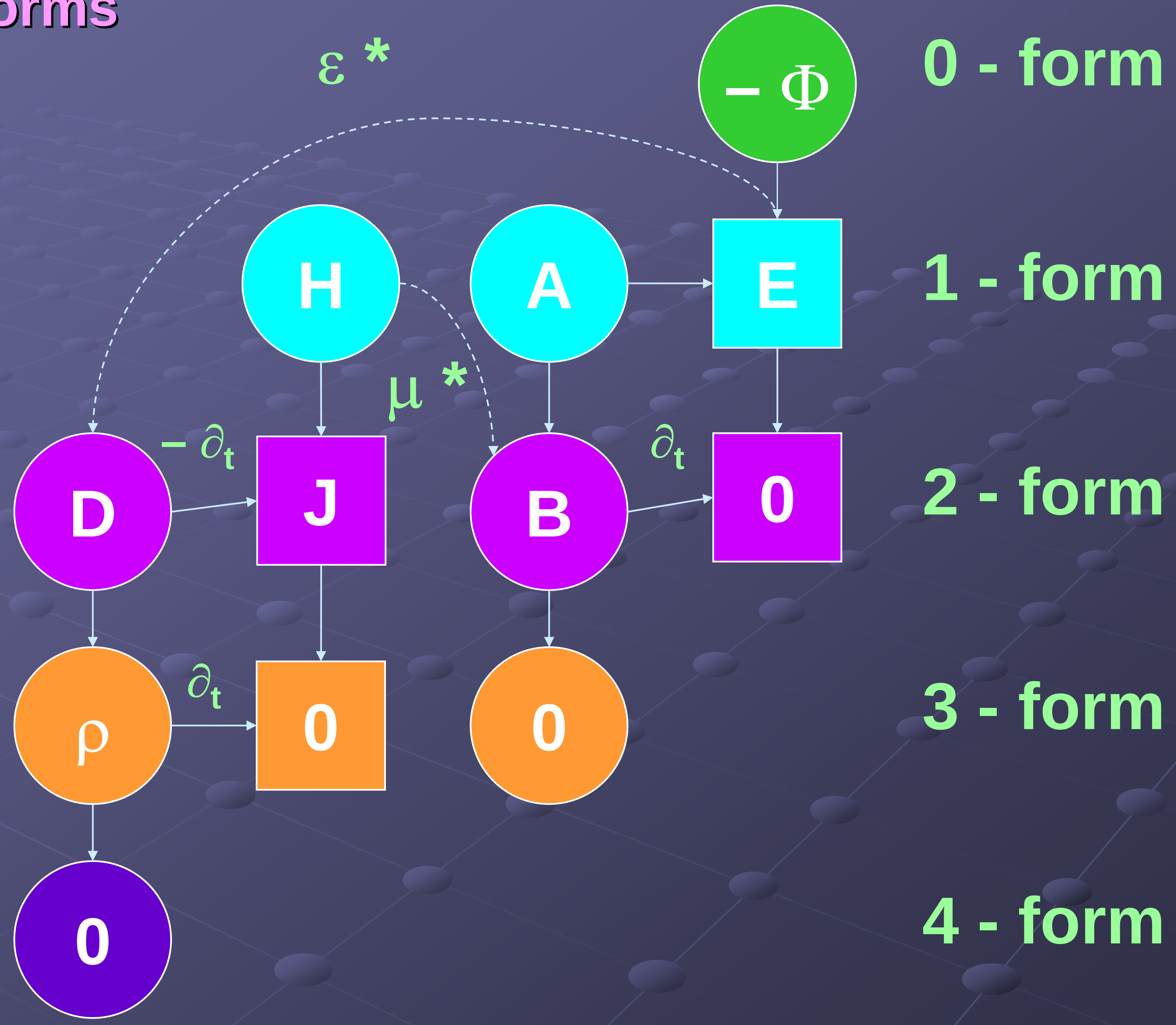
Finite element solution



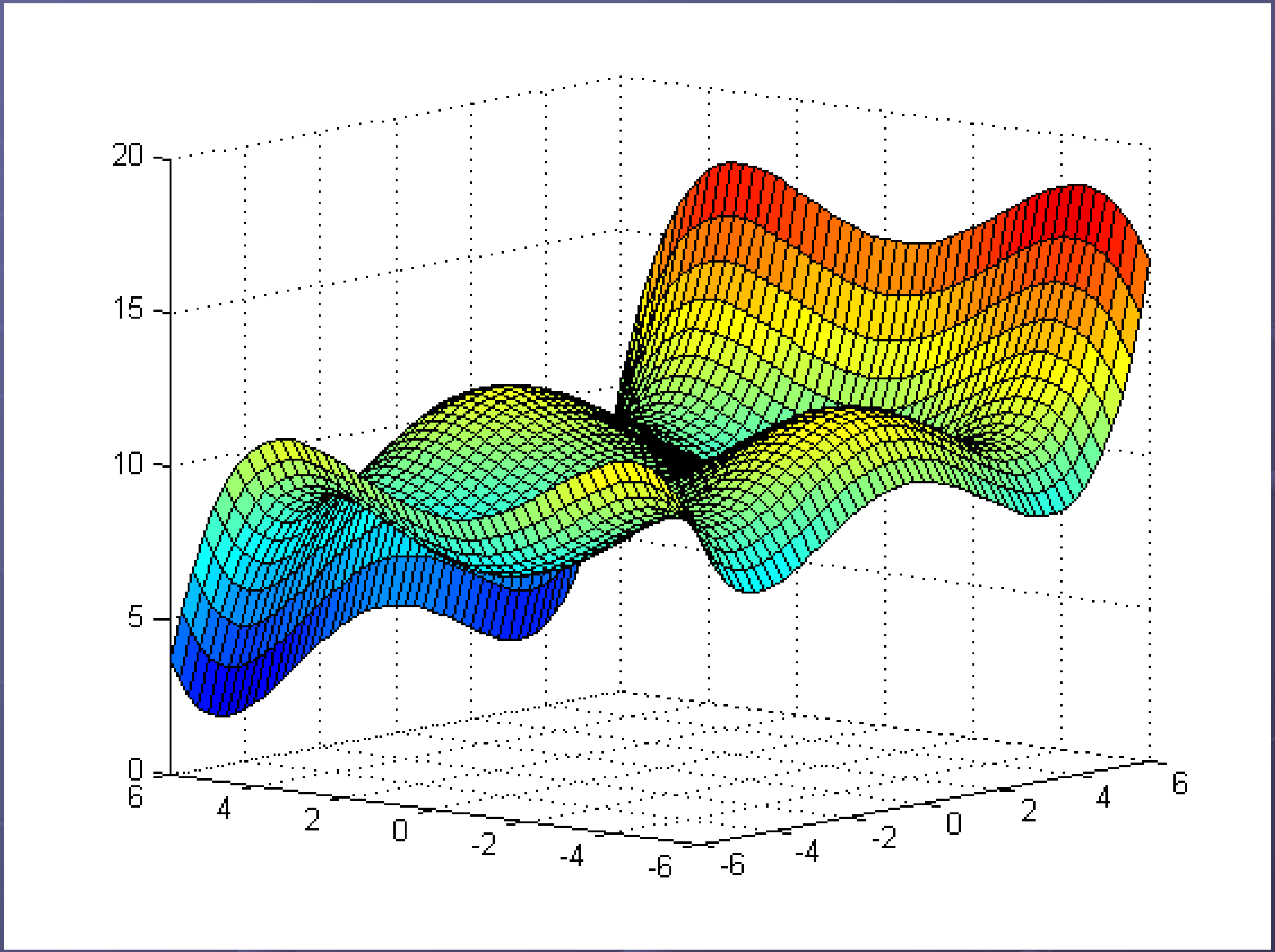
Tubes and Slices

	Resistance (per unit depth) [value × material resistivity]	**Guaranteed accuracy**	**Actual error**
Hand calculation	2.7333	22.0 %	12.4 %
TAS coarse	2.4388	9.5 %	0.3 %
TAS refined	2.4324	4.0 %	0.3 %
Finite elements	2.4316	0.9 %	-

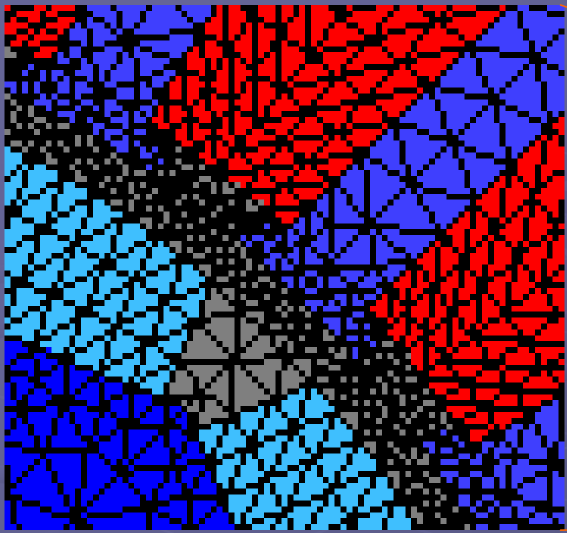
Maxwell's equations in differential forms

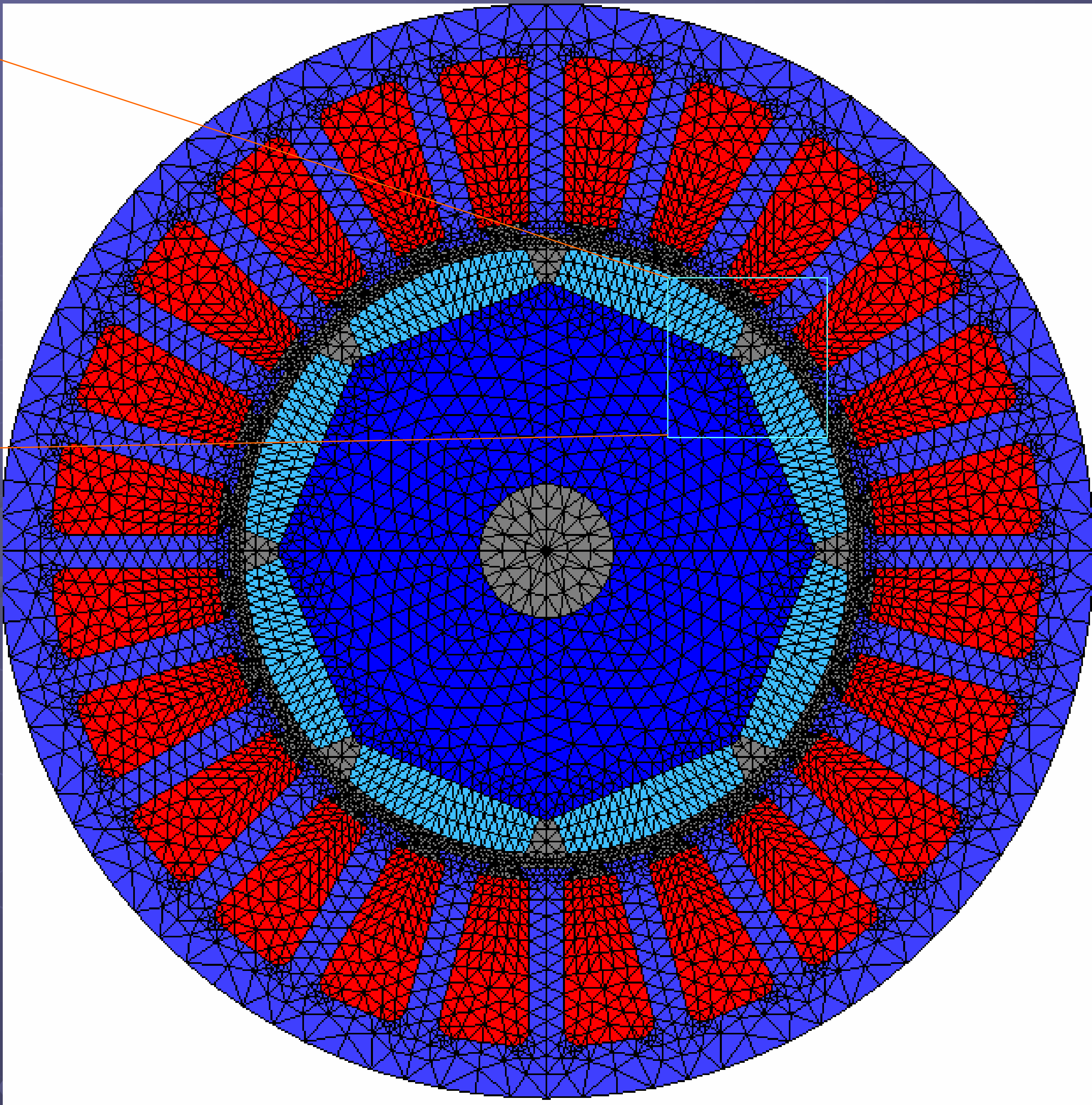


Finite-element analysis



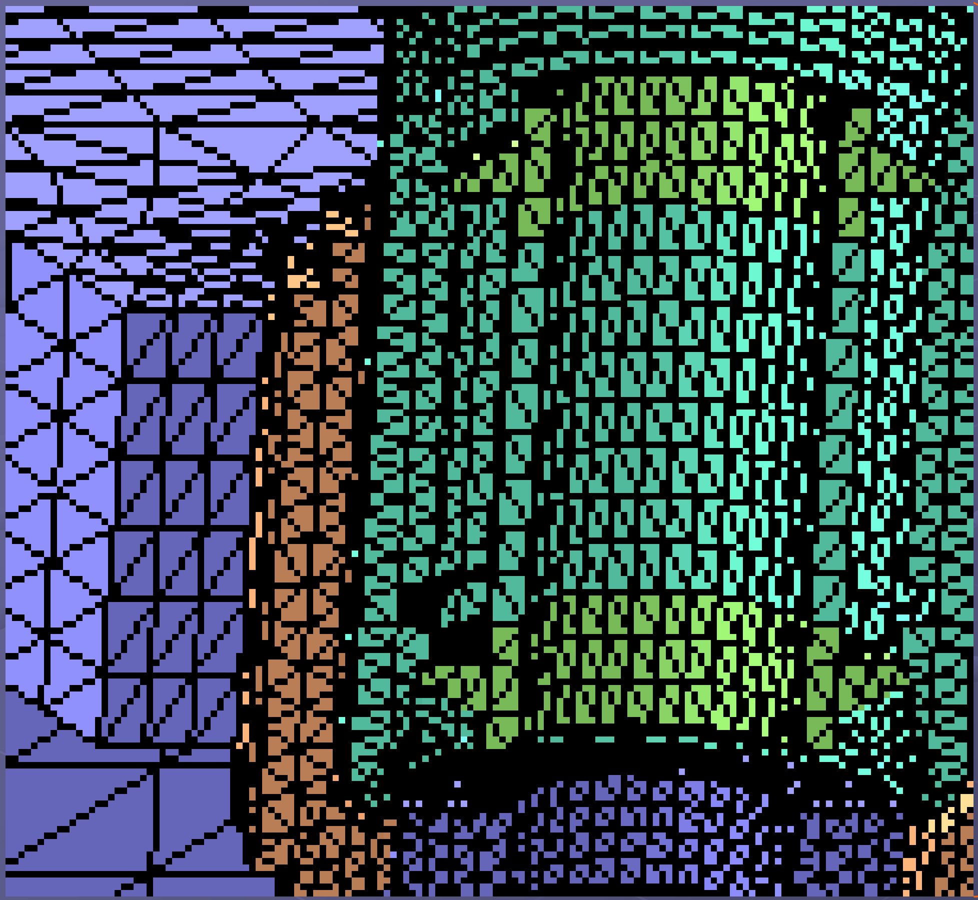
Finite-element analysis

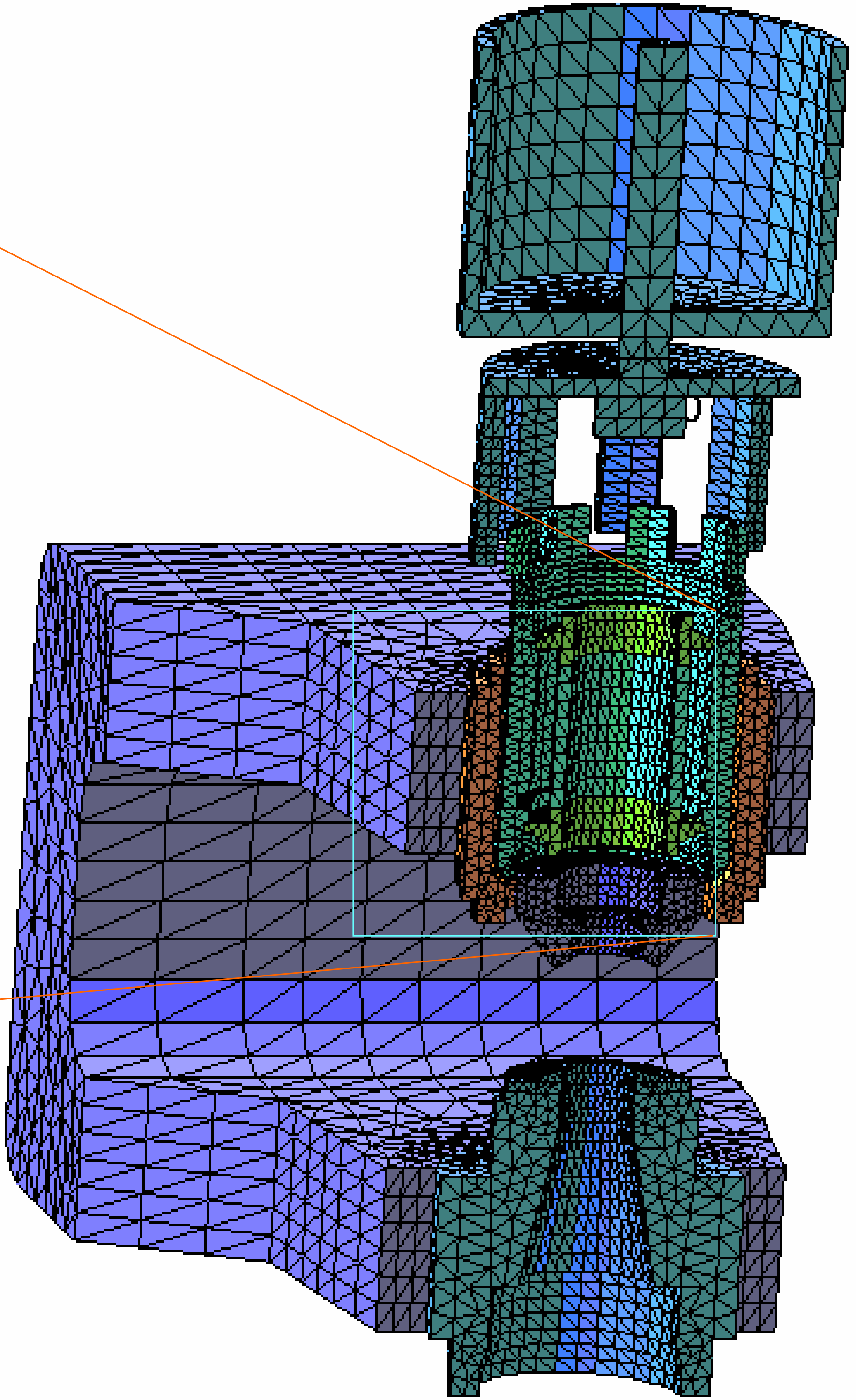




Typical triangular mesh in 2D

Finite-element analysis





3D meshes

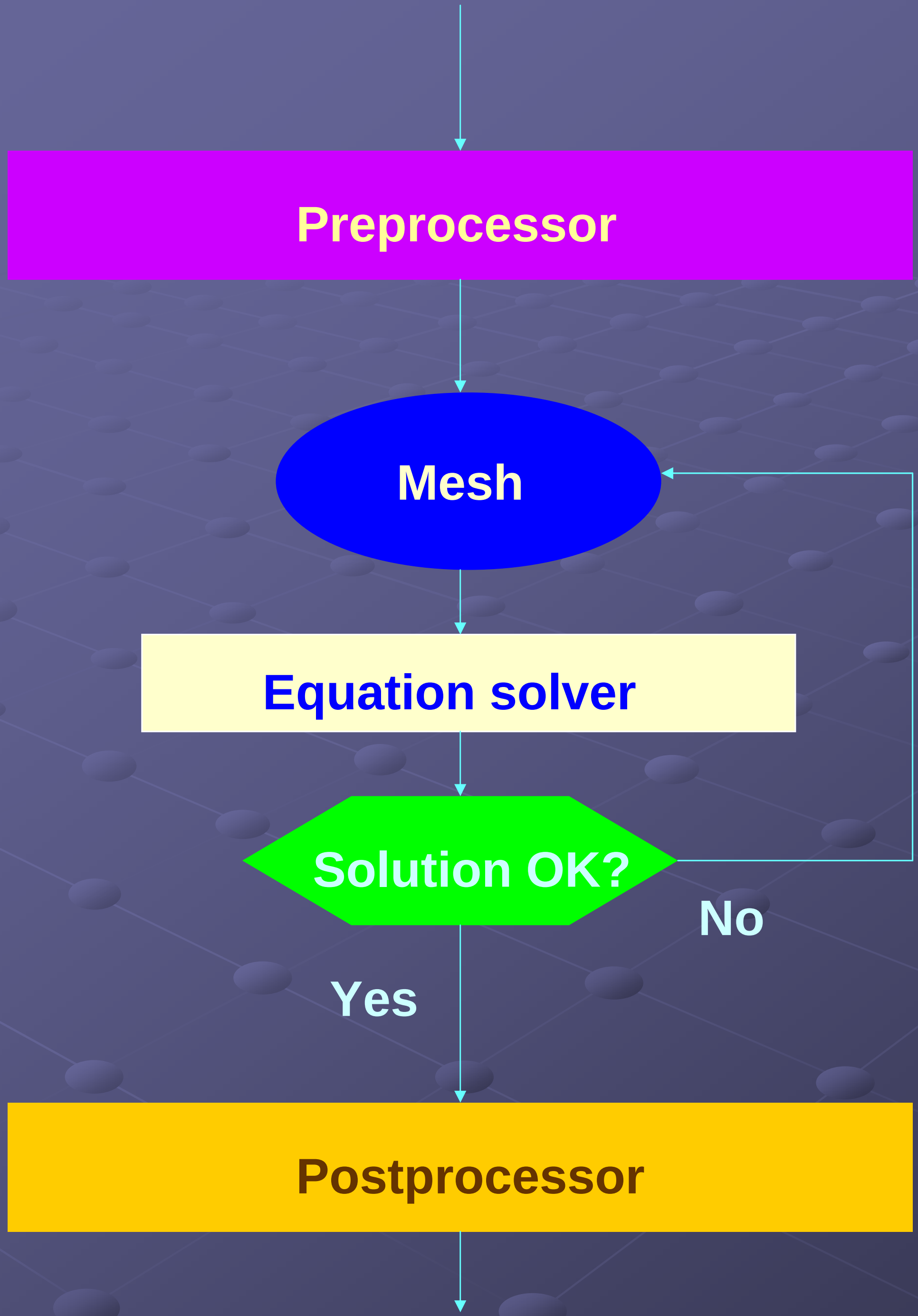
The state of the art

Contemporary software capable of solving

- 2D, axi-symmetric and 3D problems
- Eddy currents
- Non-linearity of materials
- Anisotropy and hysteresis
- Motion effects
- Static, steady-state and transient solutions
- Coupling to mechanical and thermal effects
- Connections to driving circuitry

