Reduced-Complexity Near-Optimum Genetic Algorithm Assisted Multiuser Detection for Synchronous Multicarrier CDMA

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Abstract—**In this contribution, a Genetic Algorithm (GA) assisted Multiuser Detector (MUD) designed for MC-CDMA is investigated in the context of frequency selective Rayleigh fading channels. The achievable BER performance of the GA assisted MUD as well as its near-far resistance are investigated for a range of parameter values. It is shown that the proposed GA assisted MUD is capable of significantly reducing the complexity in comparison to that of Verdu's optimum MUD. For example, when supporting $K = 20$ users, the number of likelihood function evaluations is reduced by a factor of 1300.**

I. INTRODUCTION

Multicarrier CDMA (MC-CDMA) [1–3] is a novel transmission technique, which combines DS-CDMA and Orthogonal Frequency Division Multiplexing (OFDM) [4–7]. In MC-CDMA systems, instead of applying spreading sequences in the time domain for spreading each bit, we employ spreading sequences in the frequency domain. Hence, we are capable of achieving frequency diversity gain at the cost of a reduced spreading gain. Numerous Multiuser Detection (MUD) schemes have been proposed in the literature [8–10]. The Minimum Mean Square Error (MMSE) MUD has been described for example in [3,8], while an Interference Cancellation (IC) based MUD has been proposed in [3,9].

In [11], the Maximum Likehood (ML) MUD designed for MC-CDMA has been considered. In this specific MUD, the receiver constructs all the possible combinations of the transmitted signals of all users and employs the estimated channel transfer function for generating all the possible received signals, in order to find the one, which has the smallest Euclidean distance from the received signal. Hence, the ML detection based MUD designed for MC-CDMA is capable of achieving a near-single-user performance. However, it requires the calculation of the 2^K number of possible received signal combinations of the K users supported in conjunction with Binary Phase Shift Keying (BPSK) modulation. In other words, the ML detection based MUD's complexity will increase exponentially with the number of users K. Hence the complexity imposed will become excessive, when the number of users K is high. Therefore, in this treatise we will invoke Genetic Algorithms (GA) [12, 13] for reducing the complexity of the ML detection based MUD employed in MC-CDMA systems. The GA-based MUD was first proposed by Juntti *et al.* [14] for a synchronous DS-CDMA system communicating over an Additive White Gaussian Noise (AWGN) channel. Yen *et al.* [3] [15–17] further improved the performance of the GA-based MUD, demonstrating that the performance of the GA-based MUD approaches the single-user performance bound at a significantly lower computational complexity, than that of Verdu's optimum MUD [18]. In our investigations we assume that each subcarrier obeys independent Rayleigh fading. More explicitly, we will investigate the performance of this specific GA assisted MUD as a function of the affordable detection complexity.

Acknowledgements: The work reported in this paper has formed part of the Wireless Enabling Techniques work area of the Core 2 Research Programme of the Virtual Centre of Excellence in Mobile and Personal Communications, Mobile VCE, www.mobilevce.com, whose funding support, including that of EPSRC, is gratefully acknowledged. Fully detailed technical reports on this research are available to Industrial Members of Mobile VCE.

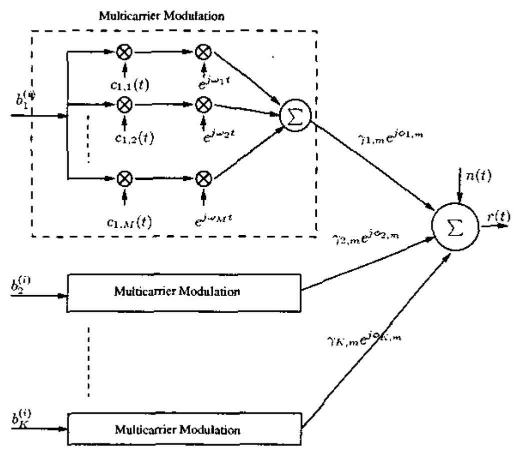

Fig. 1. The transmitter of a MC-CDMA system

This contribution is organized as follows. Section II describes the model of the synchronous MC-CDMA system considered. Section III highlights the operating principle of the GA assisted MUD designed for a synchronous MC-CDMA system, while Section IV characterises the achievable performance. Finally, Section V offers our conclusions.

