TURBO DETECTION OF CHANNEL-CODED SPACE-TIME SIGNALS USING SPHERE PACKING MODULATION

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Abstract - A recently proposed space-time signal construction method that combines orthogonal design with sphere packing, referred to here as (STBC-SP), has shown useful performance improvements over Alamouti's conventional orthogonal design. In recent years, iterative decoding algorithms have attained substantial performance improvements in the context of wireless communication systems. In this paper, we demonstrate that the performance of STBC-SP systems can be further improved by concatenating sphere packing aided modulation with channel coding and performing demapping as well as channel decoding iteratively. The sphere packing demapper is modified for the sake of accepting *a priori* information that is obtained from the channel decoder. Bit-wise mutual information measures were also employed for the sake of searching for the optimum bits-to-symbol mapping. We present simulation results for the proposed scheme communicating over a correlated Rayleigh fading channel. At a BER of 10^{-5}, the proposed turbo-detected STBC-SP scheme employing the optimum mapping was capable of achieving a coding gain of approximately $19dB$ over the identical-throughput 1 bit/symbol uncoded STBC-SP benchmarker scheme. The proposed scheme also achieved a coding gain of approximately $2dB$ over the 1 bit/symbol channel-coded STBC-SP benchmarker scheme that employed Gray mapping.

1. INTRODUCTION

The adverse effects of channel fading may be significantly reduced by employing space-time coding invoking multiple antennas [1]. Alamouti [2] discovered an appealingly simple transmit diversity scheme employing two transmit antennas. This low-complexity design inspired Tarokh *et al.* [3, 4] to generalise Alamouti's transmit diversity scheme using the principle of orthogonal design to an arbitrary number of transmit antennas. Since then, the pursuit of designing better space-time modulation schemes has attracted considerable further attention [2]. The concept of combining orthogonal transmit diversity designs with the principle of sphere packing was introduced by Su *et al.* in [5]. Orthogonal transmit diversity designs can be described recursively [6] as follows. Let $G_1(x_1) = x_1 I_1$, and

$$G_{2^k}(x_1, \ldots, x_{k+1}) = \begin{bmatrix} G_{2^{k-1}}(x_1, \ldots, x_k) & x_{k+1} I_{2^{k-1}} \\ -x_{k+1}^* I_{2^{k-1}} & G_{2^{k-1}}^{\mathcal{H}}(x_1, \ldots, x_k) \end{bmatrix},$$

for $k = 1, 2, 3, \ldots$,where x_{k+1}^* is the complex conjugate of x_{k+1}, $G_{2^{k-1}}^{\mathcal{H}}(x_1, \ldots, x_k)$ is the Hermitian of $G_{2^{k-1}}(x_1, \ldots, x_k)$ and $I_{2^{k-1}}$ is a $(2^{k-1} \times 2^{k-1})$ identity matrix. Then, $G_{2^k}(x_1, x_2, \ldots, x_{k+1})$ constitutes an orthogonal design of size $(2^k \times 2^k)$, which maps the complex variables representing $(x_1, x_2, \ldots, x_{k+1})$ to 2^k transmit antennas. In other words, $x_1, x_2, \ldots, x_{k+1}$ represent $k+1$ complex modulated symbols to be transmitted from 2^k transmit antennas in $T = 2^k$ time slots. It was shown in [5] that the diversity product (coding advantage) of an orthogonal transmit diversity scheme is determined by the minimum Euclidean distance of the vectors $(x_1, x_2, \ldots, x_{k+1})$. Therefore, in order to maximise the achievable coding advantage, it was proposed in [5] to use sphere packing schemes that have the best known minimum Euclidean distance in the $2(k+1)$-dimensional real-valued Euclidean space $R^{2(k+1)}$ [7]. The results of [5] demonstrated that the proposed Sphere Packing (SP) aided Space-Time Block Coded (STBC) system of Section 2, referred to here as STBC-SP, was capable of outperforming the conventional orthogonal design based STBC schemes of [2, 3].