Geometric modellers can handle most practical shapes

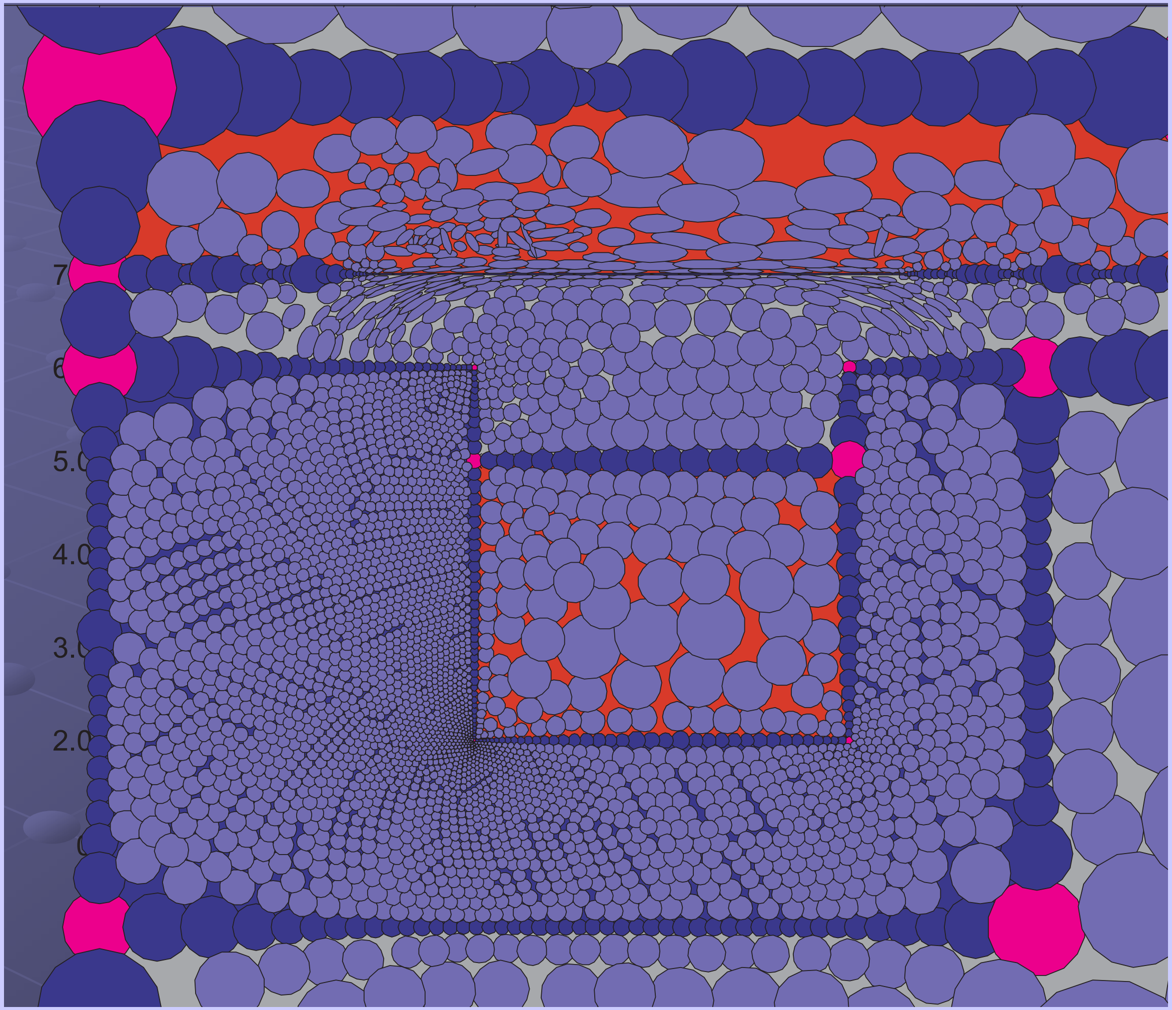
Adaptive meshing



- Increase number of elements (more nodes)
- Increase order of approximation
- Reposition nodes
- Neural network approach
- Bubble meshing
- A combination of the above

Adaptive meshing

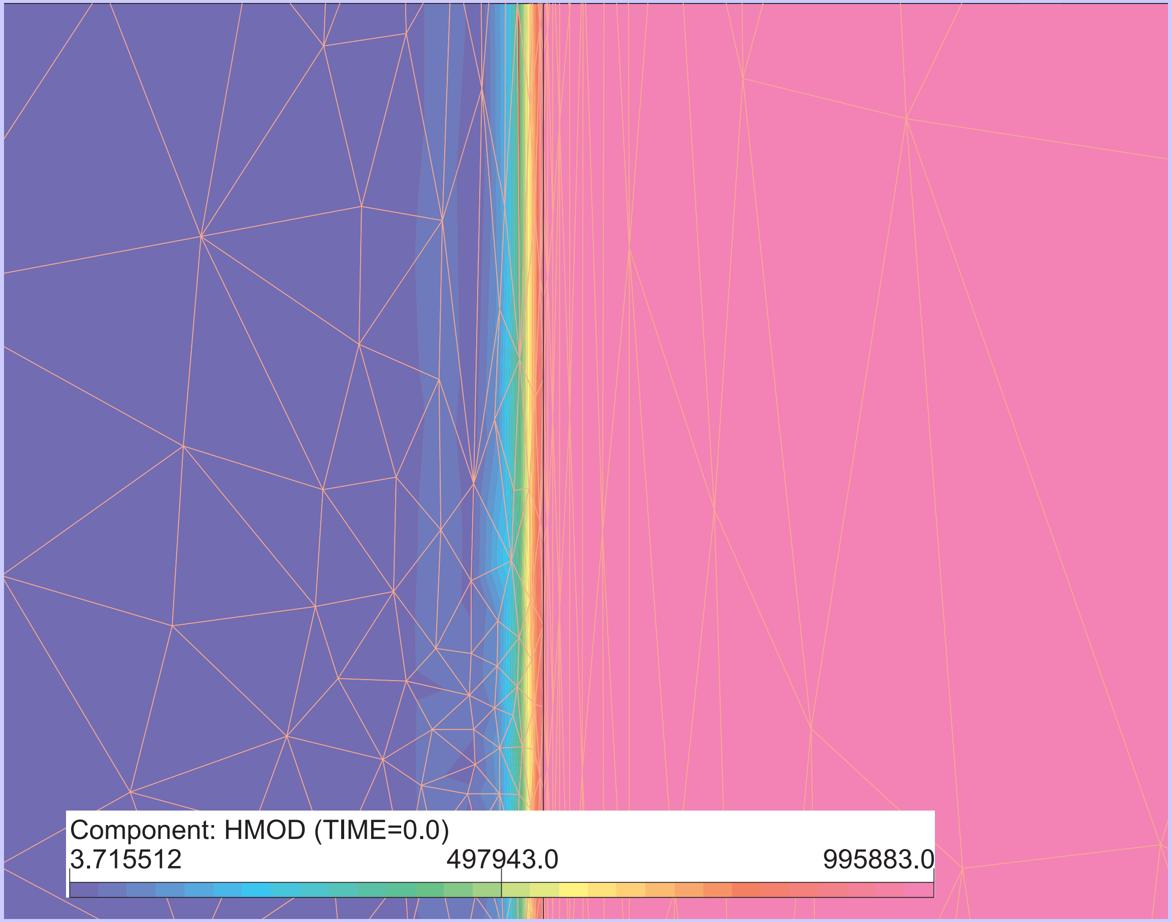
Coin recognition sensor



Adaptive meshing in 2D using dynamic bubbles

Adaptive meshing

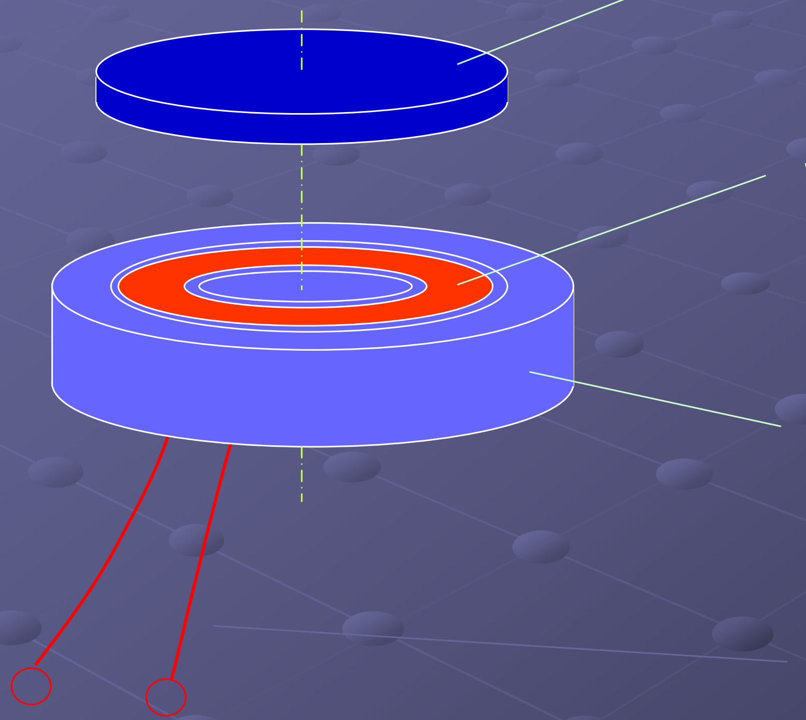
Interface between conducting plate and air



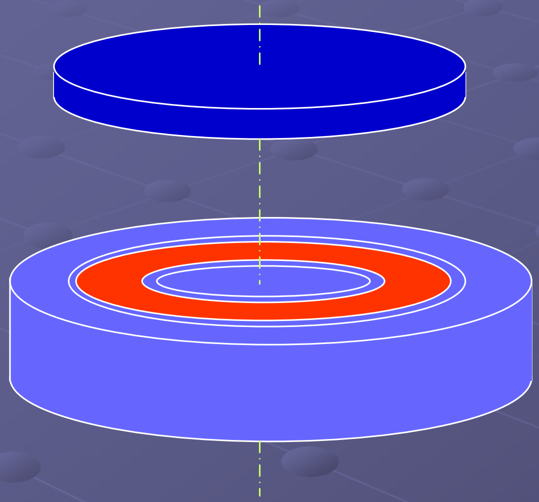
Adaptive meshing in 2D using dynamic bubbles

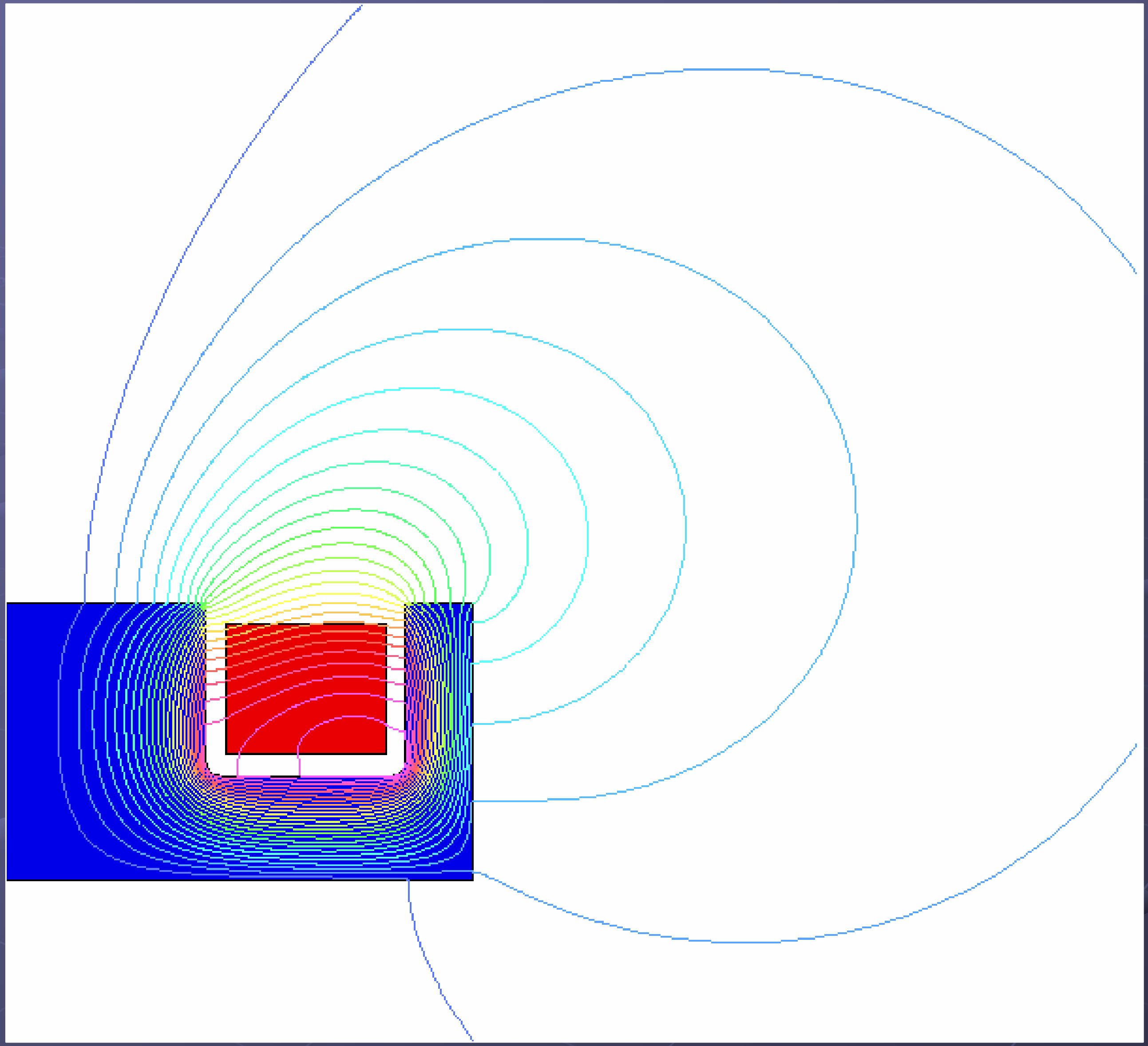
Coin recognition sensor

Analysis of performance



Coin recognition sensor



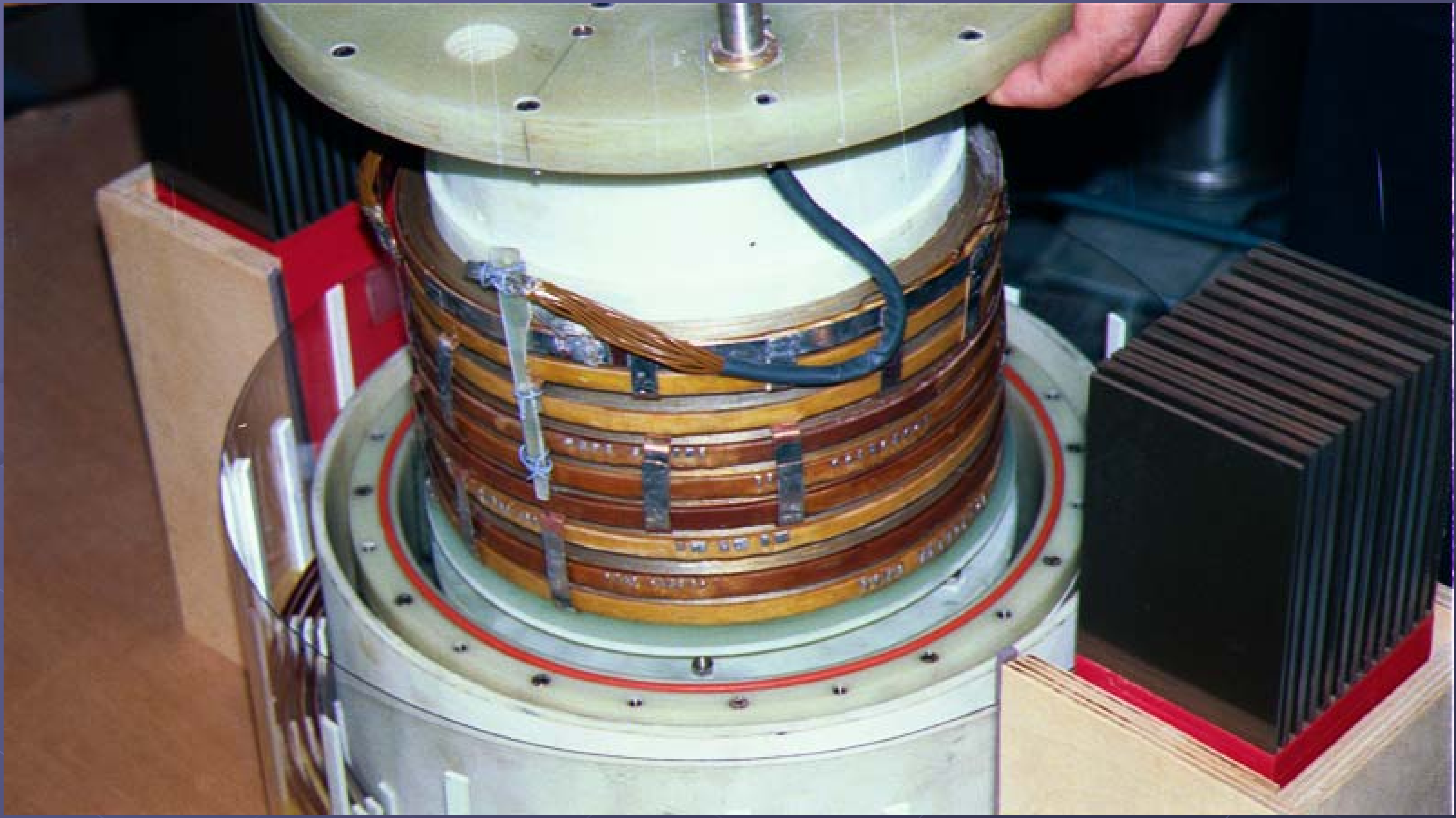


New materials

High Temperature Superconductors

- ceramic materials discovered in 1986
- conductivity 10^6 better than copper
- operate at liquid nitrogen temperature (78K)
- cheap technology (often compared to water cooling)
- current density 10 times larger than in copper windings
- great potential in electric power applications (generators, motors, fault current limiters, transformers, flywheels, cables, etc.), as losses and/or size are significantly reduced
- present a modelling challenge because of very highly non-linear characteristics and anisotropic properties of materials, and due to unconventional designs