II. TRANSMITTER

Let us consider the bit-synchronous MC-CDMA system illustrated in Figure 1, which employs *both time- and frequency-domain spreading*. More explicitly, observe in Figure 1 that the ith bit $b_k^{(i)}$ of the kth user is spread in the frequency-domain to M parallel subcarriers, each conveying an N-chip time-domain spreading sequence $c_{k,m}(t), \quad m = 1, \ldots, M$ spanning the bit-duration $(0, T_b)$. Hence we have $T_b/T_c = N$, where T_b and T_c are the bit duration and chip duration, respectively. Each of the M N-chip time-domain spreading sequences is mapped to a different subcarrier. In other words, a single-carrier system occupying the same bandwidth as the multicarrier system considered would use a spreading sequence having NM chips/bit, and both of these systems have a processing gain of NM. Hence, the transmitted signal of the kth user associated with the mth subcarrier can be expressed in an equivalent lowpass representation as:

$$s_{k,m}(t) = \sqrt{\frac{2E_{bk}}{M}} c_{k,m}(t) b_k^{(i)} e^{jw_m t}, \quad (1)$$

where E_{bk} is the kth user's signal energy per bit. Furthermore, $b_k^{(i)} \in (1, -1)$, $k = 1, \ldots, K$ denotes the ith transmitted bit of the kth user, while the kth user's signature waveform is $c_{k,m}(t)$, $k = 1, \ldots, K$, $m = 1, \ldots, M$ that is assigned to the mth subcarrier, which again has a length of N chips, yielding:

$$c_{k,m}(t) = \sum_{n=0}^{N-1} c_{k,m}^{(n)} p(t - nT_c), \quad m = 1, \ldots, M, \quad (2)$$

where T_c is the chip duration, N is the number of chips per bit associated with each subcarrier and we have $T_b/T_c = N$. Again, the total processing gain is NM, while $p(t)$ is the rectangular chip waveform employed, which can be expressed as:

$$p(t) = \begin{cases} 1 & 0 \leq t < T_c \\ 0 & otherwise. \end{cases} \quad (3)$$

Without loss of generality, we assume that the signature waveform $c_{k,m}(t)$ used for spreading the bits to N chips in the time-domain and mapping them to a total of M subcarriers in the frequency-domain for all the K users has unity energy, which can be written as:

$$\int_0^{T_b} c_{k,m}^2(t) dt = 1 \quad k = 1, \ldots, K, \; m = 1, \ldots, M. \quad (4)$$

Each user's signal $s_{k,m}(t)$ transmitted on the mth subcarrier is assumed to propagate over an independent non-dispersive single-path Rayleigh fading channel and the fading envelope of each path is statistically independent for all the users. Hence, the single-tap narrowband Channel Impulse Response (CIR) of the kth user on the mth subcarrier can be expressed as: $\gamma_{k,m} e^{j\phi_{k,m}}$, where the amplitude $\gamma_{k,m}$ is a Rayleigh distributed random variable, while the phase $\phi_{k,m}$ is uniformly distributed between $[0, 2\pi)$.

Having described the transmitter and the channel, the received signal on the mth subcarrier can be expressed as:

$$r_m(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} \sqrt{\frac{2E_{bk}}{M}} c_{k,m}(t - iT_b) \gamma_{k,m} b_k^{(i)} e^{(jw_m t + \phi_{k,m})} + n(t), \quad (5)$$

where K is the number of users supported and $n(t)$ is the Gaussian noise process with a variance of $N_0/2$.

III. GA ASSISTED MUD FOR MC-CDMA

Figure 2 shows the schematic of the GA assisted MUD employed in a synchronous MC-CDMA system. The first step of the receiver's operation is the demodulation of all the subcarrier signals. This is followed by Matched Filtering (MF) for each of the K users and the outputs of the K users' matched filters are input to the GA-based MUD. It is more convenient to express the associated signals in matrix and vectorial formats, when the sum of the transmitted signals of all users can be expressed as:

$$r_m(t) = \mathbf{C_m W_m A b + n}, \quad (6)$$

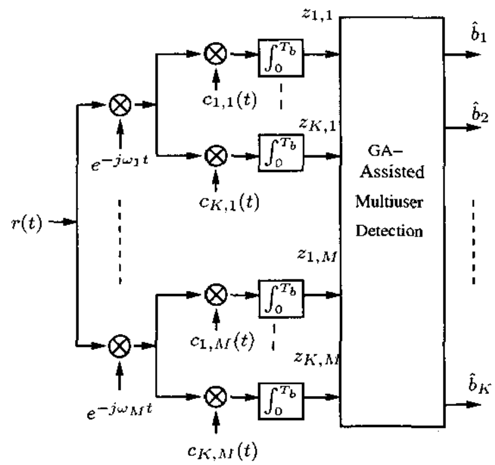

Fig. 2. Schematic of the GA assisted MUD aided MC-CDMA base station receiver

where we have

$$\begin{aligned} \mathbf{C_m} &= [\mathbf{c_{1,m}(t)}, \ldots, \mathbf{c_{K,m}(t)}] \\ \mathbf{W_m} &= \text{diag}[\gamma_{\mathbf{1,m}} \mathbf{e^{j\phi 1,m}}, \ldots, \gamma_{\mathbf{K,m}} \mathbf{e^{j\phi K,m}}] \\ \mathbf{A} &= \text{diag}[\sqrt{\tfrac{\mathbf{2E_{b1}}}{\mathbf{M}}}, \ldots, \sqrt{\tfrac{\mathbf{2E_{bK}}}{\mathbf{M}}}] \\ \mathbf{b} &= [\mathbf{b_1}, \ldots, \mathbf{b_K}]^{\mathbf{T}} \\ \mathbf{n} &= [\mathbf{n_1}, \ldots, \mathbf{n_K}]^{\mathbf{T}}. \end{aligned} \quad (7)$$

Based on Equation 6, the output vector $\mathbf{Z_m}$ of the bank of matched filters portrayed in Figure 2 can be formulated as [18]:

$$\begin{aligned} \mathbf{Z_m} &= [z_{1,m}, \ldots, z_{K,m}] \\ &= \mathbf{R_m W_m A b + n} \end{aligned} \quad (8)$$

where we have

$$\mathbf{R_m} = \begin{bmatrix} \rho_{11}^{(m)} & \rho_{12}^{(m)} & \cdots & \rho_{1K}^{(m)} \\ \rho_{21}^{(m)} & \rho_{22}^{(m)} & \cdots & \rho_{2K}^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{K1}^{(m)} & \rho_{K2}^{(m)} & \cdots & \rho_{KK}^{(m)} \end{bmatrix}, \quad (9)$$

and the elements $\rho_{jk}^{(m)}$ of the matrix $\mathbf{R_m}$ are the auto- and cross-correlation of the spreading code, which can be expressed as:

$$\rho_{jk}^{(m)} = \int_0^{T_b} c_{j,m}(t) c_{k,m}(t) dt. \quad (10)$$

According to [15] [18], the optimum multiuser detector of the mth subcarrier will maximize the following objective function:

$$\Omega_m(\mathbf{b}) = 2\text{Re}[\mathbf{b^T A W_m^* Z_m}] - \mathbf{b^T A W_m R_m W_m^* A b}, \quad (11)$$

where the superscript $*$ indicates the conjugate complex version of a matrix. Therefore, combining the contributions of a total of M parallel subcarriers, the objective function to be maximized in the context of an optimum multiuser detected MC-CDMA system can be ex-

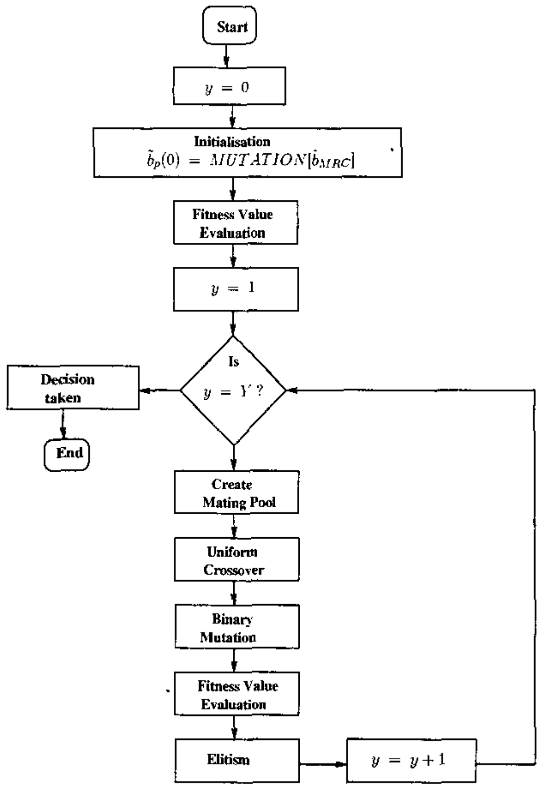

Fig. 3. Flowchart of a genetic algorithm assisted MUD in the context of the synchronous MC-CDMA base station receiver

pressed as:

$$\begin{aligned}\Omega(\mathbf{b}) &= \sum_{m=1}^{M} \Omega_m(\mathbf{b}) \\ &= \sum_{m=1}^{M} \{2\text{Re}[\mathbf{b}^T \mathbf{A} \mathbf{W}_m^* \mathbf{Z}_m] - \\ &\quad \mathbf{b}^T \mathbf{A} \mathbf{W}_m \mathbf{R}_m \mathbf{W}_m^* \mathbf{A} \mathbf{b}\}. \end{aligned} \tag{12}$$