Iterative decoding of spectrally efficient modulation schemes was considered by several authors [1, 8–10]. In [11], the employment of the turbo principle was considered for iterative soft demapping in the context of multilevel modulation schemes combined with channel decoding, where a soft demapper was used between the

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multilevel demodulator and the channel decoder.

Motivated by the performance improvements reported in [5] and [11], we propose a novel system that exploits the advantages of both iterative demapping and decoding as well as those of the STBC-SP scheme of [5]. The STBC-SP demapper of [11] was modified for the sake of accepting the *a priori* information passed to it from the channel decoder as extrinsic information. Given a certain effective throughput, our simulation results demonstrate that the proposed turbo detection aided STBC-SP scheme is capable of providing attractive performance improvements over established orthogonal STBC designs, constituted by the STBC-SP scheme of [5] as well as over its separately channel-decoded counterpart.

This paper is organised as follows. In Section 2, a brief description of orthogonal design with sphere packing modulation is presented, followed by a brief system overview in Section 3. Section 4 shows how the STBC-SP demapper is modified for exploiting the *a priori* knowledge provided by the channel decoder. Simulation results and our discussions are provided in Section 5. Finally, we conclude in Section 6.

2. ORTHOGONAL DESIGN WITH SPHERE PACKING MODULATION

In this contribution, space-time systems employing two transmit antennas are considered, where the space-time signal is given by [2]

$$G_2(x_1, x_2) = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \qquad (1)$$

where the rows and columns of Equation (1) represent the temporal and spatial dimensions, corresponding to two consecutive time slots and two transmit antennas, respectively. According to Alamouti's design [2] for example, x_1 and x_2 represent BPSK modulated symbols transmitted in the 1^{st} and 2^{nd} time slots. Let us assume that there are L legitimate space-time signals $G_2(x_{l,1}, x_{l,2})$, $l = 0, 1, \ldots, L-1$ that the encoder may choose the signal from, that has to be transmitted over the two antennas in two consecutive time slots, where the throughput of the system is given by $(log_2 L)/2$ bits per channel use. The aim is to design $x_{l,1}$ and $x_{l,2}$ jointly, such that they have the best minimum Euclidean distance from all other $(L-1)$ legitimate transmitted space-time signals, since this minimises the system's error probability. Let $(a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4})$, $l = 0, 1, \ldots, L-1$, be phasor points from the four-dimensional real-valued Euclidean space R^4, where each of the four elements $a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}$ gives one coordinate of the phasor points. Subsequently, $x_{l,1}$ and $x_{l,2}$ may be written as

$$\begin{aligned} \{x_{l,1}, x_{l,2}\} &= T(a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}) \\ &= \left\{a_{l,1} + ja_{l,2}, a_{l,3} + ja_{l,4}\right\}. \end{aligned} \qquad (2)$$

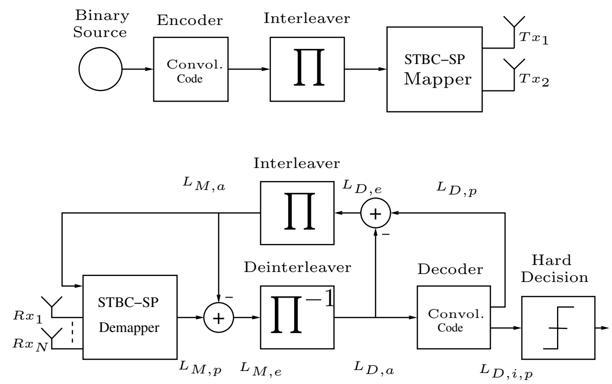

Figure 1: Turbo Detection STBC-SP System.