HTS transformer
built and tested at Southampton 1998/99



HTS transformer
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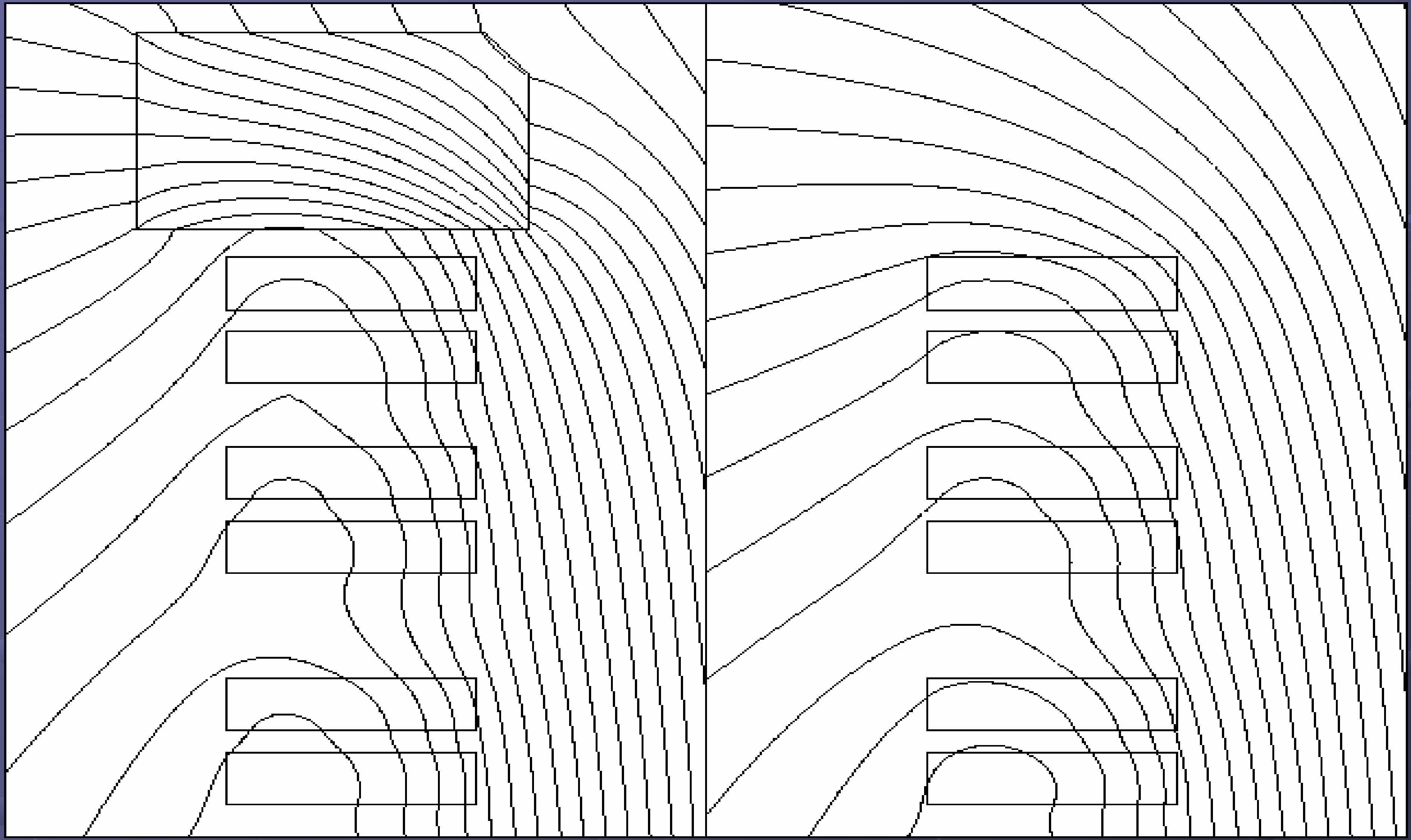
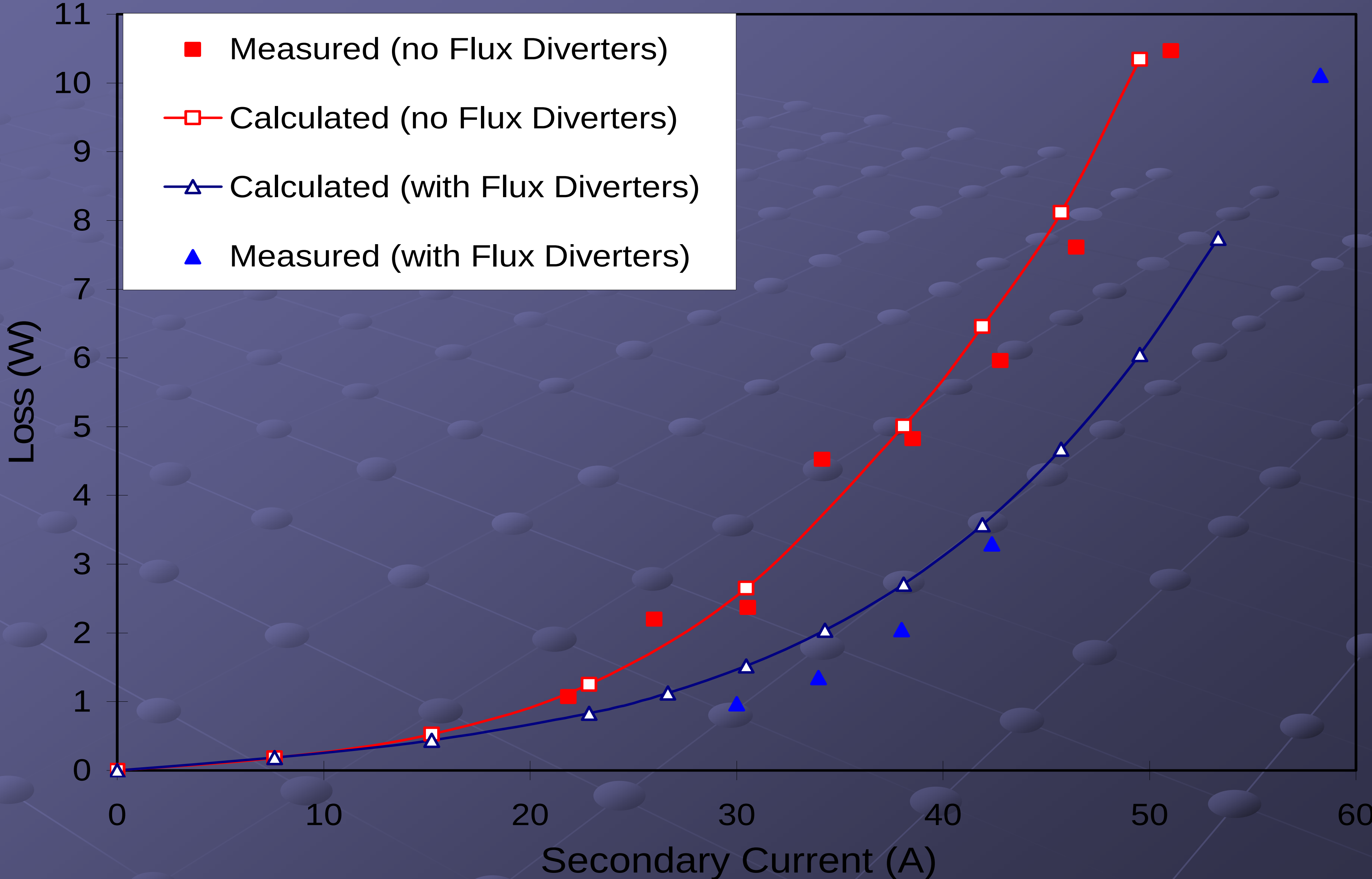


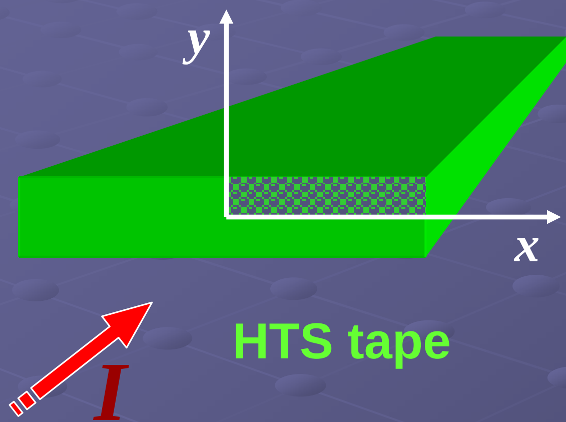
Field plots with and without flux diverters

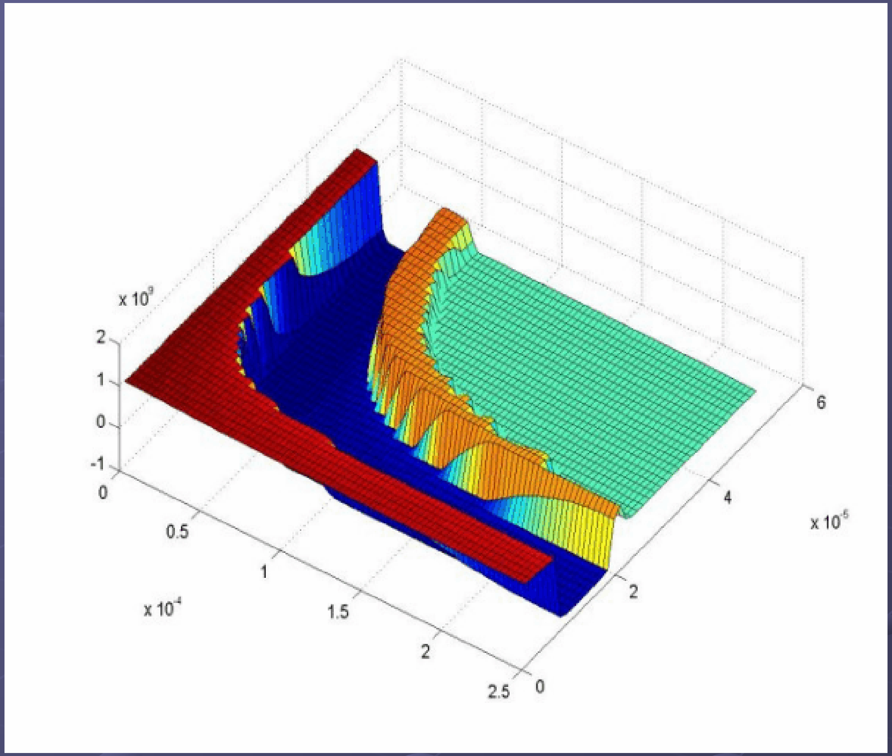
HTS transformer
built and tested at Southampton 1998/99



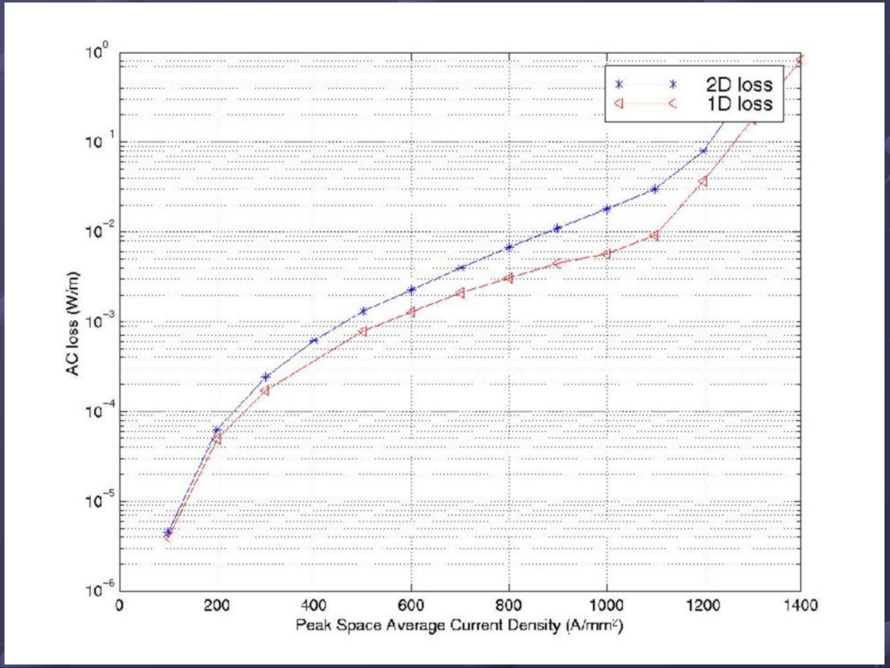
Field and current penetration in HTS tape

Diffusion of current density into HTS tape



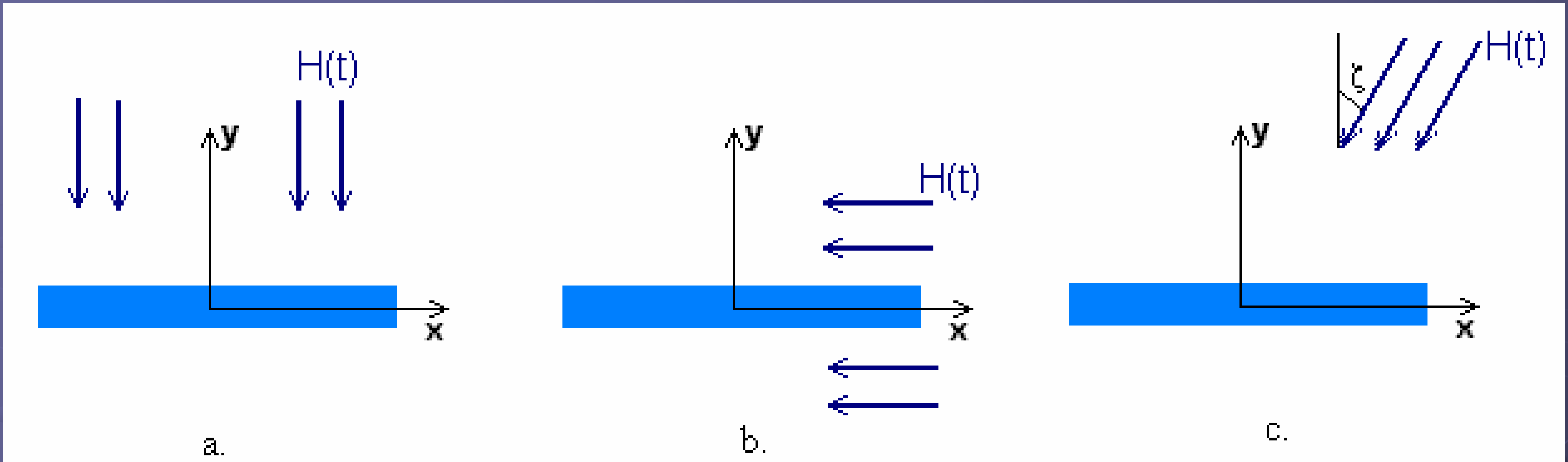


Flow of transport current through an HTS tape



AC loss as a function of average current density

HTS tape subjected to an external magnetic field



Rhyner model:

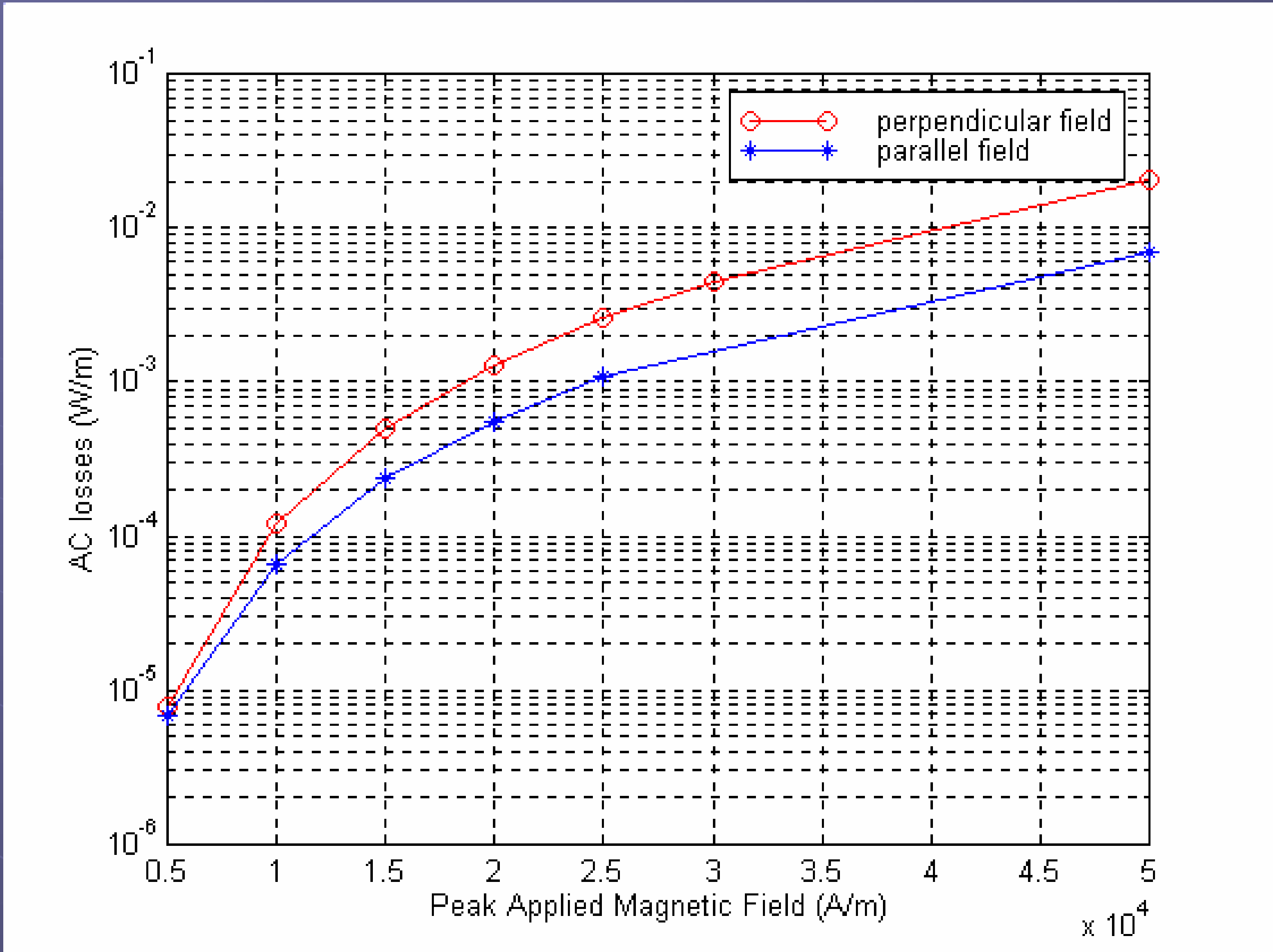
$$E = E_c \left(J / J_c \right)^{\alpha} \quad , \quad \rho = \rho_c \left(J / J_c \right)^{\alpha - 1}.$$

The critical current density J_c corresponds to an electric field E_c of 100 $\mu V m^{-1}$, and $\rho_c = E_c / J_c$.

The power law contains the linear and critical state extremes ($\alpha = 1$ and $\alpha \rightarrow \infty$ respectively).

In practice $\alpha \approx 10 - 20$ and thus the system is very non-linear.

