On the Uplink Performance of Band-Limited DS-CDMA Systems using RAKE-Receiver Over Nakagami-m Channels

H.Wei and L.Hanzo[1]

Dept. of ECS, University of Southampton, SO17 1BJ, UK.
Tel: +44-23-8059-3125, Fax: +44-23-8059-4508
Email:lh[1]@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

Abstract

In this paper, we investigated the BER performance of DS-CDMA using various chip-waveforms, which include three time-limited chip-waveforms and two band-limited chip-waveforms. Closed-form formulae were derived for evaluating the achievable bit-error rate performance with the aid of the standard Gaussian approximation.

1. INTRODUCTION

The signalling pulse design or waveform design plays an important role in determining the properties of digital communication systems. In [1], Amoroso introduced various definitions of the bandwidth of digital signalling schemes. In the context of CDMA based communications, numerous valuable studies have been conducted [2–4] for the sake of finding various attractive chip waveforms.

Cho and Lehnert [3] investigated the performance of several band-limited chip-waveforms in the context of DS-CDMA systems, when communicating over a non-dispersive AWGN channel. For example, a time-domain chip-waveform associated with a raised cosine spectrum was studied in [3, 5] while the so-called optimum chip-waveform was investigated [3]. In [4], Yang and Hanzo investigated the performance of generalized multicarrier DS-CDMA using various time-limited chip-waveforms, namely the rectangular, half-sine, as well as the raised-cosine chip-waveforms and closed-form formulae were derived for evaluating the BER performance, when communicating over a dispersive Nakagami-m channel. Based on the above-mentioned studies, in this treatise we investigate the performance of band-limited DS-CDMA systems in conjunction with different chip-waveform designs, when communicating over a dispersive Nakagami-m channel.

This paper is organized as follows. Section 2 describes the CDMA communication system designed for communicating over a dispersive Nakagami-m channel, while Section 3 will analyze the BER performance of the band-limited DS-CDMA system in conjunction with different waveforms, which include both time-limited and band-limited chip-waveforms. Finally, Section 4 provides our numerical results and Section 5 offers our conclusions.

The financial support of the Mobile VCE, UK is gratefully acknowledged.

2. SYSTEM MODEL

2.1. System Model

Let us consider an asynchronous K-user DS-CDMA communication system, where each user is assigned a unique signature waveform $\mathbf{c}_k(t) = \sum_{i=0}^{G-1} c_{ki}\psi_{T_c}(t - iT_c)$. The sequence $c_{ki} \in \frac{1}{\sqrt{N}}\{+1, -1, +j, -j\}$ represents the random Pseudo-Noise (PN) spreading sequence of the kth user and G is the processing gain, which obeys $G = T_d/T_c$, and $\psi_{T_c}(t)$ represents the chip-waveform having an energy of $\int_{-\infty}^{+\infty} \psi_{T_c}^2(t)dt = T_c$. For convenience, we define the normalized chip-waveform $\hat{\psi}(t) = \psi_{T_c}(t/T_c)$. For the band-limited system considered, the normalized chip-waveform $\hat{\psi}(t)$ satisfies:

$$\int_{-\infty}^{+\infty} \hat{\psi}(t - n_1)\hat{\psi}^*(t - n_2)dt = \delta(n_1, n_2), \qquad (1)$$

where $\delta(n_1, n_2) = 1$ for $n_1 = n_2$, and 0 for $n_1 \neq n_2$. From Eq.(1), we have:

$$\sum_{n=-\infty}^{\infty} \hat{\psi}^2(n) = 1. \qquad (2)$$

Consequently, when the K users' signals are transmitted over a frequency-selective fading channel, the received complex-valued low-pass equivalent signal at a given base station can be expressed as:

$$\begin{aligned} R(t) &= \sum_{k=1}^{K}\sum_{l=0}^{L_p-1} \sqrt{2P_k}\mathbf{c}_k(t - lT_c - \tau_k)b_k(t - lT_c - \tau_k) \\ &\quad \times h_{kl}\exp(j\theta_{kl}) + N(t), \end{aligned} \qquad (3)$$

where $N(t)$ is the complex-valued low-pass-equivalent AWGN having a double-sided spectral density of N_0 and τ_k is the propagation delay of user k, while L_p is the total number of resolvable paths.