In the four-dimensional real-valued Euclidean space R^4, the lattice D_4 is defined as a sphere packing having the best minimum Euclidean distance from all other $(L-1)$ legitimate constellation points in R^4 [7]. More specifically, D_4 may be defined as a lattice that consists of all legitimate points having integer coordinates $[a_1\ a_2\ a_3\ a_4]$ such that $a_1 + a_2 + a_3 + a_4 = k$, where k is an even integer. Assuming that $S = \{[a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}] \in \mathrm{R}^4 : 0 \le l \le L-1\}$ constitutes a set of L legitimate constellation points from the lattice D_4 having a total energy of $E \triangleq \sum_{l=0}^{L-1}(|a_{l,1}|^2 + |a_{l,2}|^2 + |a_{l,3}|^2 + |a_{l,4}|^2)$, and upon introducing the notation

$$C_l = \sqrt{\frac{2L}{E}} G_2(x_{l,1}, x_{l,2}), \qquad l = 0, 1, \ldots, L-1, \quad (3)$$

we have a set of space-time signals, $\{C_l : 0 \le l \le L-1\}$, whose diversity product is determined by the minimum Euclidean distance of the set of L legitimate constellation points in S.

3. SYSTEM OVERVIEW

The schematic of the entire system is shown in Figure 1, where the transmitted source bits are convolutionally encoded and then interleaved by a random bit interleaver. A rate $R = \frac{1}{2}$ recursive systematic convolutional code having a constraint length of $K = 5$ and octal generator polynomials (G_r, G)=$(35, 23)$ was employed. After channel interleaving, the STBC-SP modulator first maps B channel-coded bits $\mathbf{b} = b_{0,\ldots,B-1} \in \{0,1\}$ to a sphere packing modulated symbol $s \in S$ such that we have $s = map_{sp}(\mathbf{b})$, where $B = log_2 L$. The mapper then maps the sphere packing modulated symbol s to a space-time signal $C_l = \sqrt{\frac{2L}{E}} G_2(x_{l,1}, x_{l,2})$, $0 \le l \le L-1$, using Equation (2). Subsequently, each space-time signal is transmitted over $T = 2$ time slots using two transmit antennas, as shown in Equation (1).

In this treatise, we considered a correlated narrowband Rayleigh fading channel, associated with a normalised Doppler frequency of $f_D = 0.1$. The complex fading envelope is assumed to be constant across the transmission period of a space-time coded symbol spanning $T = 2$ time slots. The complex Additive White Gaussian Noise (AWGN) of $n = n_I + jn_Q$ is also added to the received signal, where n_I and n_Q are two independent zero mean Gaussian random variables having a variance of $\sigma_n^2 = \sigma_{n_I}^2 = \sigma_{n_Q}^2 = N_0/2$ per dimension, where $N_0/2$ represents the double-sided noise power spectral density expressed in W/Hz.

As shown in Figure 1, the received complex-valued symbols are demapped to their Log-Likelihood Ratio (LLR) representation for each of the B coded bits per STBC-SP symbol. The *a priori* LLR values of the demodulator are subtracted from the *a posteriori* LLR values for the sake of generating the extrinsic LLR values $L_{M,e}$, and then the LLRs $L_{M,e}$ are deinterleaved by a soft-bit deinterleaver, as seen in Figure 1. Next, the soft bits $L_{D,a}$ are passed to the convolutional decoder in order to compute the *a posteriori* LLR values $L_{D,p}$ provided by the Max Log MAP algorithm [12] for all the channel-coded bits. During the last iteration, only the LLR values $L_{D,i,p}$ of the original uncoded systematic information bits are required, which are passed to a hard decision decoder in order to determine the estimated transmitted source bits. The extrinsic information $L_{D,e}$, is generated by subtracting the *a priori* information from the *a posteriori* information according to $L_{D,p} - L_{D,a}$, which is then fed back to the STBC-SP demapper as the *a priori* information $L_{M,a}$ after appropriately reordering them using the interleaver of Figure 1. The STBC-SP demapper exploits the *a priori* information for the sake of providing improved *a posteriori* LLR values, which are then passed to the channel decoder and in turn back to the STBC-SP demodulator for further iterations.