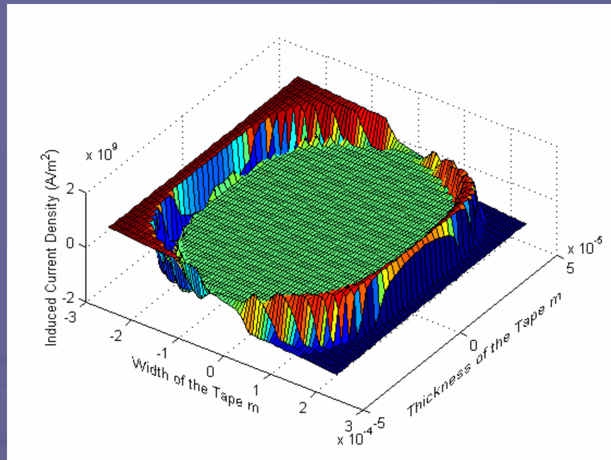
HTS tape subjected to an external magnetic field

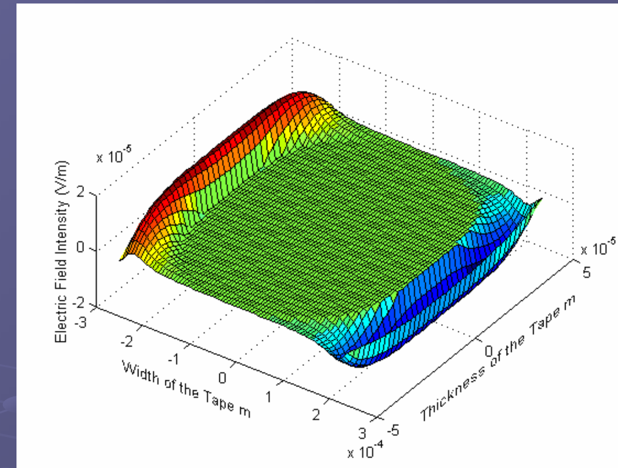


AC loss as a function of H_m (applied peak magnetic field strength)

Field angle

0º

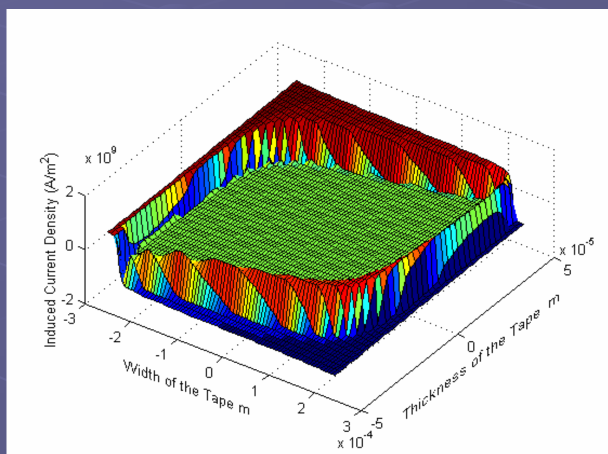


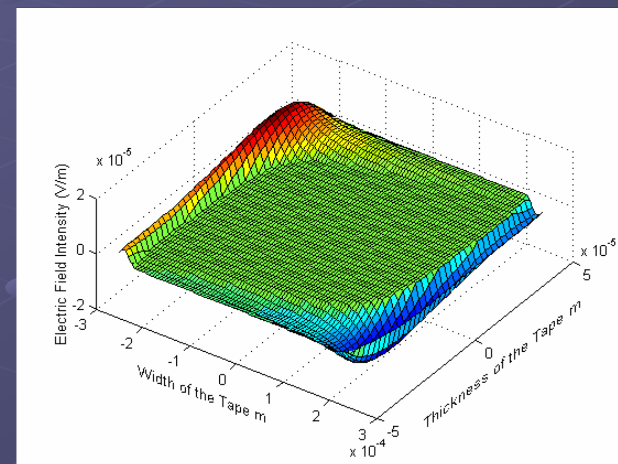


Current

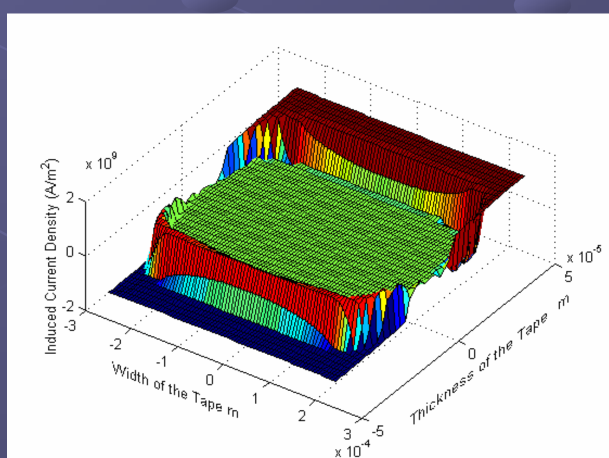
45º

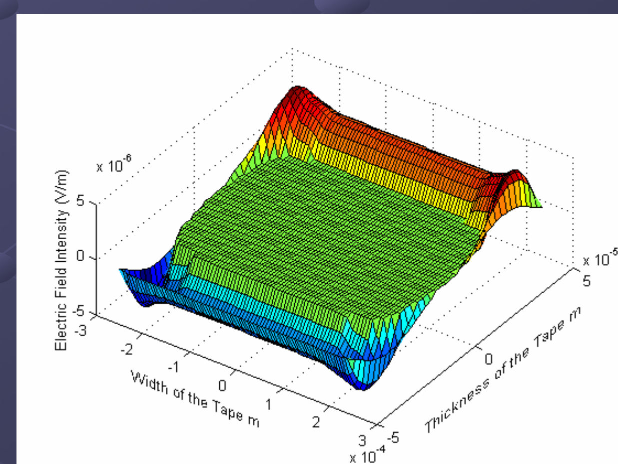
Electric field



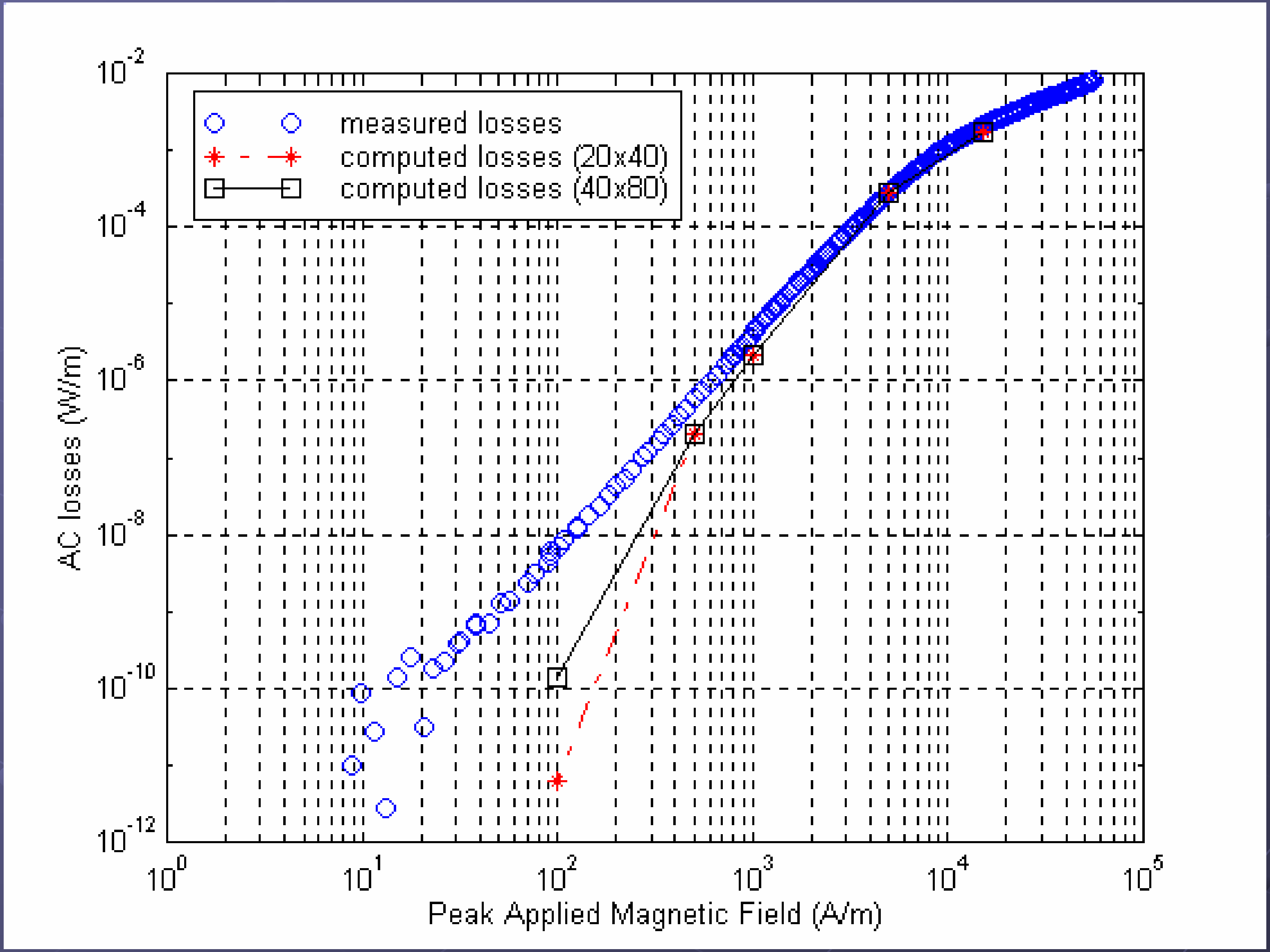


90º





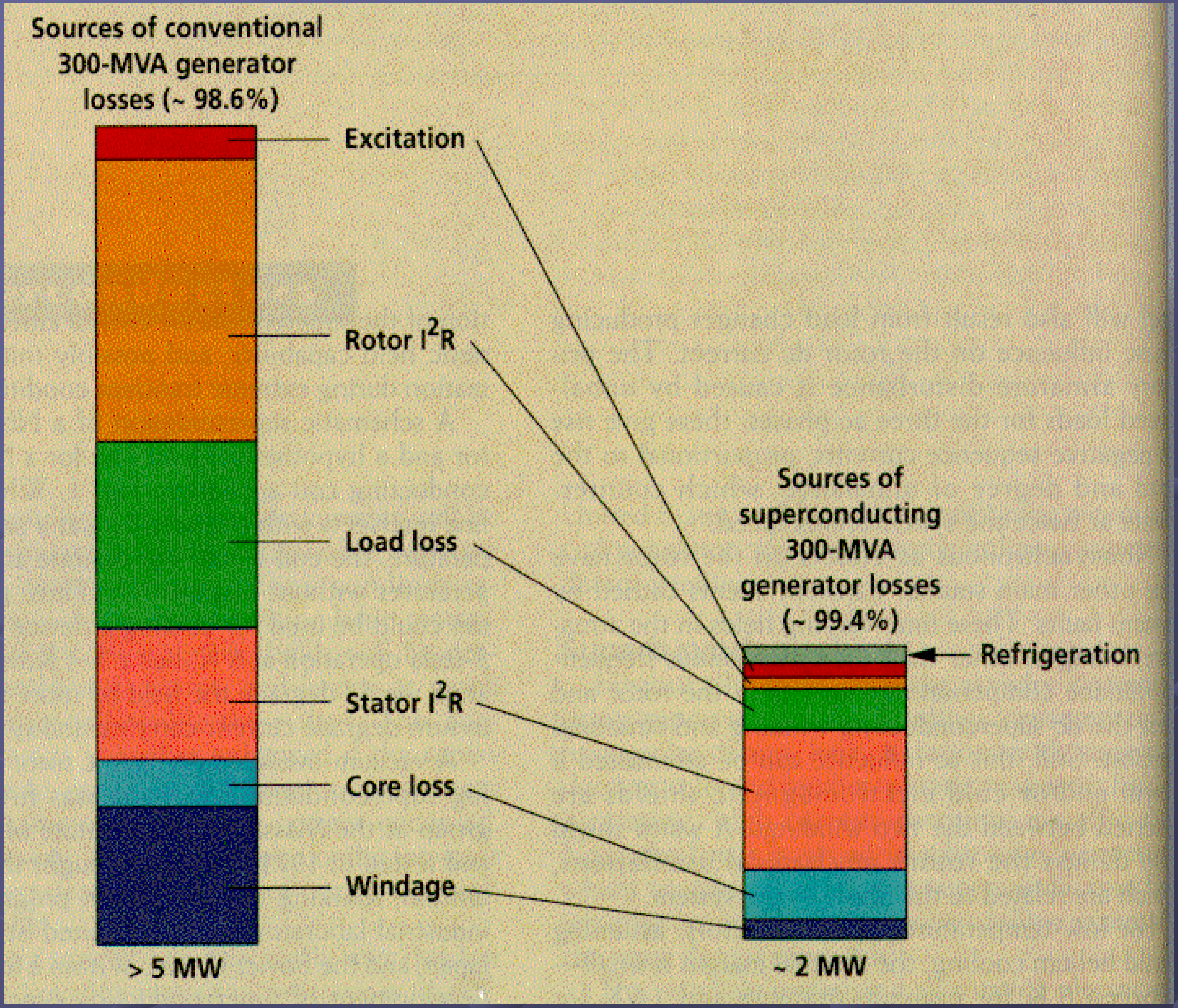
Experimental verification



Superconducting generators and motors

Why ?

Superconducting generators and motors



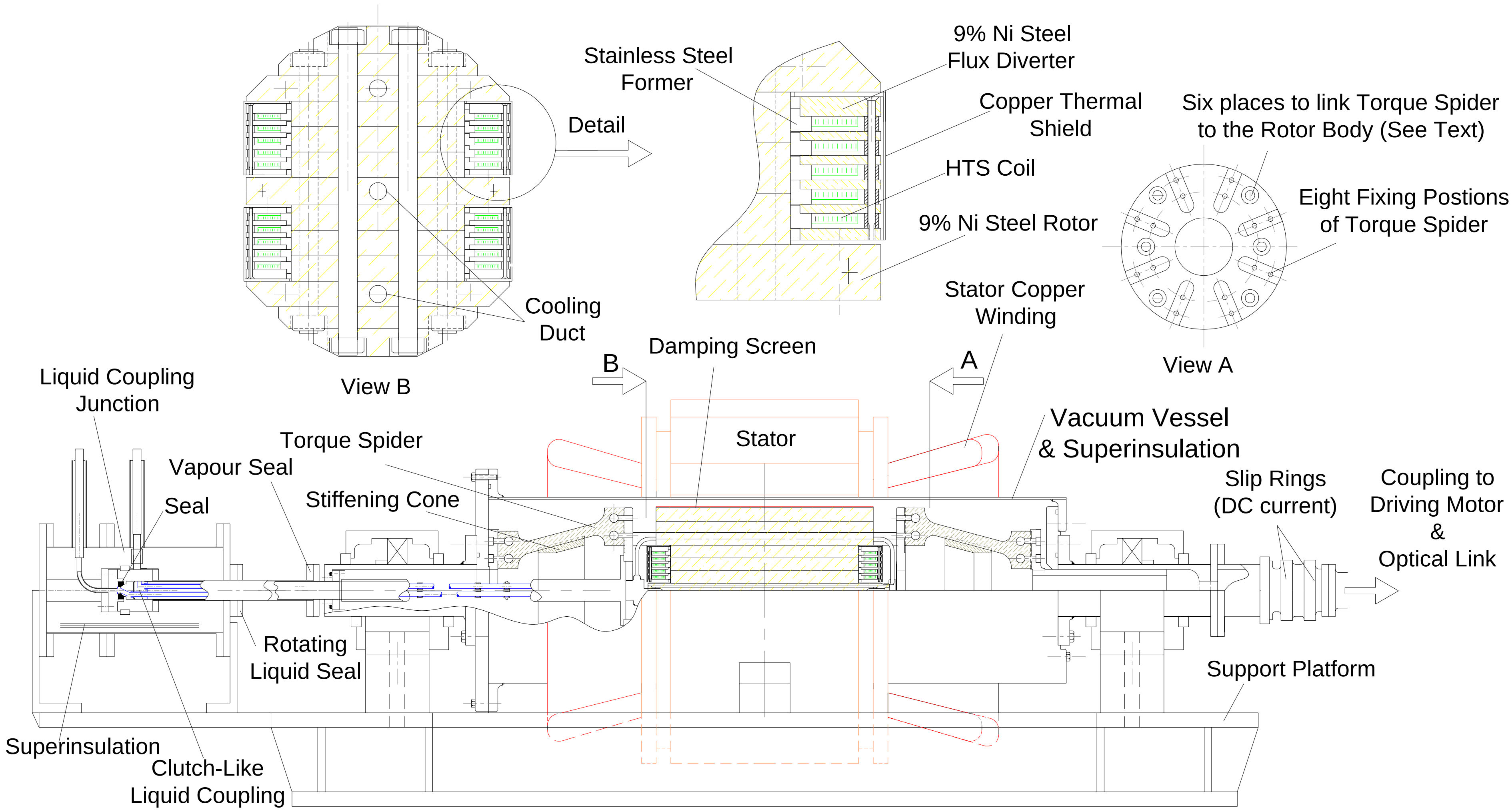
Losses in conventional and superconducting designs

Superconducting generators and motors

and / or

smaller size !

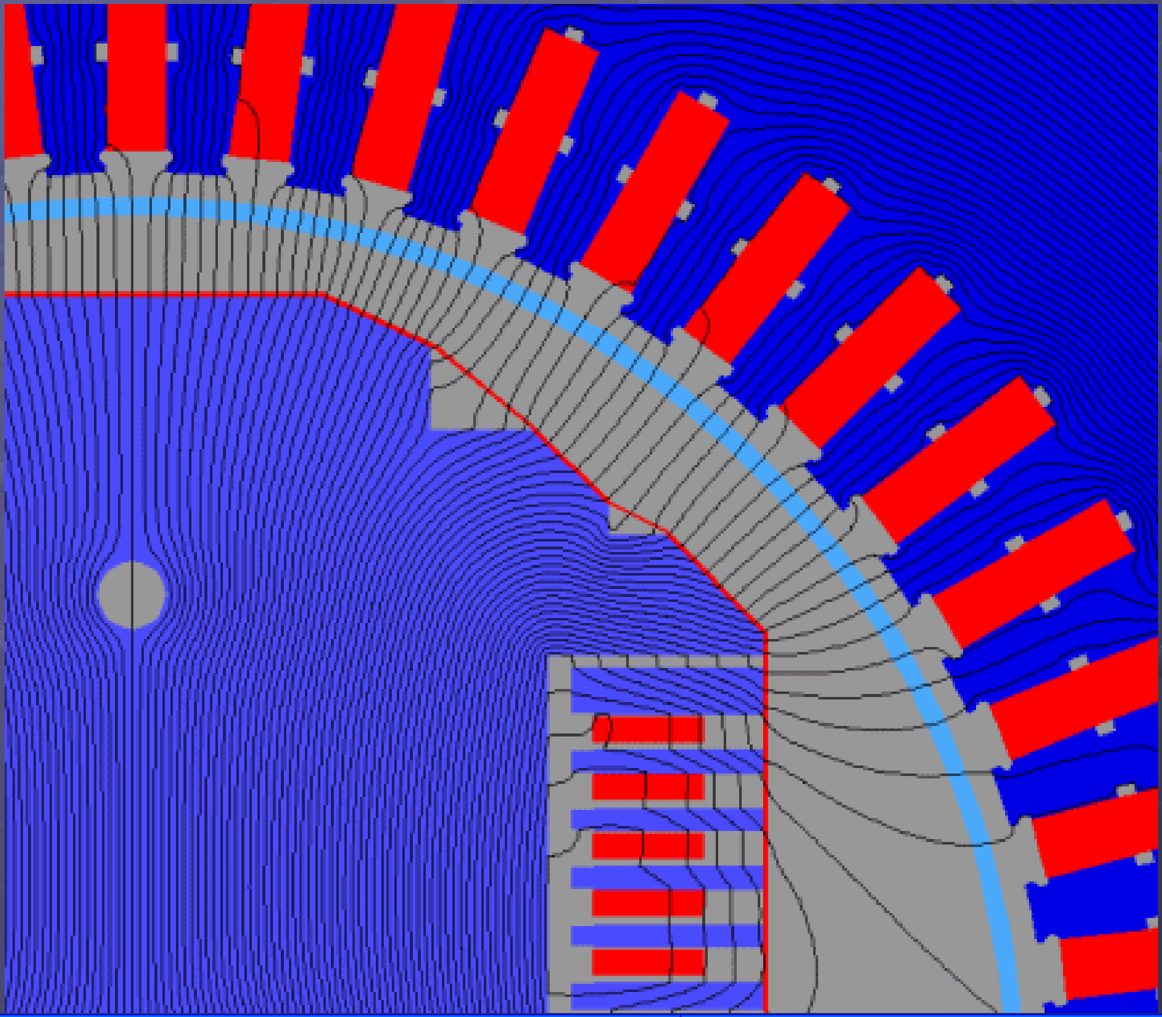
Machine Design



Modelling of Eddy-Current Loss

No-load losses

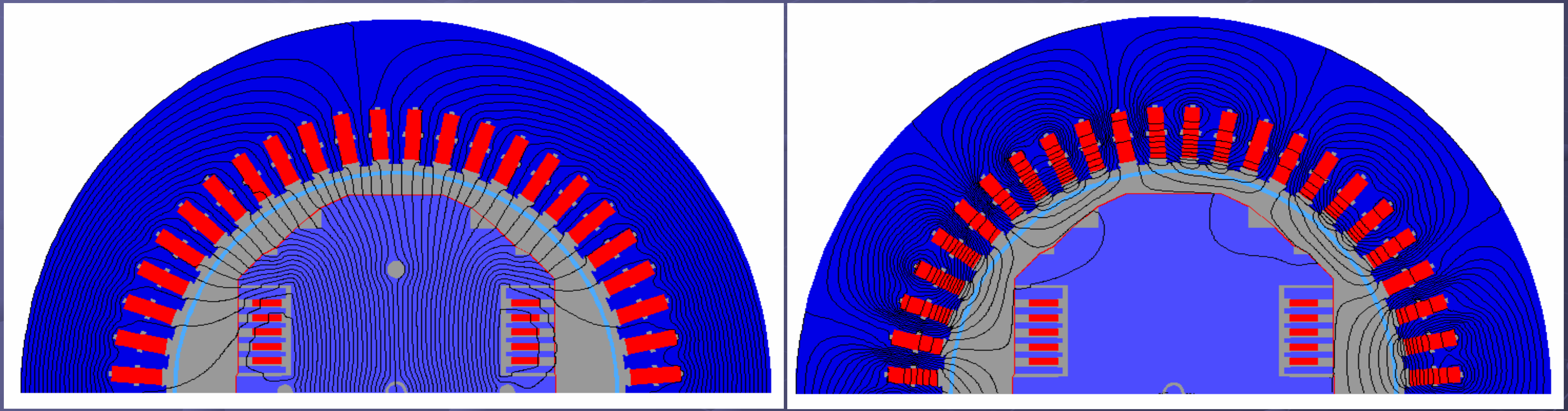
- Eddy currents occur as 48th time harmonic
- Transient losses were estimated and subtracted
- Total no-load loss found to be **0.264 W**