Hence the decision rule for Verdu's optimum CDMA multiuser detection scheme based on the maximum likelihood criterion is to choose the specific K-user bit combination $\mathbf{b}$, which maximizes the metric of Equation 12. Hence, we have to find:

$$\hat{\mathbf{b}} = \arg \left\{ \max_{\mathbf{b}} [\Omega(\mathbf{b})] \right\}. \tag{13}$$

The maximization of Equation 12 is a combinatorial optimisation problem, which requires an exhaustive search for each of the $J = 2^K$ combinations of $\mathbf{b}$, in order to find the one that maximizes the metric of Equation 12. Explicitly, since in case of binary transmissions there are $J = 2^K$ possible combinations of $\mathbf{b}$, the optimum multiuser detector has a complexity that increases exponentially with the number of users K.

Hence, we invoked a GA for finding a solution near to the maximum of the objective function defined by the metric of Equation 12 without an exhaustive search. Again, the legitimate solutions are the

Parameters	Value
Modulation scheme	BPSK
Spreading code	WALSH
Number of subcarriers M	4
$N(T_b/T_c)$	8
Processing gain MN	32
GA's selection method	Fitness-proportionate
GA's mutation method	Binary mutation
GA's crossover method	Uniform crossover
GA's mutation probability p_m	0.1
GA's crossover probability p_c	1
Mating pool size T	5
Elitism	Yes
Incest Prevention	Yes

TABLE I

THE BASIC SIMULATION PARAMETERS USED BY THE GA ASSISTED MUD AIDED MC-CDMA SYSTEM

$J = 2^K$ number of possible combinations of the K-bit vector $\mathbf{b}$. During the GA's operation, each individual of the GA [15] will take the form of a K-bit vector corresponding to the K users' transmitted bits during a single bit interval, which can be denoted for the pth individual of the GA as $\tilde{\mathbf{b}}_p(y) = [\tilde{b}_{p,1}(y), \ldots, \tilde{b}_{p,K}(y)]$, where $y,\ y = 1, \ldots, Y$ denotes the yth generation, and $p,\ p = 1, \ldots, P$ denotes the pth individual in the GA's mating pool.

We create the initial biased population with the aid of the Maximum Ratio Combining (MRC) based matched filter outputs, which are subjected to hard decision, rather than randomly generating the initial population at the commencement of a GA assisted search. Explicitly, according to [1], the MRC-combined output vector $\hat{\mathbf{b}}_{MRC}$ of the matched filter output can be expressed as: $\hat{\mathbf{b}}_{MRC} = [\hat{b}_{1,MRC}, \ldots, \hat{b}_{K,MRC}]$, where we have:

$$\hat{b}_{k,MRC} = \sum_{m=1}^{M} z_{k,m} \gamma_{k,m} e^{-j\phi_{k,m}}. \tag{14}$$

Having generated $\hat{\mathbf{b}}_{MRC}$, we adopt a 'mutated' version of the hard decision vector $\hat{\mathbf{b}}_{MRC}$ for creating each individual in the initial population, where each bit of the vector $\hat{\mathbf{b}}_{MRC}$ is toggled according to the mutation probability used. Hence, the first individual of the population, namely $\tilde{\mathbf{b}}_p(0)$, can be written as:

$$\tilde{\mathbf{b}}_p(0) = \text{MUTATION}[\hat{\mathbf{b}}_{MRC}]. \tag{15}$$

To elaborate a little further, Figure 3 shows the flowchart of the GA assisted MUD, which follows the philosophy of [3, 16]. We will characterise the performance of the GA assisted MUD in the following section.