4. ITERATIVE DEMAPPING

Assuming perfect channel estimation, the complex-valued received channel symbols are first diversity-combined in order to extract the estimates $\tilde{x}_1$ and $\tilde{x}_2$ of the most likely transmitted symbols $x_{l,1}$ and $x_{l,2}$ [2][1, pp.400 – 401], resulting in

$$\begin{aligned} \tilde{x}_1 &= h \cdot x_{l,1} + \acute{w} && (4) \\ \tilde{x}_2 &= h \cdot x_{l,2} + \acute{w}, && (5) \end{aligned}$$

where h is real and Rayleigh distributed representing the channel coefficient and $\acute{w}$ is a zero-mean complex Gaussian random variable with variance $\sigma_{\acute{w}}^2 = h \cdot \sigma_n^2$. A received sphere packing symbol r is then constructed from the estimates $\tilde{x}_1$ and $\tilde{x}_2$ using Equation (2) as

$$r = T^{-1}(\tilde{x}_1, \tilde{x}_2), \quad (6)$$

where $r = \{[\tilde{a}_1, \tilde{a_2}, \tilde{a_3}, \tilde{a_4}] \in R^4\}$. The received sphere packing symbol r can be written as

$$r = h \cdot \sqrt{\frac{2L}{E}} \cdot s + w, \quad (7)$$

where $s \in S$ and w is a four-dimensional Gaussian random variable having a variance of $\sigma_w^2 = \sigma_{\acute{w}}^2 = h \cdot \sigma_n^2$, since the symbol constellation S is four-dimensional. Accordingly, the conditional PDF $p(r/s)$ is given by

$$\begin{aligned} p(r/s) &= \frac{1}{(2\pi\sigma_w^2)^{\frac{N_D}{2}}} e^{-\frac{1}{2\sigma_w^2}(r-\alpha\cdot s)^2}, \\ &= \frac{1}{(2\pi\sigma_w^2)^{\frac{N_D}{2}}} e^{-\frac{1}{2\sigma_w^2}\left(\sum_{i=1}^{4}(\tilde{a}_i-\alpha\cdot a_i)^2\right)}, \quad (8) \end{aligned}$$

where we have $\alpha = h \cdot \sqrt{\frac{2L}{E}}$ and $N_D = 4$, since a four-dimensional symbol constellation is used.

The sphere packing symbol r carries B channel-coded bits $\mathbf{b} = b_{0,\ldots,B-1} \in \{0,1\}$. The LLR-value of bit k for $k = 0, \ldots, B-1$ can be written as [11]

$$L(b_k/r) = L_a(b_k) + \ln \frac{\sum_{s\in S_1^k} p(r/s) \cdot e^{\sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)}}{\sum_{s\in S_0^k} p(r/s) \cdot e^{\sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)}}, \quad (9)$$

where S_1^k and S_0^k are subsets of the symbol constellation S such that $S_1^k \triangleq \{s \in S : b_k = 1\}$ and likewise, $S_0^k \triangleq \{s \in S : b_k = 0\}$. In other words, S_i^k represents all symbols of the set S, where we have $b_k \in \{0,1\}$, $k = 0, \ldots, B-1$. Using Equation (8), we can write Equation (9) as

$$\begin{aligned} & L(b_k/r) \\ = \; & L_a(b_k) \\ + \; & \ln \frac{\sum_{s\in S_1^k} \exp\left[-\frac{1}{2\sigma_w^2}(r-\alpha\cdot s)^2 + \sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)\right]}{\sum_{s\in S_0^k} \exp\left[-\frac{1}{2\sigma_w^2}(r-\alpha\cdot s)^2 + \sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)\right]}, \\ = \; & L_{M,a} + L_{M,e}. \quad (10) \end{aligned}$$

Finally, the max-log approximation of Equation (10) is as follows

$$\begin{aligned} & L(b_k/r) \\ = \; & L_a(b_k) \\ + \; & max_{\sum_{s\in S_1^k}} \left[-\frac{1}{2\sigma_w^2}(r-\alpha\cdot s)^2 + \sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)\right] \\ - \; & max_{\sum_{s\in S_0^k}} \left[-\frac{1}{2\sigma_w^2}(r-\alpha\cdot s)^2 + \sum_{j=0,j\neq k}^{B-1} b_j L_a(b_j)\right]. \quad (11) \end{aligned}$$