Modelling of Eddy-Current Loss

Full-load losses

- Dominating 5th harmonic (and much smaller 7th)
- Losses due to 11th and higher harmonics negligible
- Total full-load loss found to be **2.319 W**

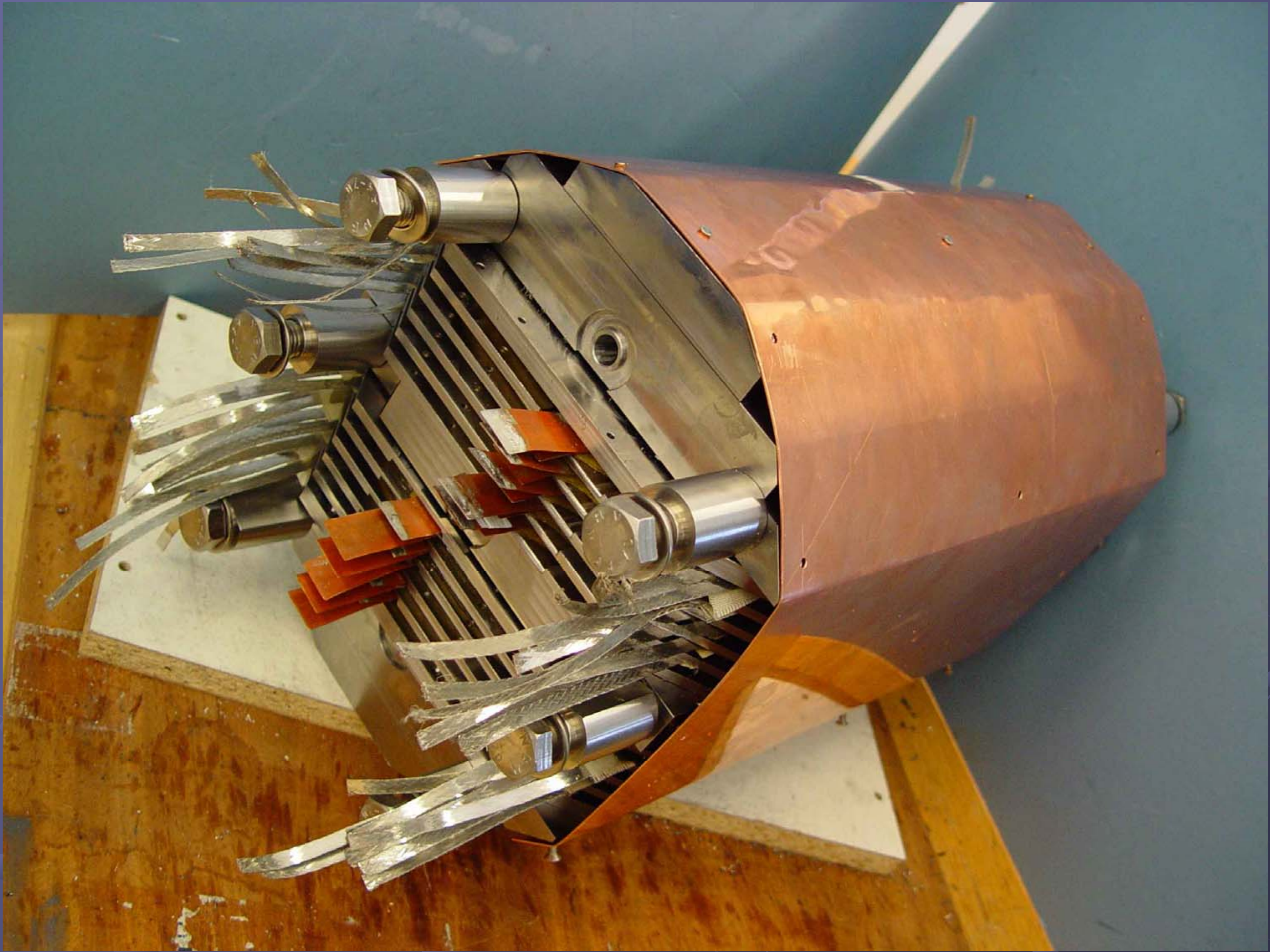


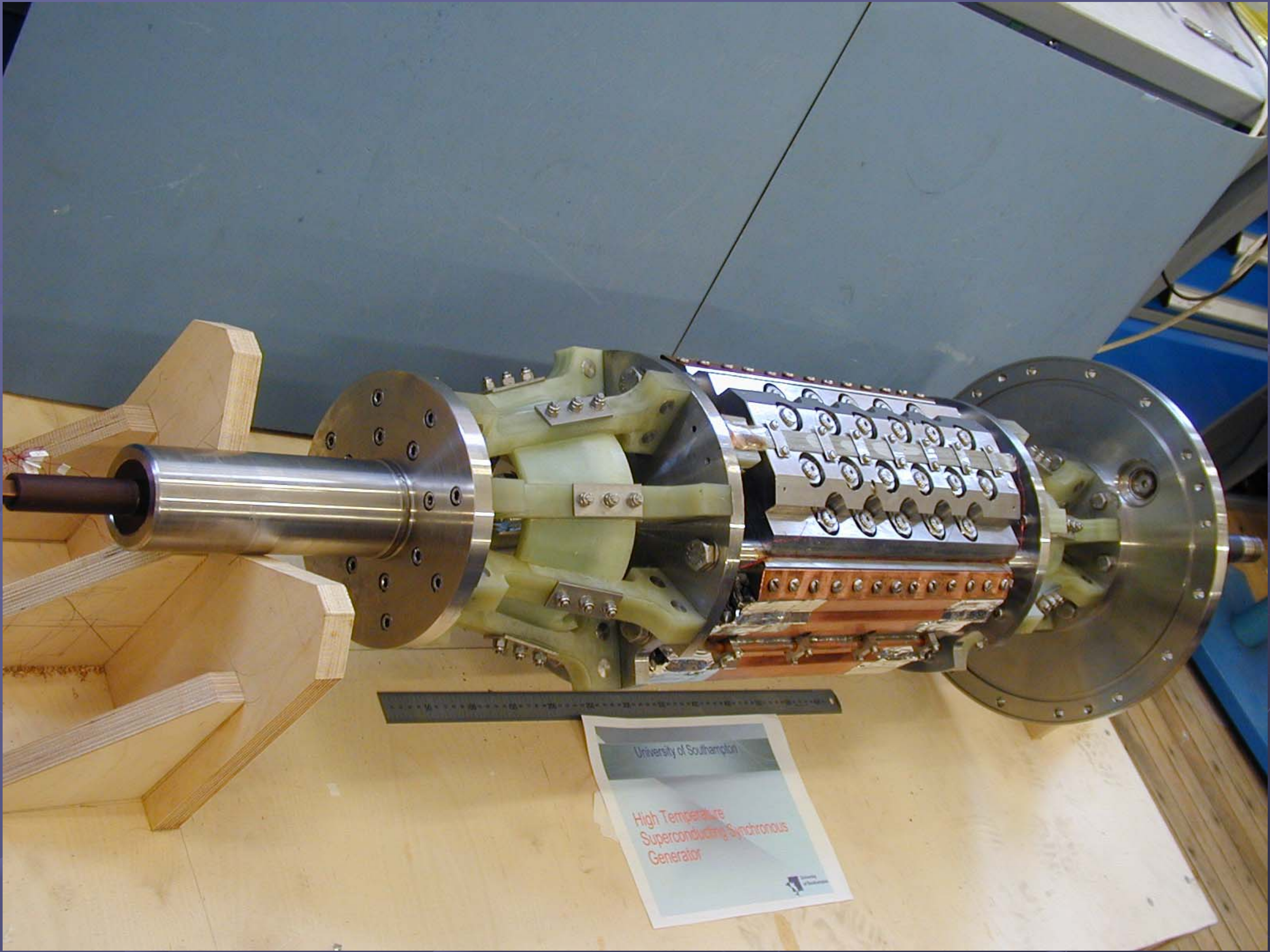
(a) DC field **(b) Additional 6th time harmonic field**

Contours of vector potential: (a) Non-linear static model and (b) Linear AC model with new current densities defined in each stator slot and incremental permeability data taken from the static model.

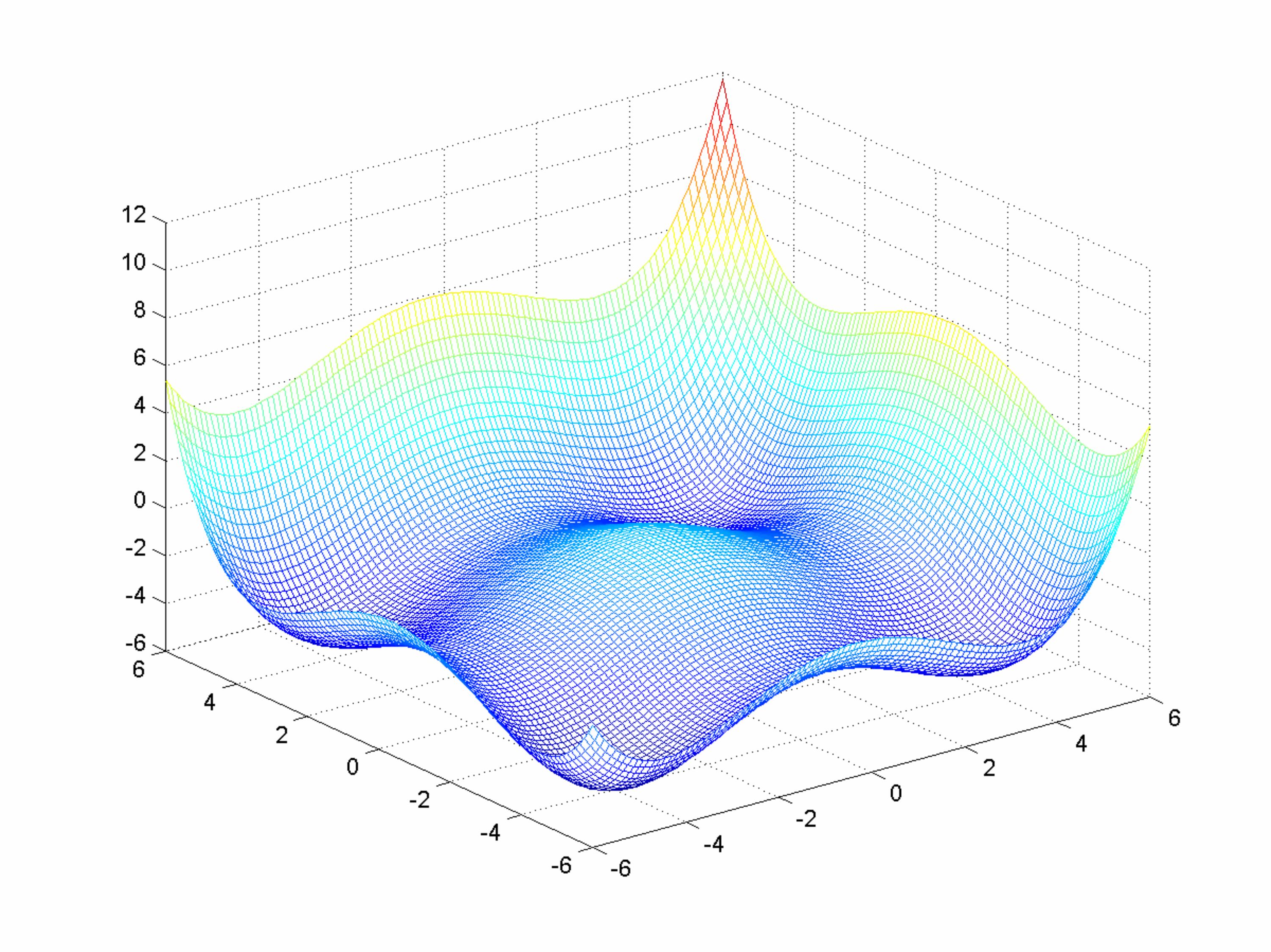
Total power loss in the cold region is 2.583 W.