2.2. Channel Model

The DS-CDMA signal experiences independent frequency-selective Nakagami-m fading. The complex low-pass equivalent representation of the Channel Impulse Response (CIR) encountered by the kth user is given by [5]:

$$h_k(t) = \sum_{l=0}^{L_p-1} h_{kl}\delta(t - lT_c)\exp(j\theta_{kl}), \qquad (4)$$

where h_{kl} represents the Nakagami-distributed fading envelope, lT_c is the relative delay of the lth path of user k with respect to the main path, while L_p is the total number of resolvable multipath components. Furthermore, θ_{kl} is the uniformly distributed phase-shift of the lth multipath component of the channel and $\delta(t)$ is the Kronecker Delta-function. More explicitly, the L multipath attenuations $\{h_{kl}\}$ are independent Nakagami distributed random variables having a Probability Density Function (PDF) of [6]:

$$\begin{aligned} p(h_{kl}) &= M(h_{kl}, m_{kl}, \Omega_{kl}), \\ M(R, m, \Omega) &= \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{(-m/\Omega)R^2}, \end{aligned} \qquad (5)$$

where $\Gamma(\cdot)$ is the gamma function [5], and m_{kl} is the Nakagami-m fading parameter, which characterizes the severity of the fading for the l-th resolvable path of user k [6] . Specifically, $m_{kl} = 1$ represents Rayleigh fading, $m_{kl} \rightarrow \infty$ corresponds to the conventional Gaussian scenario and $m_{kl} = 1/2$ describes the so-called one-sided Gaussian fading, i.e. the worst-case fading condition. The Rician and log-normal distributions can also be closely approximated by the Nakagami distribution in conjunction with values of $m_{kl} > 1$. The parameter Ω_{kl} in Eq.(5) is the second moment of h_{kl}, i.e. we have $\Omega_{kl} = E[(h_{kl})^2]$. We assume a negative exponentially decaying multipath intensity profile (MIP) given by $\Omega_{kl} = \Omega_{k0}e^{-\eta l}$, $\eta \geq 0, l = 0, \ldots, L_p - 1$, where Ω_{kl} is the average signal strength corresponding to the first resolvable path and η is the rate of average power decay.

3. BER ANALYSIS

Let the first user be the user-of-interest and consider a receiver using de-spreading as well as multipath diversity combining. The conventional matched filter based RAKE receiver using MRC may be invoked for detection, where we assume that the RAKE receiver is capable of combining L_r number of diversity paths.

Let us assume that we have achieved time synchronization and perfect estimates of the channel magnitudes and phases are available. The individual matched filter outputs are appropriately delayed, in order to coherently combine the L_r number of path signals processed by the RAKE combiner. The lth RAKE combiner finger's output Z_{kl} is sampled at $t = T + lT_c + \tau_k$, in order to detect the kth user's transmitted symbol $b_k[0]$, which is expressed as:

$$Z_{kl} = D_{kl} + I_{kl}, \qquad (6)$$

where D_{kl} represents the desired direct Line-of-Sight (LOS) component, which can be expressed as:

$$D_{kl} = \sqrt{2PT_s}b_k[0]h_{kl}^2. \qquad (7)$$

In Eq.(7) $b_k[0]$ is the first bit transmitted by the kth BPSK user and we have $b_k[0] \in \{+1, -1\}$. Hence, the interference plus noise term I_k in Eq.(6) may be expressed as:

$$I_{kl} = I_{kl}[S] + I_{kl}[M] + N_{kl}, \qquad (8)$$

where $I_{kl}[S]$ represents the multipath interference imposed by the user-of-interest. Explicitly, $I_{kl}[S]$ may be expressed as:

$$\begin{aligned} I_{kl}[S] &= \sqrt{2PT_s}h_{kl} \sum_{\substack{l_p=0 \\ l_p \neq l}}^{L_p-1} h_{kl_p}\exp(j\theta_{kl_p}) \\ &\times \left\{ \sum_{n=-\infty}^{\infty} \rho_{kk}(n)\hat{\psi}(n) \right\}, \end{aligned} \qquad (9)$$

where the term $\rho_{kk}(n)$ is formulated as:

$$\rho_{kk}(n) = \frac{1}{G}\sum_{i=0}^{G-1} c_k[i]^* c_k[i - nG - l_p]. \qquad (10)$$

Furthermore, $I_{kl}[M]$ of Eq.(8) represents the multiuser interference inflicted by the $K - 1$ interfering users, which is expressed as:

$$\begin{aligned} I_{kl}[M] &= \sqrt{2PT_s}h_{kl} \sum_{\substack{k'=1 \\ k' \neq k}}^{K} \sum_{l_p=0}^{L_p-1} h_{k'l_p}\exp(j\theta_{kl_p}) \\ &\times \left\{ \sum_{n=-\infty}^{\infty} \rho_{kk'}(n)\hat{\psi}(n - \tau_k) \right\}, \end{aligned} \qquad (11)$$

and we have:

$$\rho_{kk'}(n) = \frac{1}{G}\sum_{i=0}^{G-1} c_k[i]^* c_{k'}[i - nG - l_p]. \qquad (12)$$