5. RESULTS AND DISCUSSION

Without loss of generality, we considered a sphere packing scheme associated with $L = 16$ using a single receiver antenna in order to demonstrate the performance improvements achieved by the proposed system. Since the space-time signal which is constructed from an orthogonal design using the sphere packing scheme of Equation (3) is multiplied by a factor that is inversely proportional to $\sqrt{E}$, namely by $\sqrt{\frac{2L}{E}}$, it is desirable to

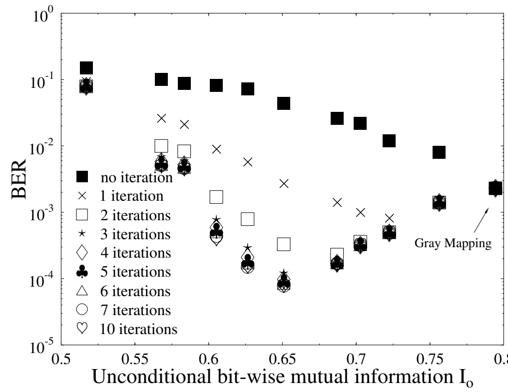

Figure 2: BER versus unconditional bit-wise mutual information I_0 between 0.5170 and 0.7944 for different STBC-SP mappings at $E_b/N_0 = 4.0dB$ in conjunction with $L = 16$, when communicating over a correlated Rayleigh fading channel having $f_D = 0.1$ and using an interleaver depth of $D = 4000$ bits.

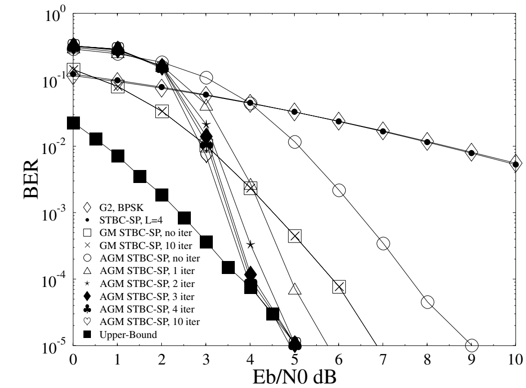

Figure 3: Performance comparison of various Gray Mapping (GM) and Anti-Gray Mapping (AGM) based convolutional-coded STBC-SP schemes in conjunction with $L = 16$ against an identical-throughput 1BPS uncoded STBC-SP scheme using $L = 4$, and against Alamouti's conventional G_2-BPSK scheme, when communicating over a correlated Rayleigh fading channel having $f_D = 0.1$ and using an interleaver depth of $D = 4000$ bits.

choose that specific $L = 16$ points from the entire set of legitimate constellation points hosted by D_4, which result in the minimum total energy. It was shown in [7] that there is a total of 24 legitimate symbols hosted by D_4 having an identical minimum energy of $E = 2$. We used a computer search for determining the optimum choice of the $L = 16$ points out of the possible 24 points, which possess the highest minimum Euclidean distance, hence minimising the error probability.

It was reported in [11] that there is a strong correlation between the average bit-wise capacity computed using no a priori information, which corresponds to the unconditional average bit-wise mutual information I_0 of the symbol constellation and the achievable BER performance when iterative demapping and turbo detection are employed. The achievable performance depends on the specific assignment of the bits to each symbol in the constellation. Different STBC-SP mapping schemes spanning a wide range of different I_0 values were investigated in Figure 2 for the sake of demonstrating this phenomenon. Figure 2 characterises the achievable BER performance against the unconditional bit-wise mutual information I_0 for different STBC-SP mappings at $E_b/N_0 = 4.0dB$ in conjunction with $L = 16$, when communicating over a correlated Rayleigh fading channel having $f_D = 0.1$ and using an interleaver depth of $D = 4000$ bits. Figure 2 also confirms the interesting fact stated in [11] that the choice of the optimum bits-to-symbol mapping scheme is dependent on the number of iterations used. For example, at $E_b/N_0 = 4.0dB$ Gray mapping is the optimum scheme when no iteration is used at $I_0 = 0.7944$. Additionally, as shown in Figure 2, the bits-to-symbol mapping scheme associated with $I_0 = 0.6509$ at $E_b/N_0 = 4.0dB$ is the optimum mapping, when three or more iterations are employed. This scheme is referred to here as the optimum bits-to-symbol mapping scheme.