ELECTRONIC
DIGITAL CALIPER




University of Southampton
High Temperature Superconducting Synchronous Generator

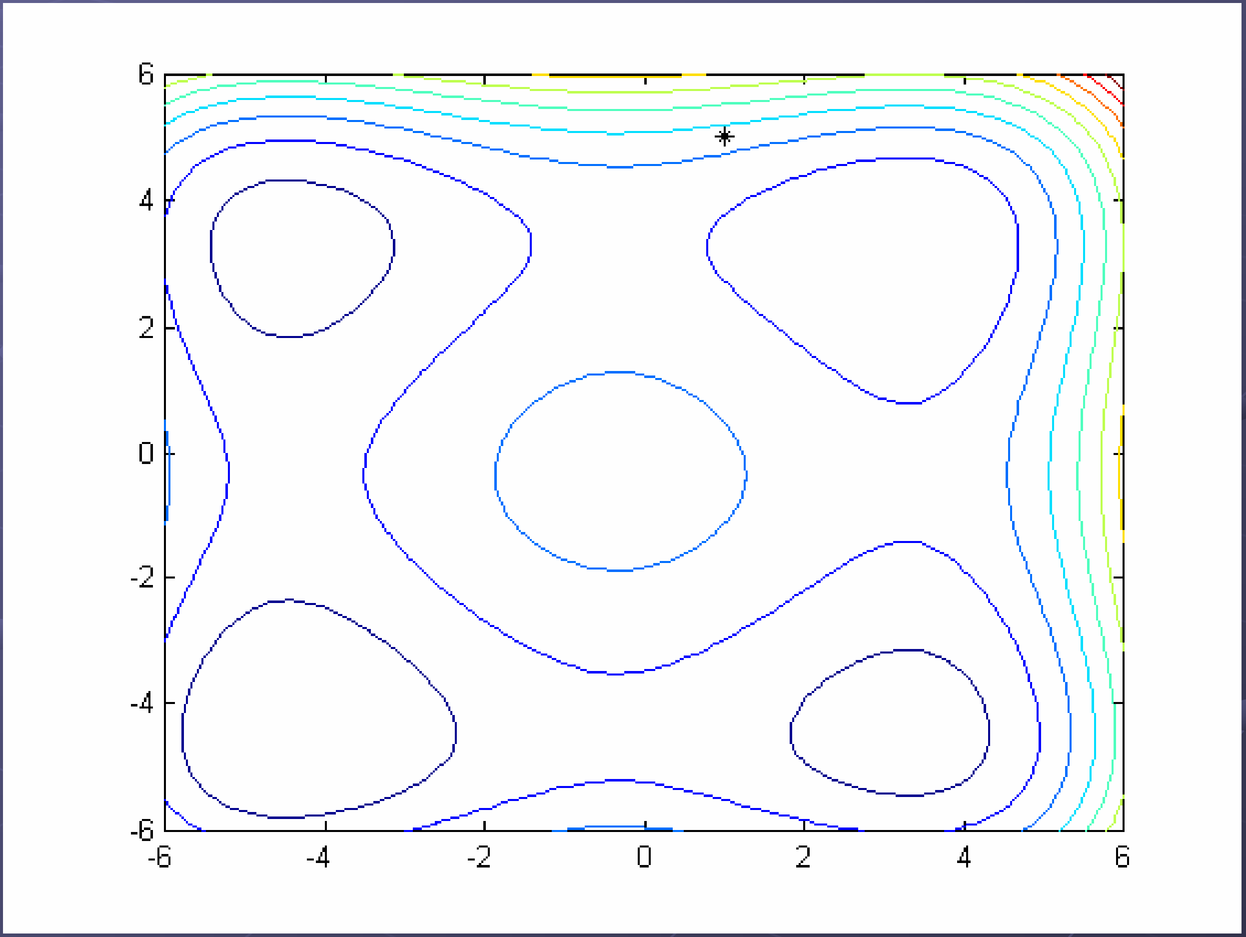
Optimization



Multiple-minima objective function

Optimization

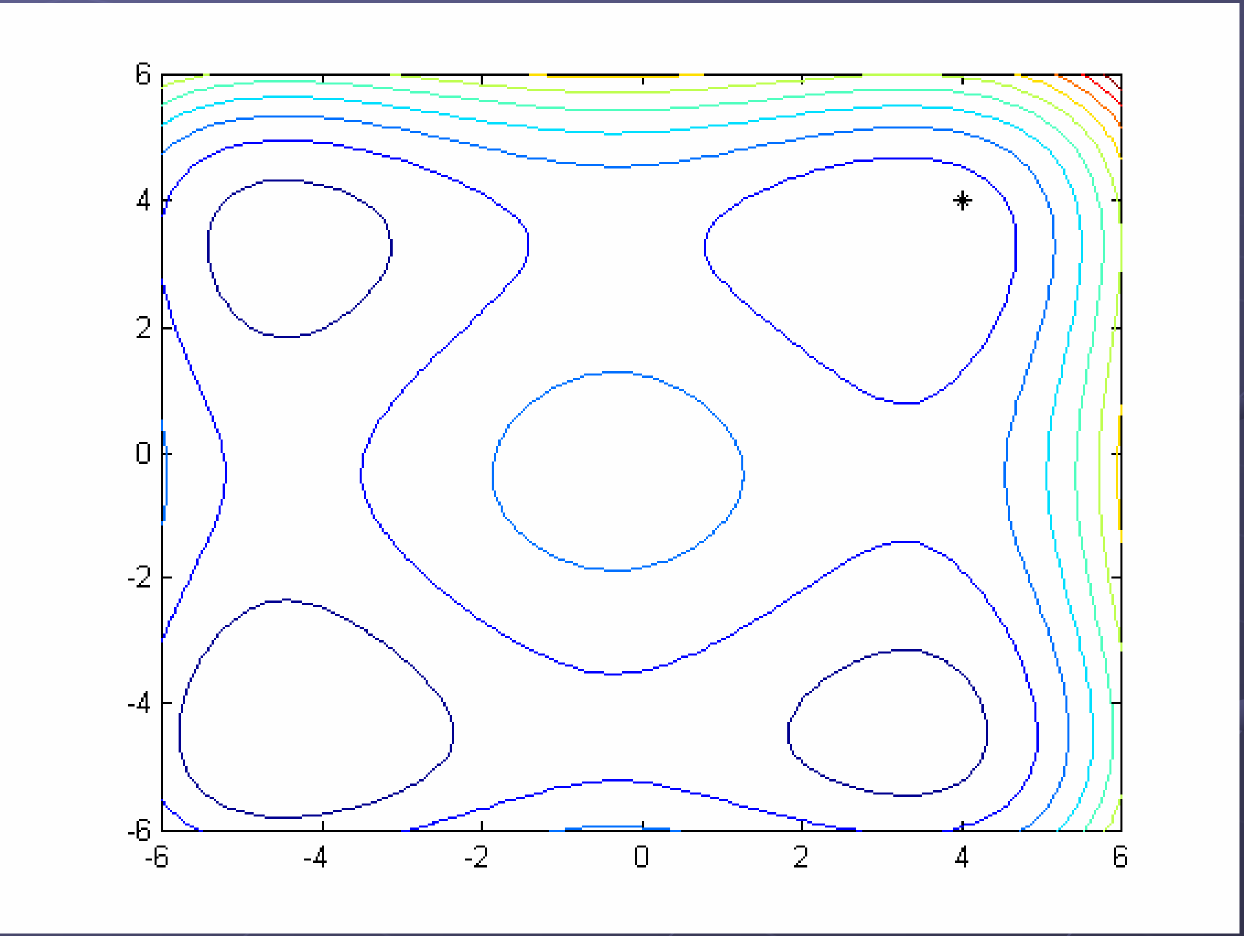
Deterministic algorithm



Multiple-minima objective function

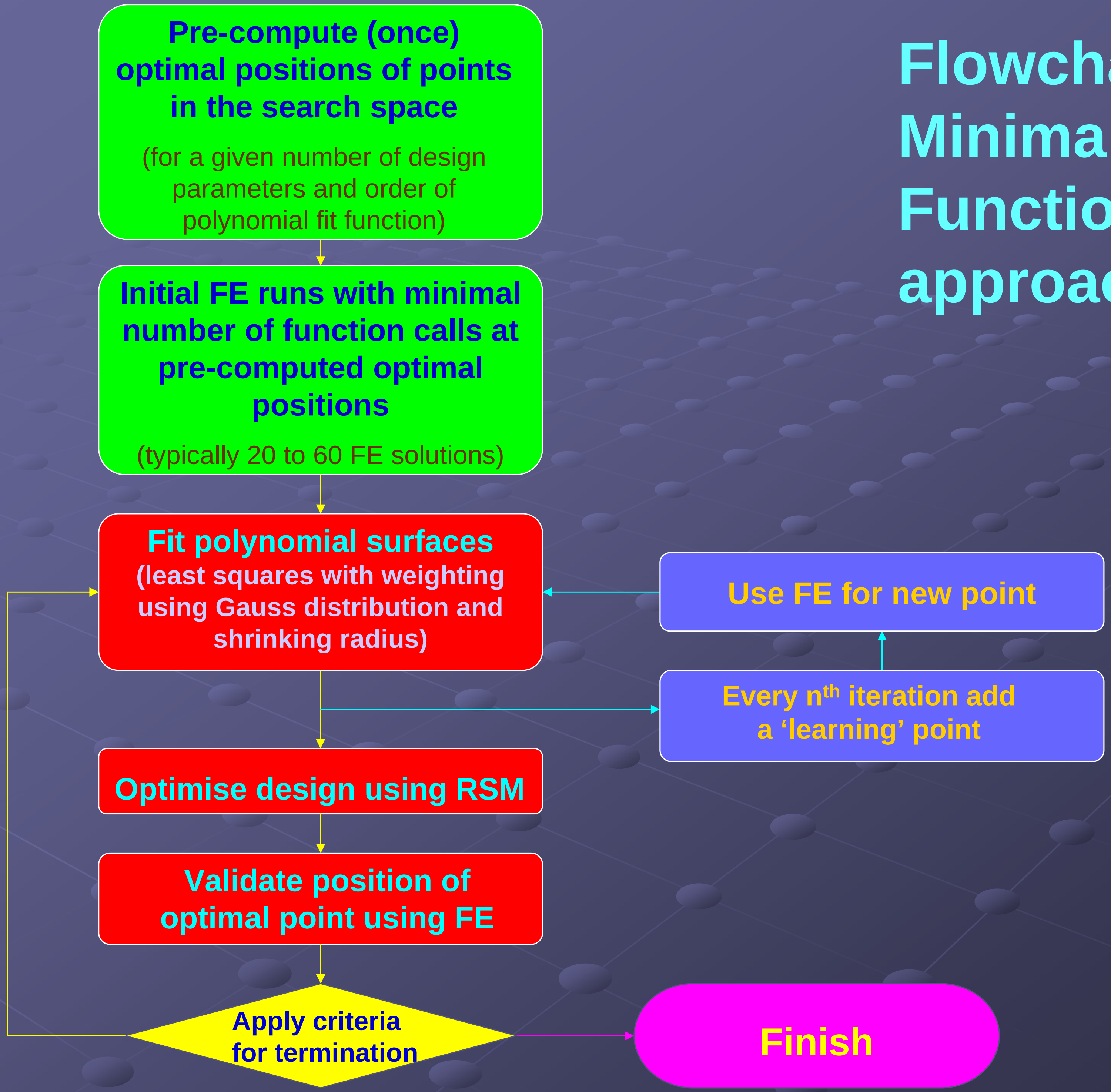
Optimization

Evolution strategy



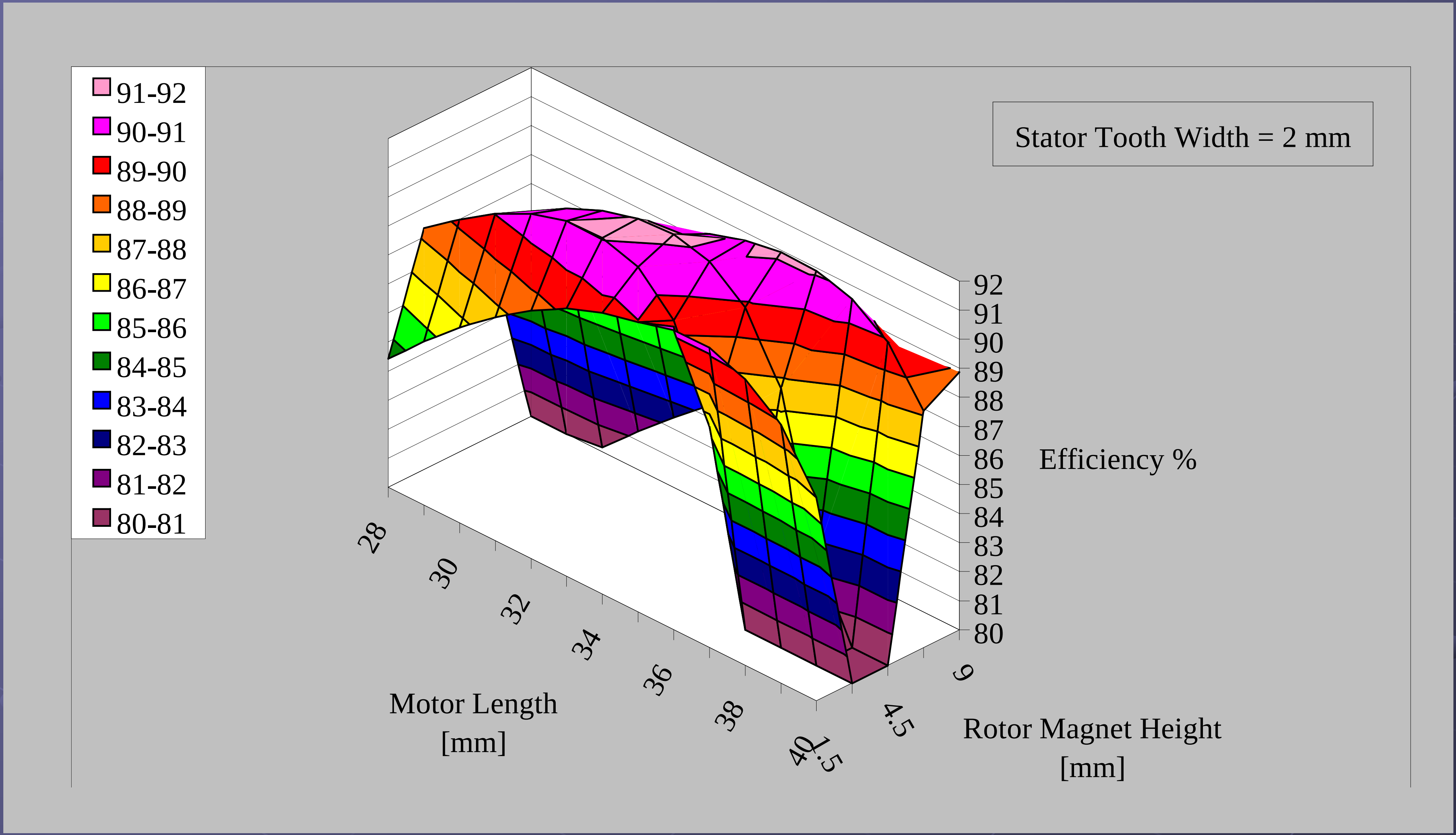
Multiple-minima objective function

Flowchart of the Minimal Function Calls approach



Optimization

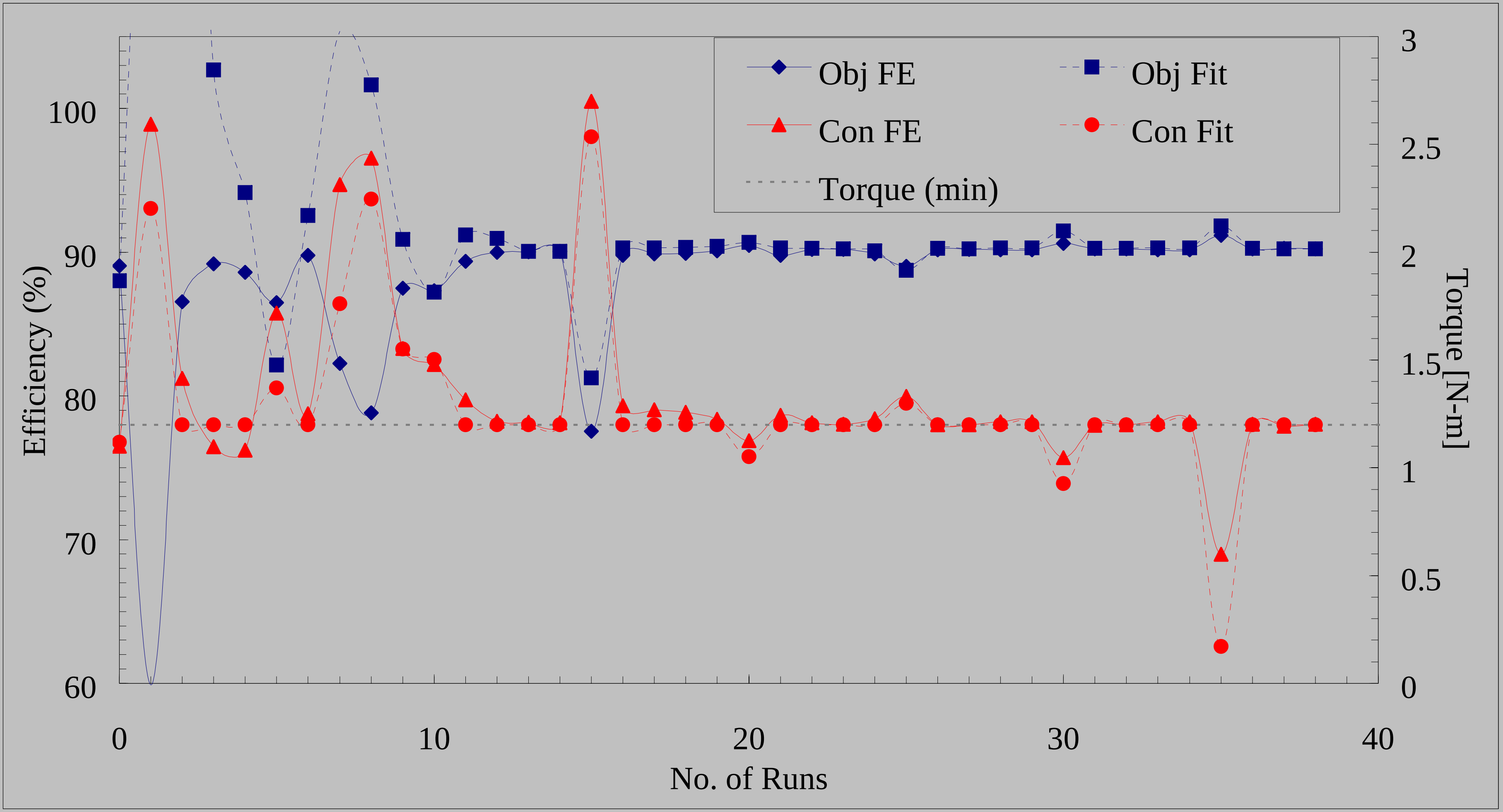
Brushless permanent magnet motor



Brushless PM motor optimisation response surface (when varying three design parameters)

Optimization

Brushless permanent magnet motor

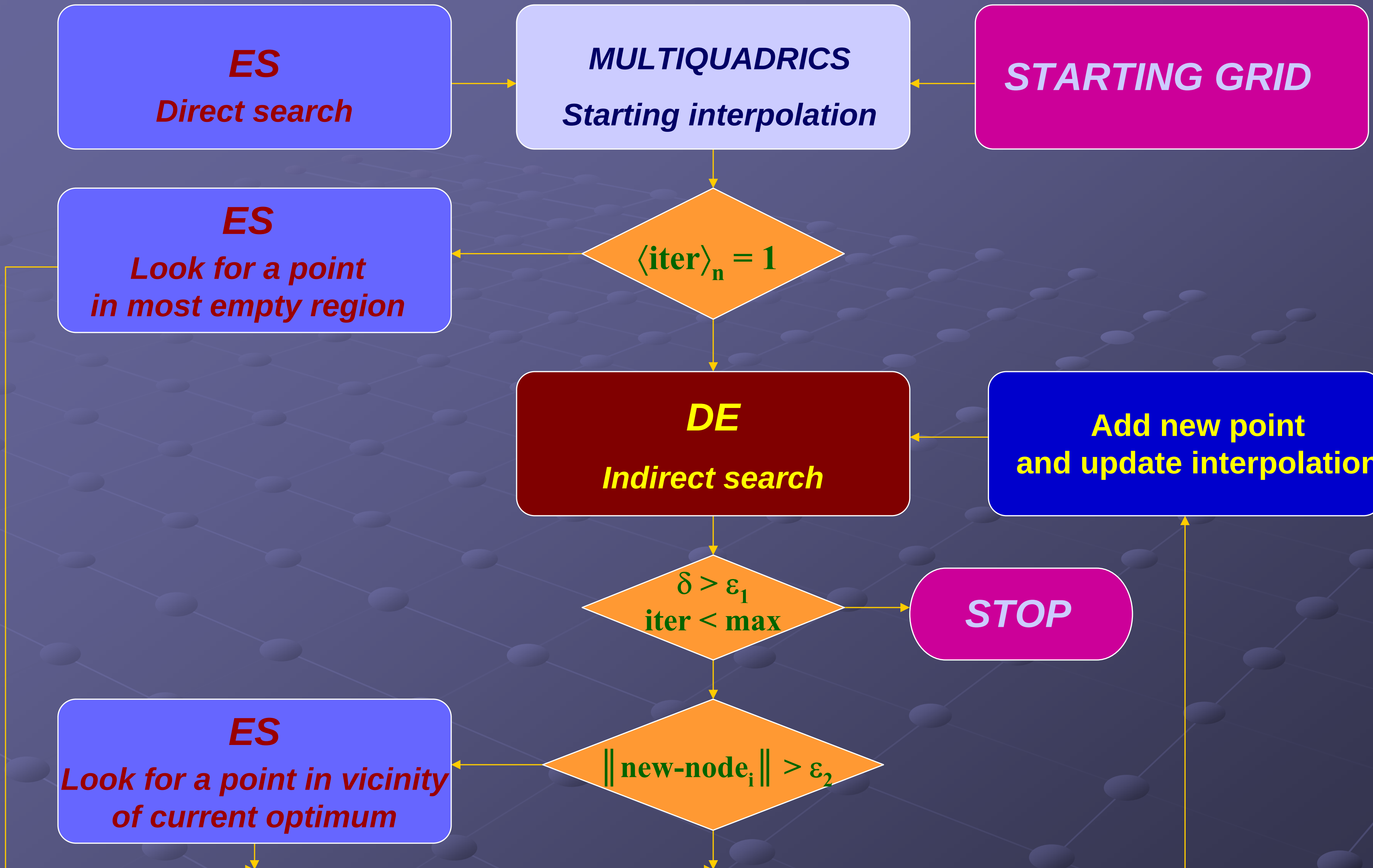


Convergence of efficiency and torque

Minimal Function Calls Approach

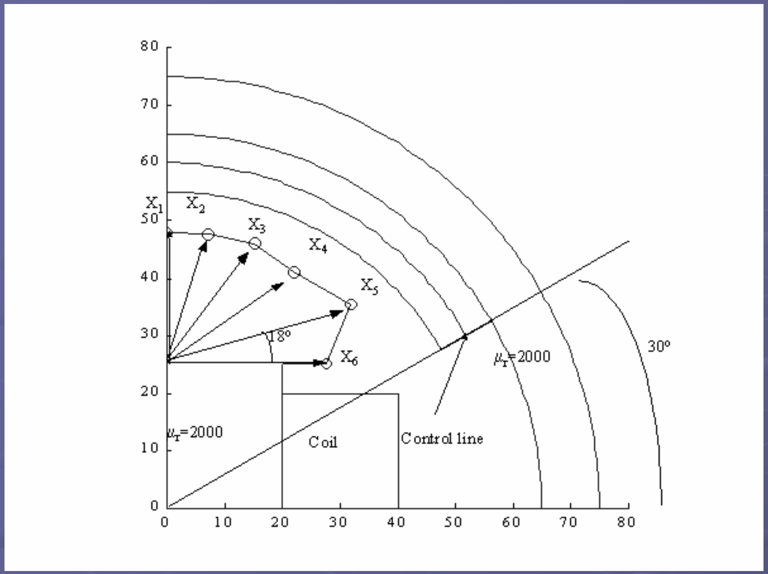
with On-Line Learning and Dynamic Weighting

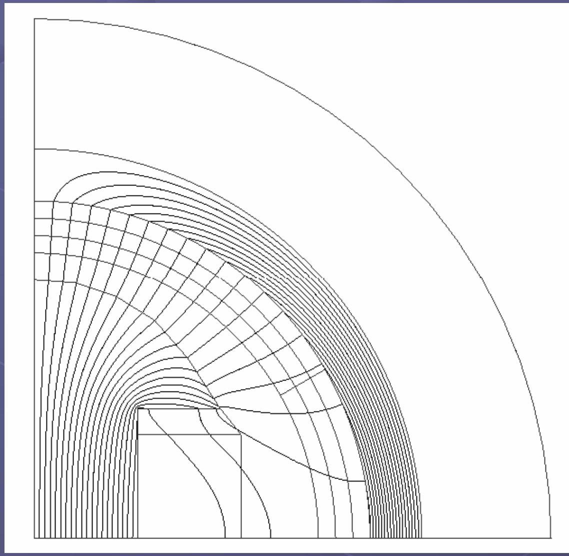
Evolution strategies



ES/DE/MQ Evolution Strategy, Differential Evolution, Multiquadrics Interpolation

Magnetiser





$$f = \sum_{k=1}^{59} \left(B_{desired,k} - B_{calculated,k} \right)^2$$

$$B_{desired,k} = B_{max} \sin\left(90^o - k\right) \qquad 1 \le k \le 59$$

Method	Starting	Optimum	n
DE1	11 random	1.235E-5	987
DE2	11 random	5.423E-5	1035
ES	1.457E-3	1.187E-5	433
ES	9.486E-2	1.318E-4	351
GBA	1.457E-3	1.238E-4	41
GBA	9.486E-2	2.433E-4	281
ES/DE/MQ	1.457E-3	1.961E-5	234
ES/DE/MQ	9.486E-2	2.125E-5	206
NF/GA/SQP		6.570E-5	189