It was shown in [3] for a random PN spreading sequence that the random variables $\rho_{kk}(n)$ and $\rho_{kk'}(n)$ may be modelled as complex Gaussian random variables having a mean of zero and a variance of $1/G$. Therefore, the variance of the term $I_{kl}[S]$ of Eq.(8), which was explicitly formulated in Eq.(9) can be expressed as:

$$\text{Var}\{I_{kl}[S]\} = 2PT_s^2 h_{kl}^2 \Omega_0[q(L_p, \eta) - 1]\frac{1}{G}\sum_{n=-\infty}^{\infty} \hat{\psi}^2(n), \qquad (13)$$

which may be simplified with the aid of Eq.(2) to:

$$\mathrm{Var}\{I_{kl}[S]\} = 2PT_s^2 h_{kl}^2 \frac{1}{G}\Omega_0[q(L_p,\eta)-1]. \quad (14)$$

Similarly, the variance of the term $I_{kl}[M]$ formulated in Eq.(11) can be expressed as:

$$\mathrm{Var}\{I_{kl}[M]\} = 2PT_s^2 h_{kl}^2 \Omega_0 K q(L_p,\eta)\frac{1}{G}E[\phi(\beta,\tau)], \quad (15)$$

where $E[\phi(\beta,\tau)] = E[\sum_{n=-\infty}^{\infty} \hat{\psi}^2(n-\tau)]$ defines the *interference factor* associated with a specific chip pulse shape, which predetermines the amount of the MAI imposed by different chip-waveforms, while τ is a random variable uniformly distributed in $[0,1]$, and $E[\phi(\beta,\tau)]$ can be expressed as a function of the excess bandwidth β, as we mentioned before.

Finally, the noise term of Eq.(8) can be expressed as:

$$N_{kl} = h_{kl}\int_0^{T_s} n(t)c[t]\cos(2\pi f_c t + \theta_{kl})dt, \quad (16)$$

which is a Gaussian random variable having zero mean and a variance of $N_0 T_s h_{kl}^2$.

The MRC's decision variable Z_k is constituted by the sum of the RAKE fingers' output, which can be expressed as:

$$Z_k = \sum_{l=0}^{L_r-1} Z_{kl}. \quad (17)$$

3.1. Probability of Error

In the analysis of this section we employ the Gaussian approximation and hence model the multiuser interference and the self-interference terms of Eq.(8) as an AWGN process having zero mean and a variance equal to the corresponding variances. More explicitly, the assumptions made in this section are as follows. The RAKE fingers' output signal Z_{kl} is a Gaussian distributed random variable having a mean of D_{kl}. Consequently, according to the analysis of the previous sections –for the random spreading codes and BPSK modulation considered, the variance of the lth RAKE finger's output samples Z_{kl} for a given set of channel amplitudes $\{h_{kl}\}$ may be approximated as:

$$\begin{aligned}\sigma_{kl}^2 &= \frac{1}{2}\times\{\mathrm{Var}\{I_{kl}[S]\} + \mathrm{Var}\{N_{kl}\} + \mathrm{Var}\{I_{kl}[N]\}\} \\ &= 2PT_s^2\Omega_0 h_{kl}^2 \times \left[\frac{Kq(L_p,\eta)E[\phi(\beta,\tau)]}{2G}\right. \\ &\quad \left. + \frac{q(L_p,\eta)-1}{2G} + \left(\frac{2\Omega_0 E_b}{N_0}\right)^{-1}\right], \end{aligned} \quad (18)$$

where $E_b = PT_s$ is the energy per bit and we have $q(L_p,\eta) = \sum_{l=0}^{L_p-1} e^{-\eta l}$. Furthermore, the MRC's output sample Z_k can be approximated by an AWGN variable having a mean value of $\mathrm{E}[Z_k] = \sum_{l=0}^{L_r-1} D_{kl}$ and a variance of $\mathrm{Var}[Z_k] = \sum_{l=0}^{L_r-1} \sigma_{kl}$ [6]. To expand further, upon using Eq.(7) and Eq.(18) we have:

$$\mathrm{E}[Z_k] = \sum_{l=0}^{L_r-1} \sqrt{2P}T_s b_k[0] h_{kl}^2, \quad (19)$$

$$\begin{aligned}\mathrm{Var}[Z_k] &= 2PT_s^2\left[\frac{Kq(L_p,\eta)E[\phi(\beta,\tau)]}{2G} + \frac{q(L_p,\eta)-1}{2G}\right. \\ &\quad \left. + \left(\frac{2\Omega_0 E_b}{N_0}\right)^{-1}\right] \times \Omega_0 \sum_{l=0}^{L_r-1} h_{kl}^2. \end{aligned} \quad (20)$$