Figure 3 compares the performance of the proposed convolutional-coded STBC-SP scheme employing the optimum bits-to-symbol mapping against that of an identical-throughput 1 Bit Per Symbol (1BPS) uncoded STBC-SP scheme and a conventional orthogonal STBC design, when communicating over a correlated Rayleigh fading channel. An interleaver depth of $D = 4000$ bits was employed and a normalised Doppler frequency of $f_D = 0.1$ was used. Observe in Figure 3 by comparing the two Gray Mapping (GM) STBC-SP curves that no BER improvement was obtained, when 10 turbo-detection iterations were employed in conjunction with Gray Mapping, which was reported also in [11]. By contrast, Anti-Gray Mapping (AGM) [11] achieved a useful performance improvement in conjunction with iterative demapping and decoding. Explicitly, Figure 3 demonstrates that a coding advantage of about $19dB$ was achieved at a BER of 10^{-5} after 10 iterations by the convolutional-coded AGM STBC-SP system over both the uncoded STBC-SP [5] and the conventional orthogonal STBC design based [2, 3] schemes for transmission over the correlated Rayleigh fading channel considered. Additionally, a coding advantage of approximately $2dB$ was attained over the 1BPS-throughput convolutional-coded GM STBC-SP scheme. The Upper-Bound curve shown in Figure 3 was obtained by feeding perfect *a*

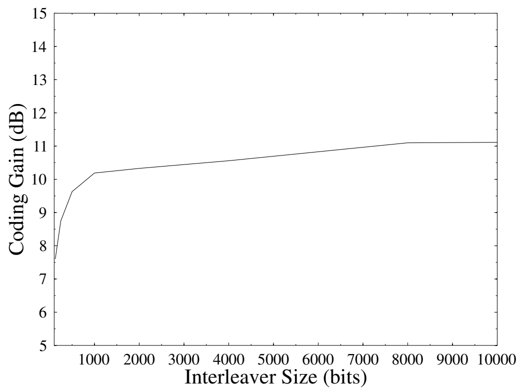

Figure 4: Coding gain of Anti-Gray Mapping (AGM) based convolutional-coded STBC-SP schemes in conjunction with $L = 16$, when communicating over a correlated Rayleigh fading channel having $f_D = 0.1$ and using different interleaver depths as compared to the 1BPS-throughput conventional orthogonal STBC design based Alamouti scheme.

priori knowledge to the STBC-SP demapper during the first iteration.

Figure 4 shows the coding gain of AGM based convolutional-coded STBC-SP schemes in conjunction with $L = 16$ employing the optimum bits-to-symbol mapping, when communicating over a correlated Rayleigh fading channel having $f_D = 0.1$ and using different interleaver depths as compared to the 1BPS-throughput conventional orthogonal STBC design based [2] scheme. The figure demonstrates that there is no significant coding gain improvement when using interleaver depths of more than 1000 bits.

6. CONCLUSION

In this paper we proposed a novel system that exploits the advantages of both iterative demapping and turbo detection [11] as well as those of the STBC-SP scheme of [5]. Our investigations demonstrated that significant performance improvements may be achieved, when the AGM STBC-SP is combined with convolutional coding and iterative demapping as compared to the Gray-Mapping based systems. Subsequently, the bit-wise unconditional mutual information I_0 was used to search for optimum bits-to-symbol mapping schemes. Several STBC-SP mapping schemes covering a wide range of different I_0 values were investigated. When using the optimum bits-to-symbol mapping scheme and 10 turbo detection iterations, a gain of about $2dB$ was obtained over the Gray Mapping scheme. The achievable performance was also investigated for different interleaver depths. Our future research may include the use of Extrinsic Information Transfer (EXIT) Charts [13] to search for other mapping and channel code combinations that converge at lower E_b/N_0 values.

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