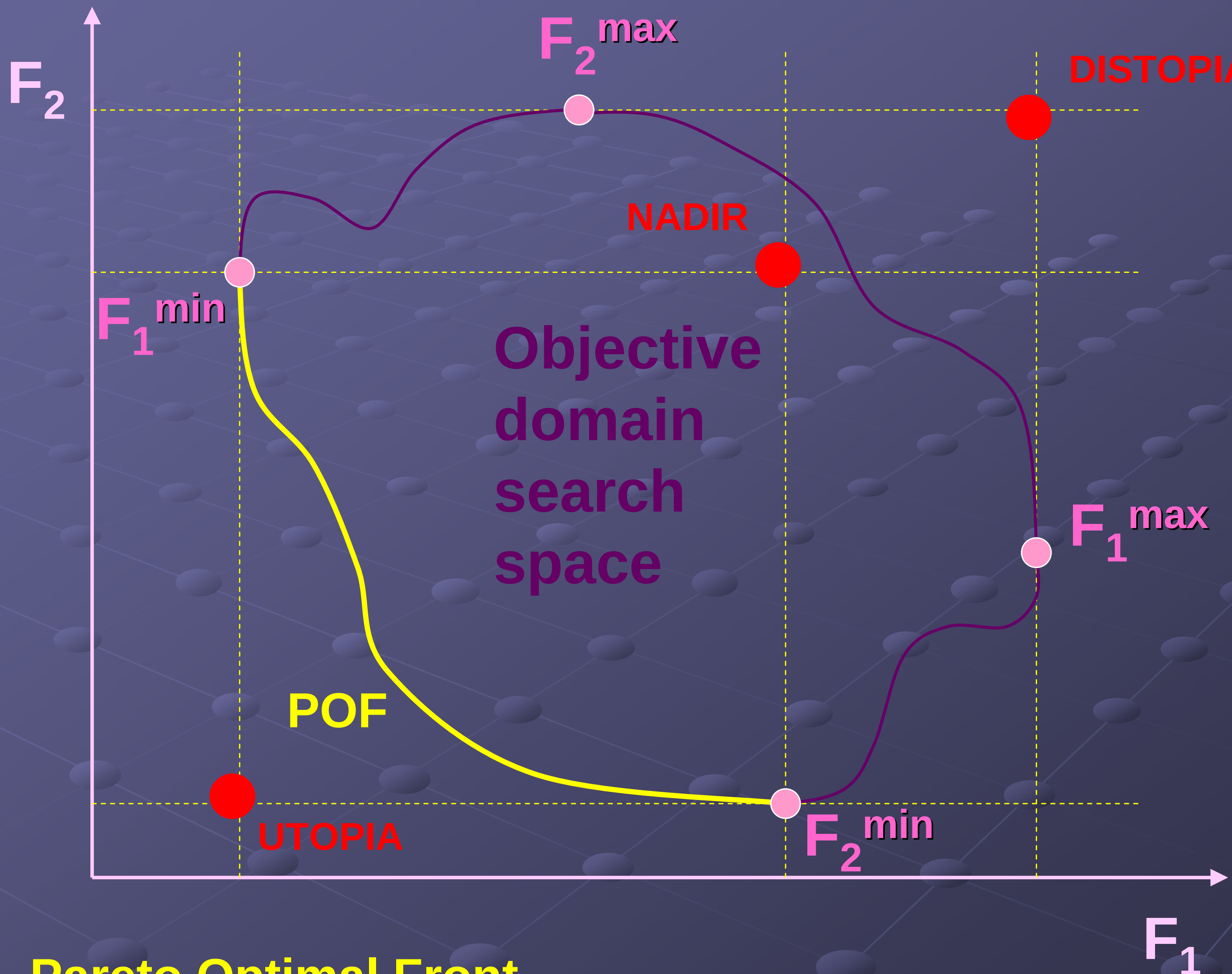
Unconstrained optimisation

Method	Optimum	N
ES/DE/MQ	1.58E-5	246
NF/GA/SQP	4.65E-5	155

Constrained optimisation

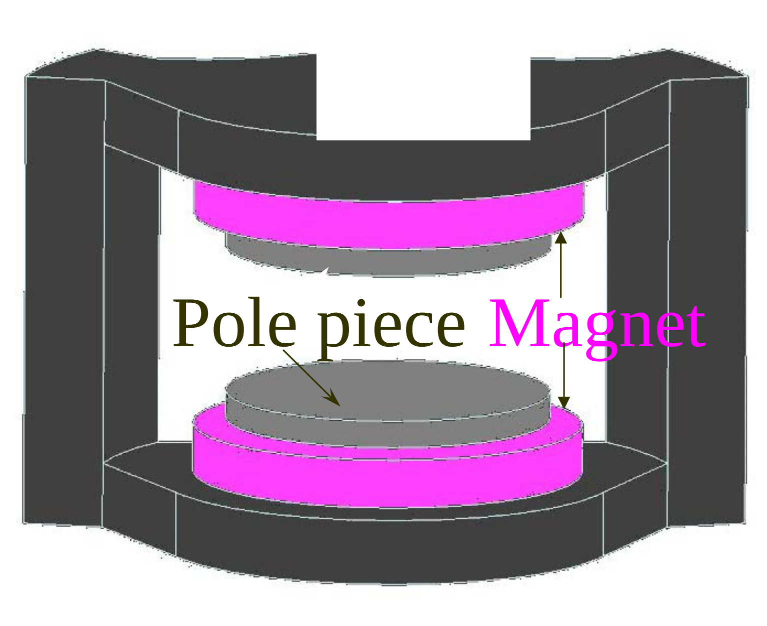
NF – Neuro-Fuzzy modelling
GA – Genetic Algorithm
SQP – Sequential Quadratic Programming

Pareto Optimisation



POF – Pareto Optimal Front

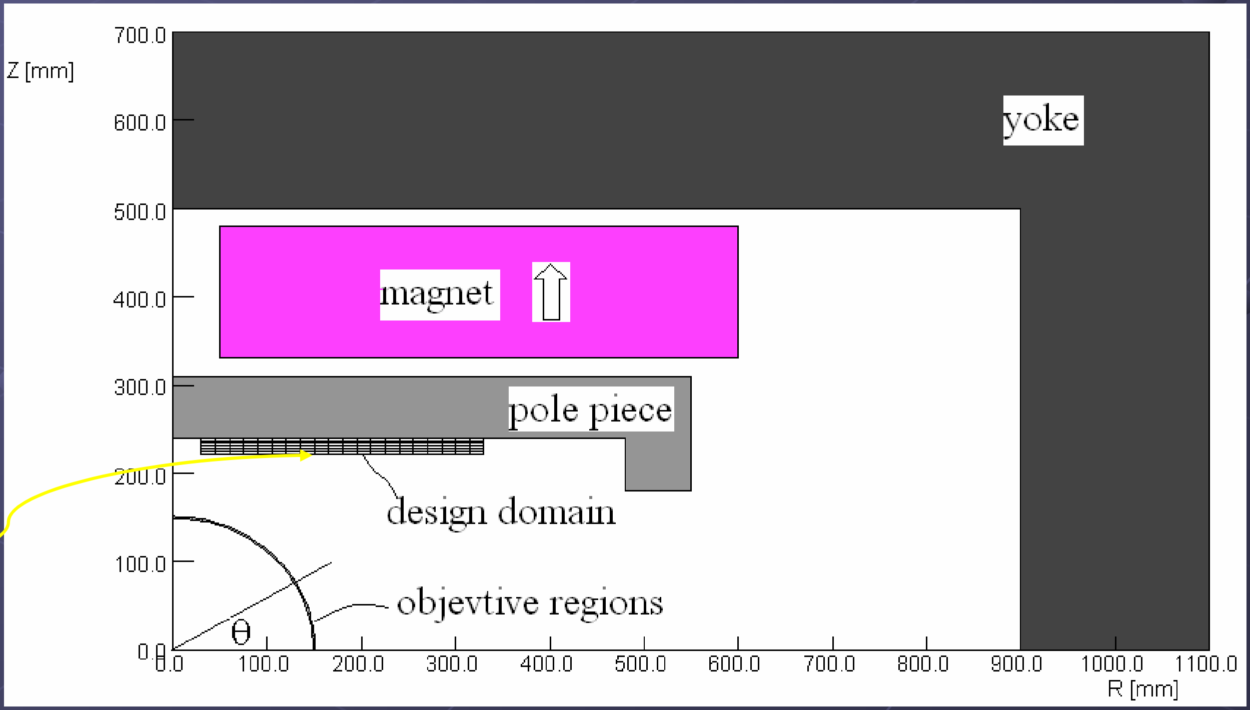
Optimized shimming magnet distribution of MRI system



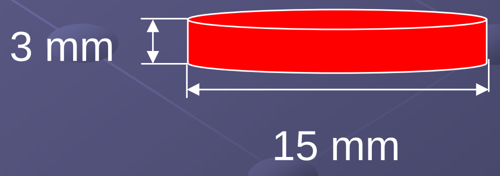
Objective function:

$$F = \sum_{i=1}^{45} (B_{zi} - B_{zo})^2, \quad \mathbf{M}(x,y) = \mathbf{M}_s(P)p^n$$

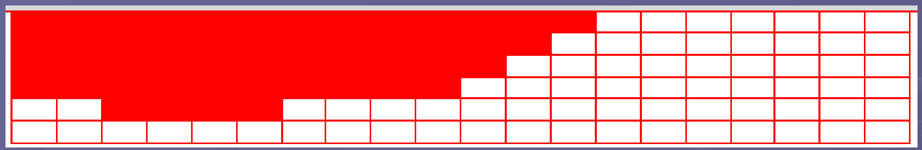
Simplified axi-symmetric model



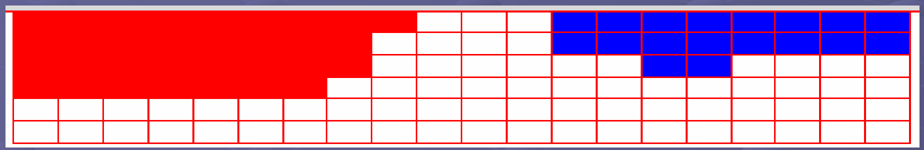
Shimming magnet (Br=0.222 T)



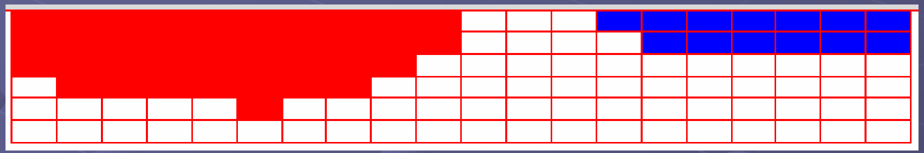
Changes of shimming magnet distribution during optimisation



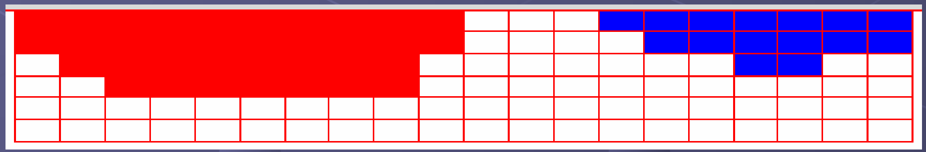
1 iteration



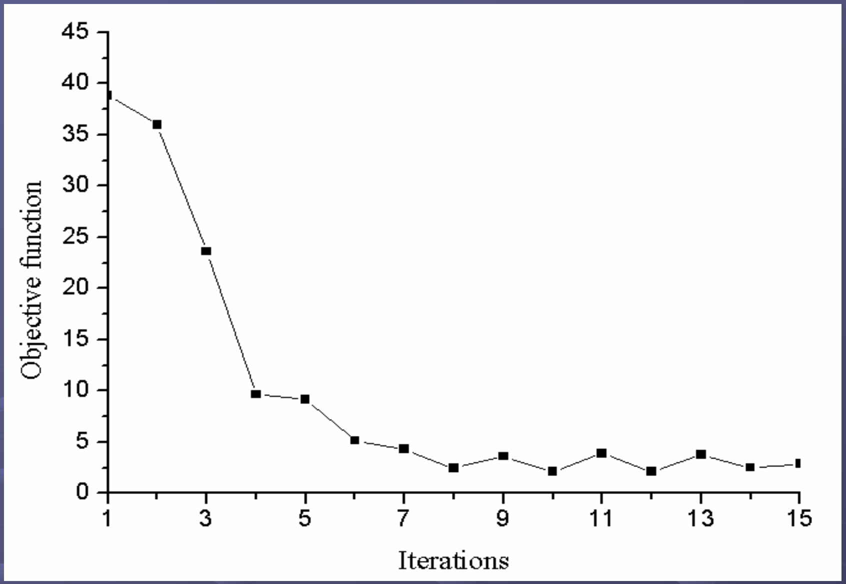
3 iterations



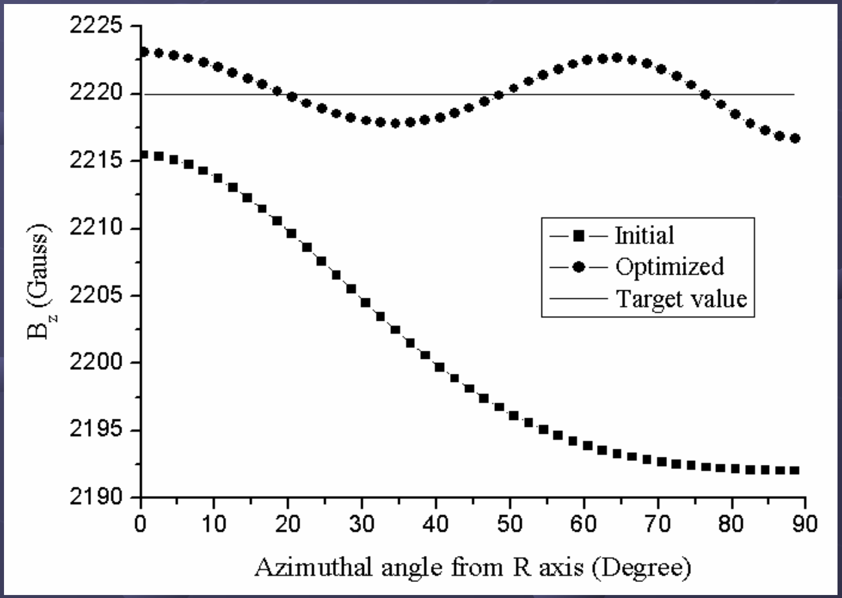
3 iterations



10 iterations



Convergence



Flux distributions

Applying Continuum Design Sensitivity Analysis to optimisation

- Features
  - No need for invalid material states between a void and a solid
  - No need for penalty terms in material parameterisation as well as in the augmented objective function

- Advantages
  - Easy implementation
  - Fast convergence

Calculation of forces

MST Maxwell Stress Tensor

Classical derivation starts from the Lorentz force expression describing the interactions of currents and magnetic fields, i.e. the J x B force.

VWP Virtual Work Principle

Based on the mechanical concepts of forces being related to the change in stored energy as a body changes its position in a magnetic field. In practice "virtual" displacement is simulated by a small physical displacement and the differential can be computed through a finite difference approach.

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Requires at least two solutions.

Implementation problems

MST Maxwell Stress Tensor

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Force computed by integrating flux density distributions over a surface enclosing the volume. As flux densities are discretised, they are not smooth – the value of the integral depends on where the contour is drawn.

Many variants of the basic MST exist.

VWP Virtual Work Principle

Based on on the mechanical concepts of forces being related to the change in stored energy as a body changes its position in a magnetic field. In practice “virtual” displacement is simulated by a small physical displacement and the differential can be computed through a finite difference approach.

Requires at least two solutions.

Coulomb’s approach: may be viewed as an implementation of discrete sensitivity approach in the finite element formulation.

Expression of force

$$\mathbf{F}_{iron} = \int_{\gamma} [(\upsilon_1 - \upsilon_2)\mathbf{B}_1 \cdot \mathbf{B}_2] V_n d\gamma - \frac{1}{2}\int_{\gamma} [\mathbf{H}_1 \cdot d\mathbf{B}_1 - \mathbf{H}_2 \cdot d\mathbf{B}_2] V_n d\gamma$$

Force due to magnetisation in the material

$$\mathbf{F}_{magnet} = \int_{\gamma} [(\mathbf{M}_2 - \mathbf{M}_1) \cdot \mathbf{B}_2] V_n d\gamma - \frac{1}{2}\int_{\gamma} [\mathbf{M}_2 \cdot d\mathbf{B}_2 - \mathbf{M}_1 \cdot d\mathbf{B}_1] V_n d\gamma$$

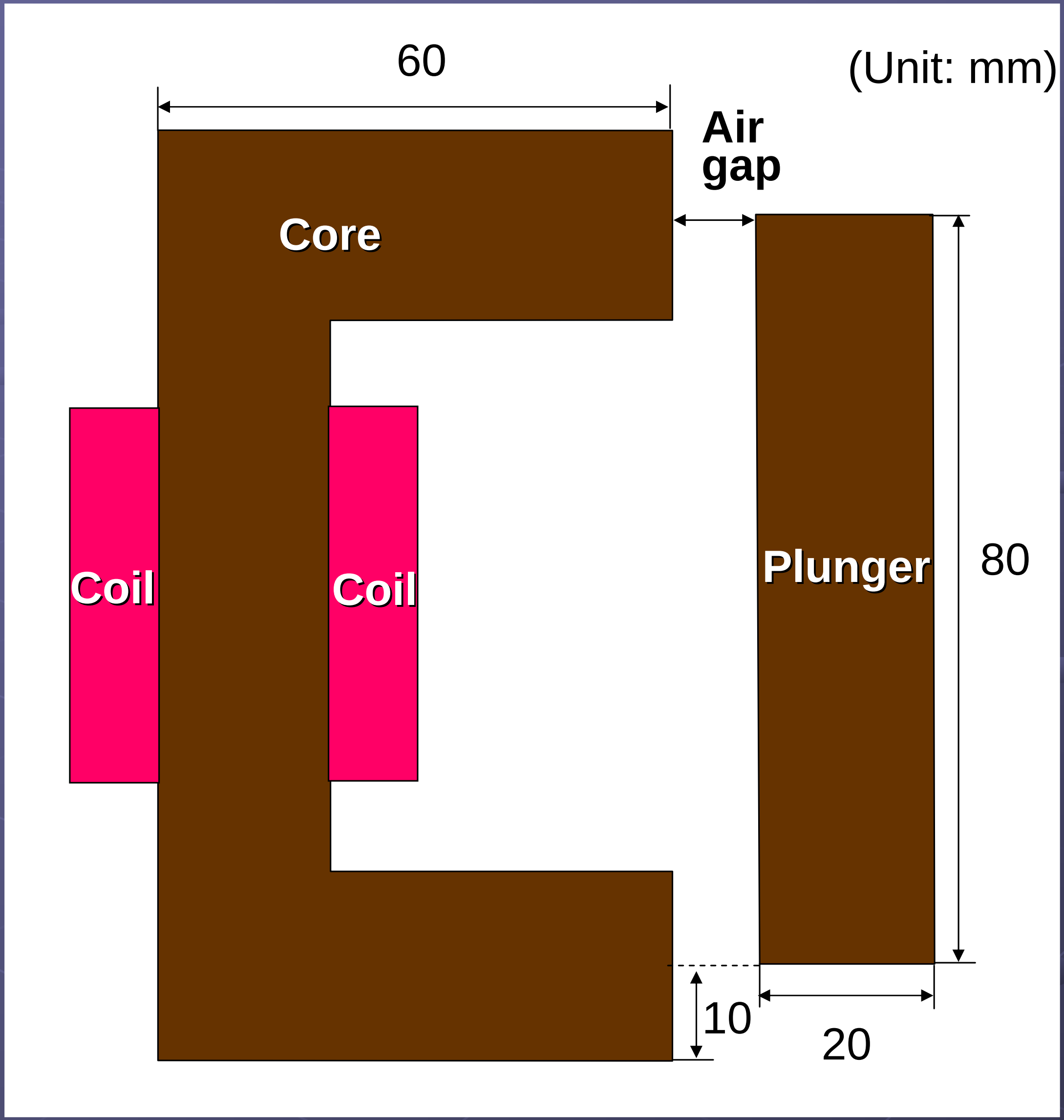
Force due to permanent magnet magnetisations

$$\mathbf{F}_{conductor} = \int_{\gamma} [(\mathbf{J}_2 - \mathbf{J}_1) \cdot \mathbf{A}_2] V_n d\gamma$$