IV. PERFORMANCE OF GA ASSISTED MUD AIDED SYNCHRONOUS MC-CDMA

The basic parameters of the GA used in our simulations are listed in Table I. Let us first relate the achievable performance of the proposed GA-aided MUD to that of the well-understood $\mathcal{M}$-algorithm (MA) in Figure 4, when communicating over a non-dispersive Gaussian channel. More specifically, the system studied employed a random spreading code having a length of $N = 31$ in the context of

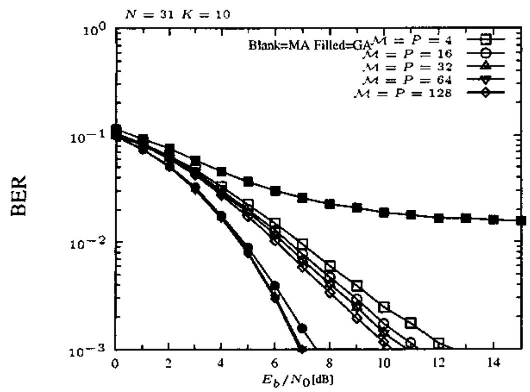

Fig. 4. BER performance of the GA and MA assisted MUDs for transmission over AWGN channel in conjunction with different detection-complexity configurations of $P \cdot Y = \mathcal{M} \cdot K$, when supporting $K = 10$ users. All the other parameters are described in Table I.

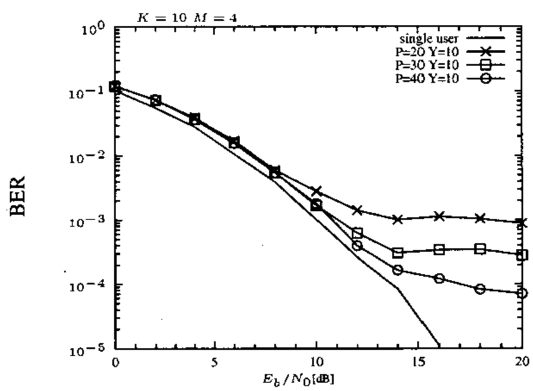

Fig. 5. BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 32-chip Walsh code. The number of users supported was $K = 10$. The number of generations was $Y = 10$ and the population size was $P = 20$, 30 and 40. The remaining parameters are specified in Table I. The number of subcarriers was $M = 4$ and each subcarrier experienced uncorrelated narrowband Rayleigh fading. The complexity reduction factor was $2^K/(P \cdot Y)$.

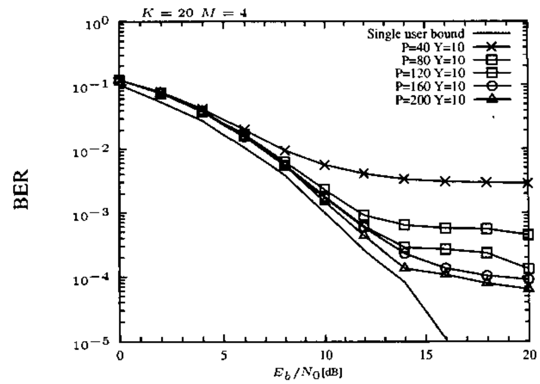

Fig. 6. BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 32-chip Walsh code. The number of users supported was $K = 20$. The number of generations was $Y = 10$ and the population size was $P = 40,\ 80,\ 120,\ 160$ and 200. The remaining parameters are specified in Table I. The number of subcarriers was $M = 4$ and each subcarrier experienced uncorrelated narrowband Rayleigh fading. The complexity reduction factor was $2^K/(P \cdot Y)$.

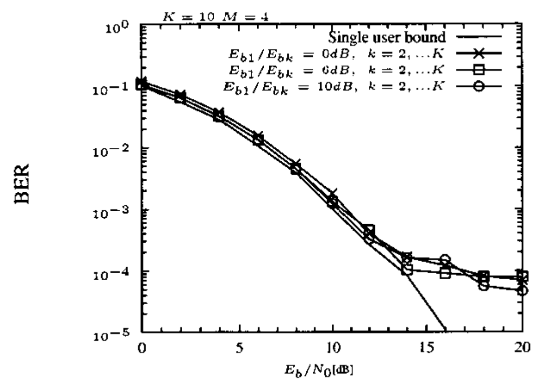

Fig. 7. BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system when the power of the interfering users was varied . The number of users supported was $K = 10$. The number of generations was $Y = 10$ and the population size was $P = 40$. The ratio of the reference user to interfering user power was $\frac{E_{b1}}{E_{bk}} = 0,\ 6,\ 10$dB, $k = 2, \ldots, K$, respectively. The remaining parameters are specified in Table I. The number of subcarriers was $M = 4$ and each subcarrier experienced uncorrelated narrowband Rayleigh fading. The complexity reduction factor was $2^K/(P \cdot Y)$.