Force due to currents on either side of interface

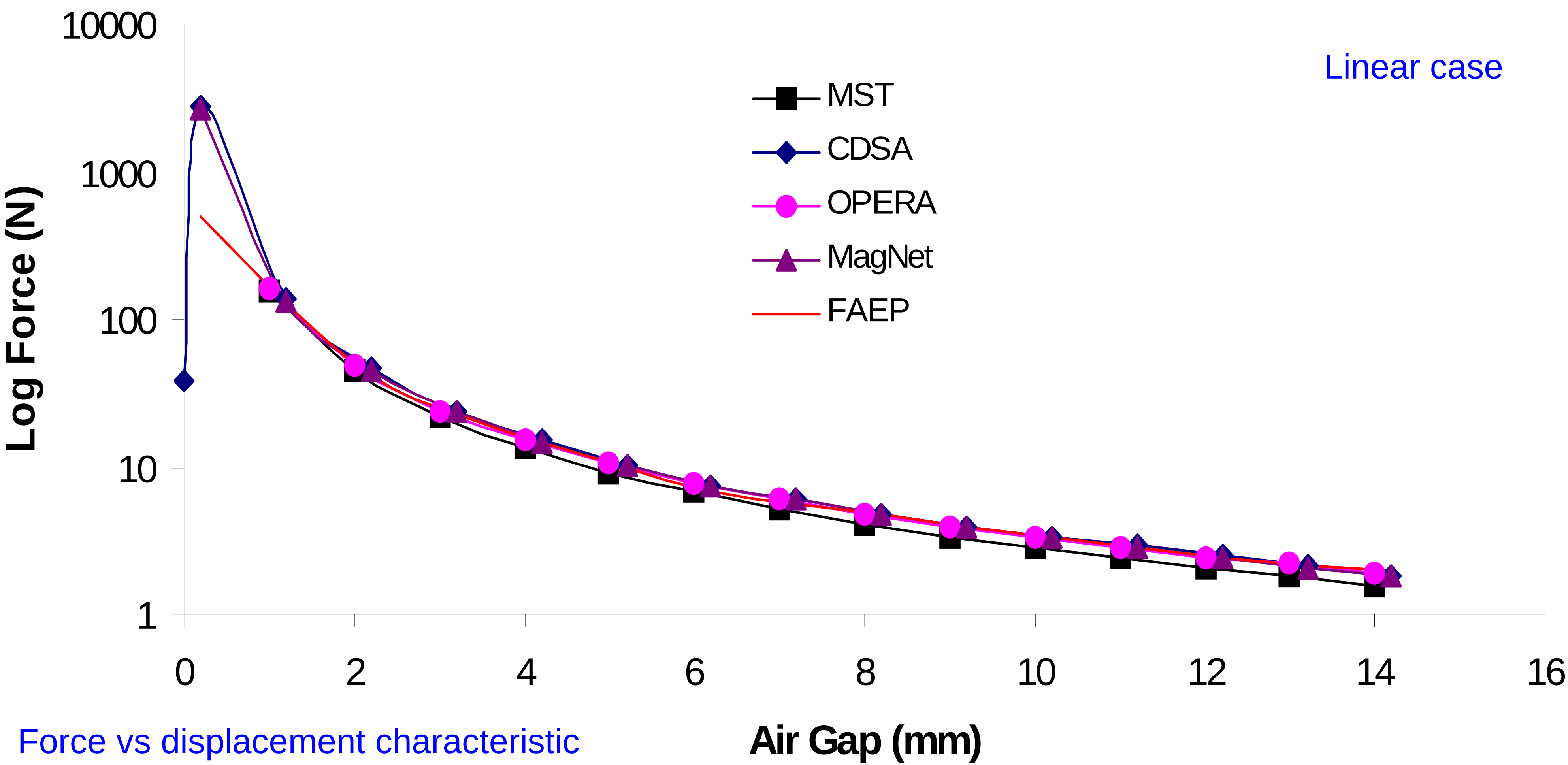
Example

C-core actuator



Example

C-core actuator

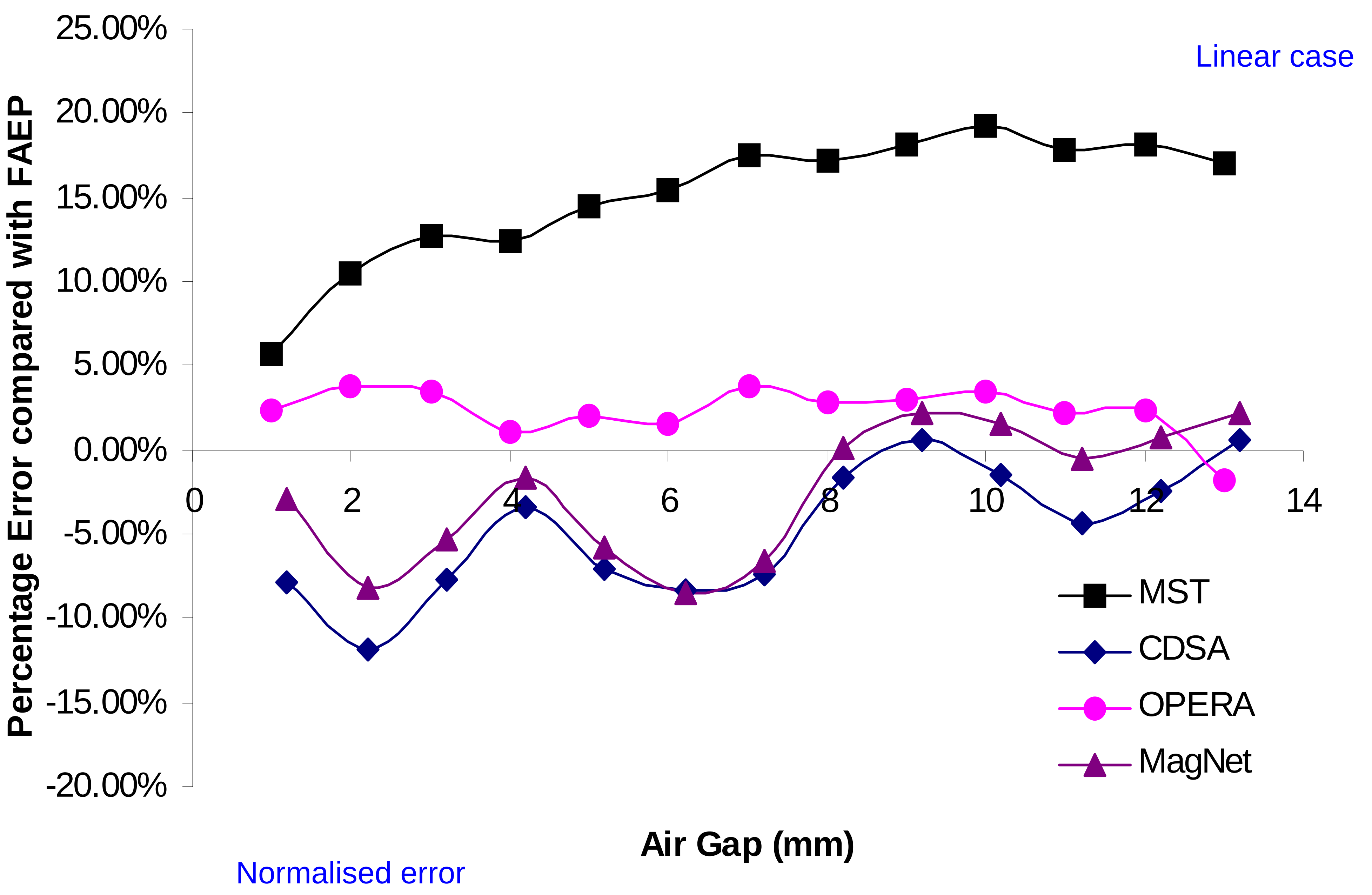


Force vs displacement characteristic

FAEP has been obtained by differentiating the co-energy vs air-gap curve and is used as a benchmark value

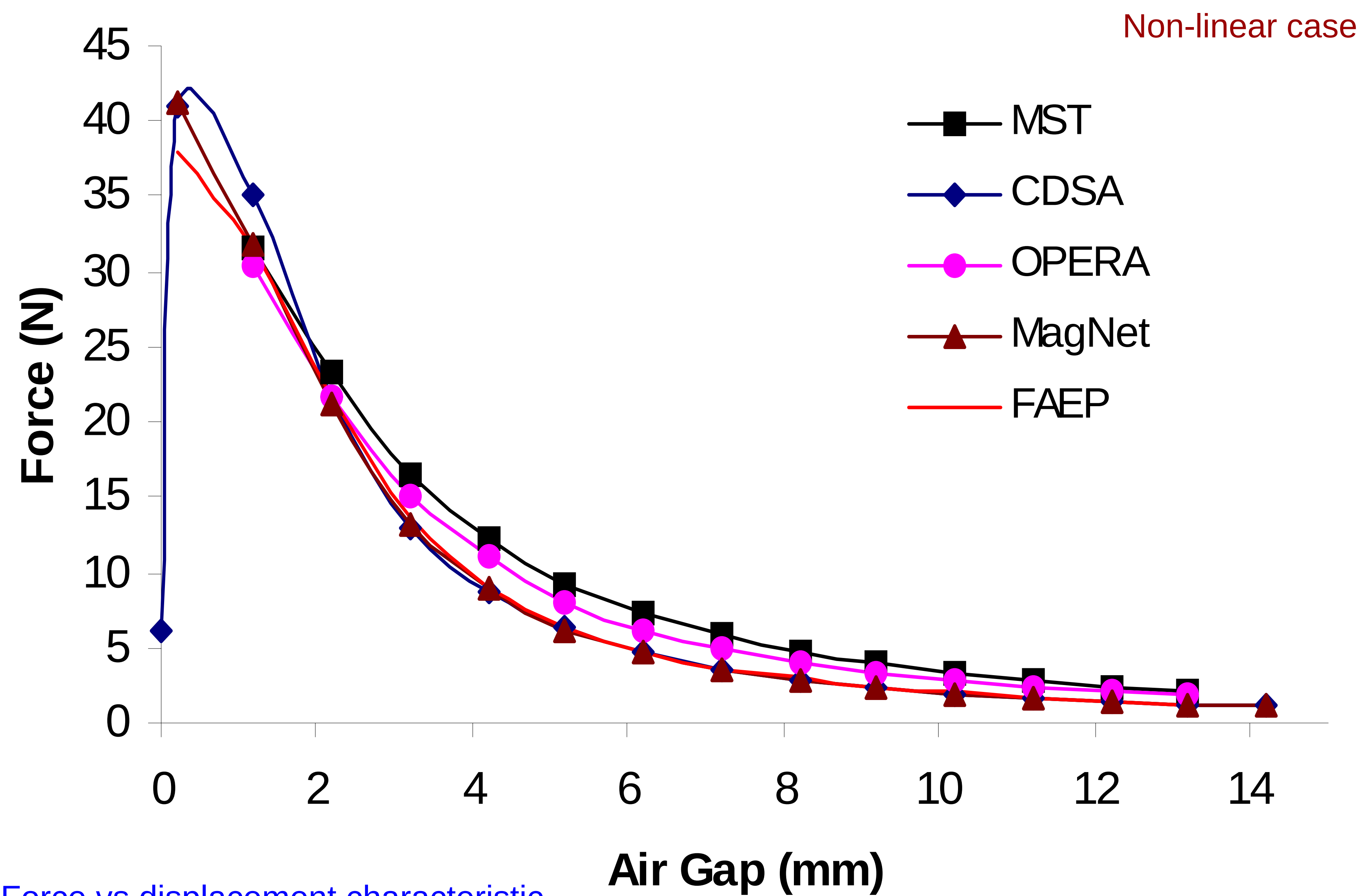
Example

C-core actuator



Example

C-core actuator



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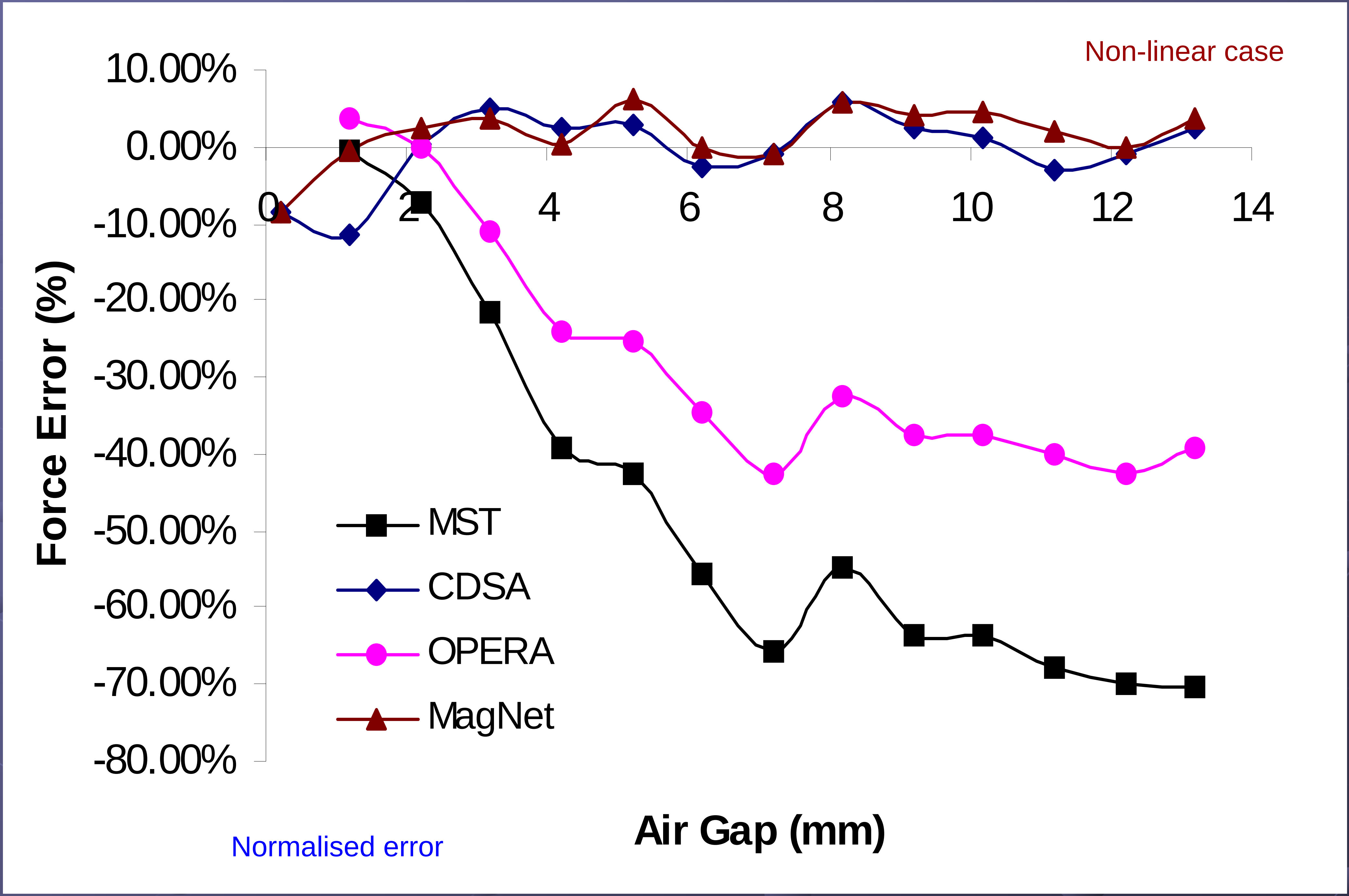
C-core actuator



Features

- Force computation algorithm
  based on continuum design sensitivity analysis
- Calculation of global forces
  as well as force distributions
- Clear indication of contributions
  from different sources of magnetic field
- Integration carried out on the surface of the body
- Implementation independent
  of the numerical analysis used
- Good agreement with other well tested methods

What drives the developments in Computational Electromagnetics?

Academia

- Intellectual curiosity
- Available research funding
- Urge to publish

Software developers

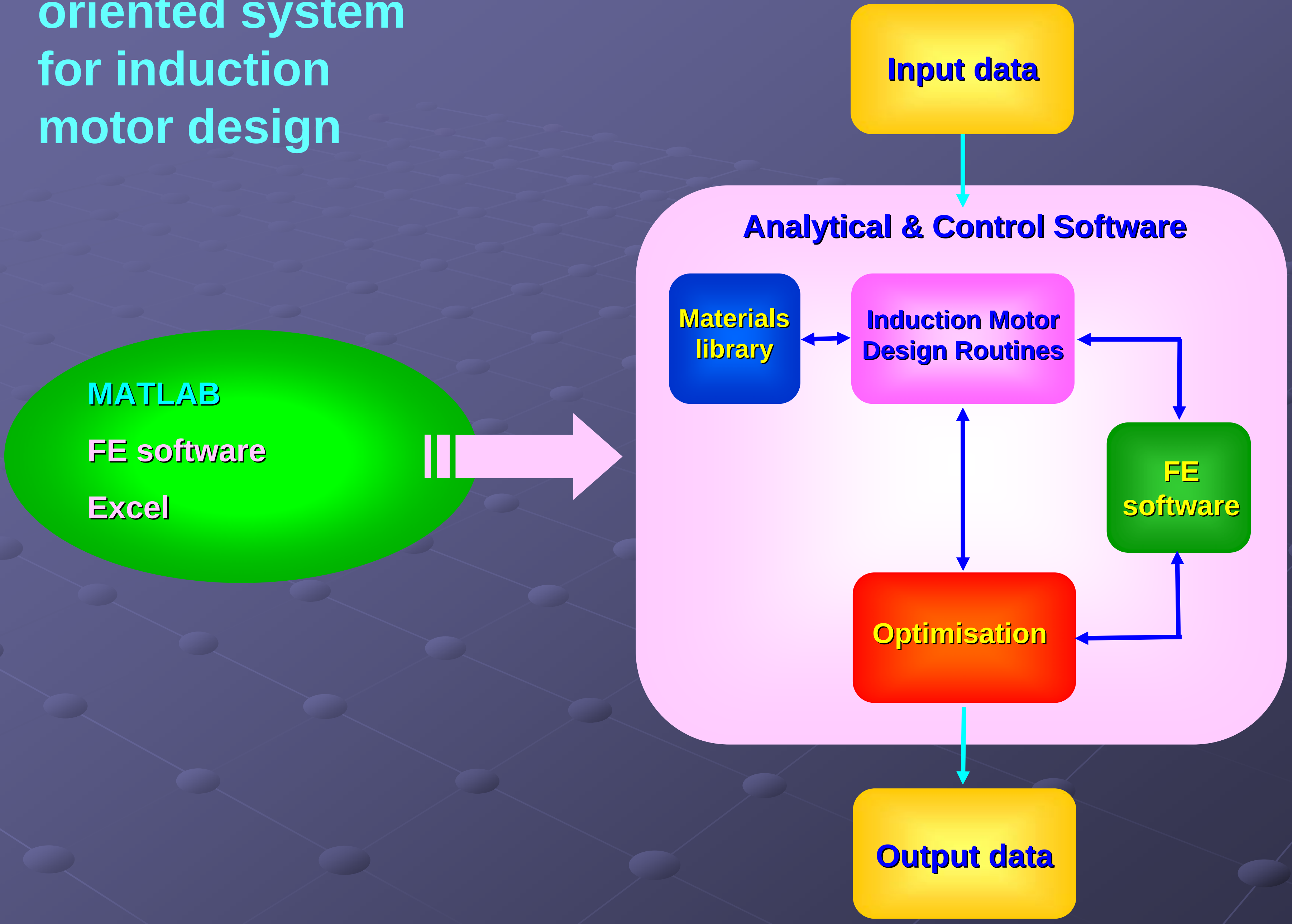
- Market
- R&D strategy
- Joint funding (e.g. Framework programmes)

Industry

- Growing need for reliable performance prediction
- Increasing role of virtual laboratories
- Needs of designers

The designer's view

Industrially oriented system for induction motor design



Hierarchical (three-layer) structure

- Approximate solutions
  (e.g. equivalent circuits, semi-empirical, design sheets)
- Extensive optimisation
- Large design space

- 2D finite element, static or steady-state
- Constrained optimisation, coupling
- Medium design space

- 3D finite-element, transient
- Fine tuning of the design
- Small design space

Knowledge base

Current and future developments

- adaptive meshing
- reliable error estimation
- high speed computing
- efficient handling of non-linearity and hysteresis
- modelling of new types of materials
- linear movement and rotation
- combined modelling of fields and circuits
- coupled problems (em + stress + temperature, etc)
- optimisation
- integrated design systems
- ...

⇒ Significant progress has been made

⇒ Tremendous effort continues

⇒ Much still needs to be done


Thank you