BPSK modulation. For the sake of comparison, the GA-aided and the $\mathcal{M}$-algorithm systems concerned were configured for maintaining a similar complexity. Specifically, the GA-aided MUD evaluates its objective function seen in Equation 11 $(P \cdot Y)$ number of times. By contrast, it can be shown that the MA-based MUD requires $(K \cdot \mathcal{M})$ number of objective function evaluations, although we note that its objective function is different from that of the GA and was not explicitly included here for reasons of space economy. As seen in Figure 4 for the specific scenario of $K = Y$, the GA-aided MUD outperforms the $\mathcal{M}$-algorithm, when we have $\mathcal{M} = \mathcal{P} \leq \infty$,which justifies our GA-based investigations.

From Figures 5 and 6 we observe that the GA assisted MUD's performance improves, when the population size P increases. For example, for E_b/N_0 values below 14dB a near-single-user performance can be achieved for $K = 10$ users, when evaluating the objective function of Equation 12, which imposes a complexity on the order of $O(P \cdot Y) = O(40 \cdot 10) = O(400)$, as seen in Fig 5. Furthermore, when the number of users K is increased to 20, the GA assisted MUD has a complexity of $O(P \cdot Y) = O(160 \cdot 10) = O(1600)$, as seen in Fig 6. We can see in Figure 7 that the GA assisted MUD is also near-far resistant, provided that we perfectly know the channel parameters. More explicitly, the GA assisted MUD exhibits a high robustness against power control errors. Figure 8 shows the BER performance as a function of the number of users K. We can infer from Figure 8 that the GA assisted MUD required a population population size P in excess of 80 for achieving a near-single-user performance, when the number of users K is higher than 14. We can observe in Figure 9 that GA assisted MUD is capable of significantly reducing the complexity of Verdu's optimum MUD. For example, the complexity was reduced by a factor of 1300, when the number of users was

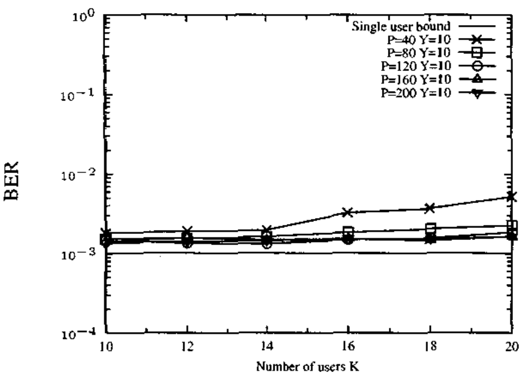

Fig. 8. BER performance of the GA assisted MUD as a function of the number of users K for the population sizes of $P = 40, 80, 120, 160, 200$, and for $Y = 10$ generations. We had $E_b/N_0 = 10$dB. The remaining parameters are specified in Table I. The number of subcarriers was $M = 4$ and each subcarrier experienced uncorrelated narrowband Rayleigh fading. The complexity reduction factor was $2^K/(P \cdot Y)$.

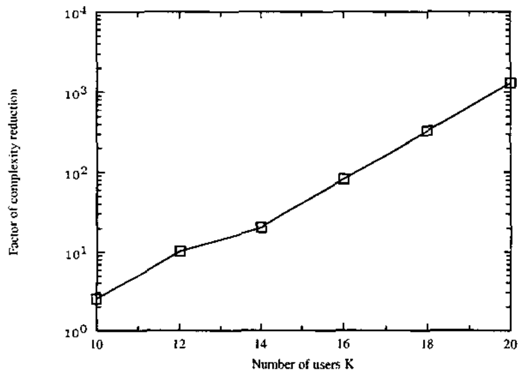

Fig. 9. The complexity reduction factor of $\frac{2^K}{P \times Y}$ was defined as the ratio of the number of objective function computations required for approaching the single-user bound at a BER of 10^{-3}, when communicating over a **synchronous** environment, where P is the population size, and Y is the number of generations, while K is the number of users supported. The remaining parameters are specified in Table I.

$K = 20$.

V. CONCLUSIONS AND FUTURE WORK

In conclusion, the GA assisted MUD is capable of significantly reducing the detection complexity in comparison to Verdu's optimum MUD, especially when the number of users supported is higher than $K = 15$. When channel coding techniques are employed, the GA assisted MUD has the potential of further reducing the complexity imposed. Our future work will comparatively study the performance versus complexity trade-offs of GA versus M-Algorithm [19] based MUDs. Another important area of further study is the employment of multilevel modulation schemes both with and without various trellis coded error protection schemes.

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