Therefore, the BER of BPSK modulation conditioned on the set of fading magnitudes $\{h_{kl}, l = 0,1,\ldots,L_r-1\}$ can be expressed as:

$$P_b(\gamma) = Q\left(\sqrt{\frac{(\mathrm{E}[Z_k])^2}{\mathrm{Var}[Z_k]}}\right) = Q\left(\sqrt{\sum_{l=0}^{L_r-1} 2\gamma_l}\right), \quad (21)$$

where $Q(x)$ represents the Gaussian Q-function, which can also be represented in its less conventional form as [6] $Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$, where $x \geq 0$. Furthermore, $2\gamma_l$ in Eq.(21) represents the output Signal to Interference plus Noise Ratio (SINR) at the lth finger of the RAKE receiver, while γ_l is given by:

$$\gamma_l = \gamma_c \cdot \frac{h_{kl}^2}{\Omega_0}. \quad (22)$$

Let us now substitute Eq.(22) into Eq.(21) and Eq.(19) as well as Eq.(20) also into Eq.(22). We can see then that the expressions under the square-root functions must be equal, which allows us to express γ_c as follows:

$$\gamma_c = \left[\frac{Kq(L_p,\eta)E[\phi(\beta,\tau)]}{G} + \frac{q(L_p,\eta)-1}{G} + \left(\frac{\Omega E_b}{N_0}\right)^{-1}\right]^{-1}. \quad (23)$$

The average BER $P_b(E)$ at a given value of E_b/N_0 can be obtained by the weighted averaging of the output γ_l, i.e. upon integrating the conditional BER of Eq.(21) after weighting it by the probability of occurrence of a specific instantaneous value of γ_l, which is quantified by the joint PDF of the instantaneous γ_l values corresponding to the L_r multipath components $\{\gamma_l : l = 0,2,\ldots,L_r-1\}$. Since the random variables $\{\gamma_l : l = 1,2,\ldots,L_r-1\}$ are assumed to be statistically independent, the average BER expressed in Eq.(21) can be formulated as [6]:

$$P_b(E) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{l=0}^{L_r-1} I_l(\overline{\gamma}_l,\theta)\,d\theta, \qquad (24)$$

where $I_l(\overline{\gamma}_l,\theta)$ is given by:

$$I_l(\overline{\gamma}_l,\theta) = \int_0^{\infty}\exp\left(-\frac{\gamma_l}{\sin^2\theta}\right)p_{\gamma_l}(\gamma_l)d\gamma_l. \qquad (25)$$

Since we have $\gamma_l = \gamma_c\cdot\frac{h_l^2}{\Omega_0}$ and h_l obeys the Nakagami-m distribution characterized by Eq.(5), it can be shown that the PDF of γ_l can be formulated as [6]:

$$p_{\gamma_l}(\gamma_l) = \left(\frac{m}{\overline{\gamma}_l}\right)^m\frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-\frac{m\gamma_l}{\overline{\gamma}_l}\right),\quad \gamma_l\geq 0, \qquad (26)$$

where $\overline{\gamma}_l = \gamma_c e^{-\eta l}$ for $l = 0,1,\ldots,L_r-1$.

Upon substituting Eq.(26) into Eq.(25) it can be shown that we have [6]:

$$I_l(\overline{\gamma}_l,\theta) = \left(\frac{m\sin^2\theta}{\overline{\gamma}_l + m\sin^2\theta}\right)^m. \qquad (27)$$

Finally, upon substituting Eq.(27) into Eq.(24), the average BER of the CDMA system considered can be written as:

$$P_b(E) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{l=0}^{L_r-1}\left(\frac{m\sin^2\theta}{\overline{\gamma}_l + m\sin^2\theta}\right)^m d\theta. \qquad (28)$$

3.2. Time-Limited and Band-Limited Waveforms

In this section, we will consider three widely-used time-limited chip-waveforms, which are the rectangular, half-sine and raised-cosine time-domain chip-waveforms, and the duration of the chip-waveform is limited $[0, T_c]$, while the corresponding frequency-domain spectral density function $Q(f)$ may be expressed with the aid of the Fourier transform $Q(f) = \mathcal{F}\{\psi_{T_c}(t)\}$. We considered two different fractional energy containment bandwidth definitions [1] for the sake of characterizing these time-limited waveforms. It is conceptually appealing to define the fractional energy containment bandwidth [1] as the normalized frequency W that satisfies $\frac{\int_{-W}^{W}Q^2(f)df}{\int_{-\infty}^{\infty}Q^2(f)df}\geq 99\%$. In [2,3], Dallas and Pavlidou investigated the interference factor, $E[\phi(\beta,\tau)]$ defined in the context of Eq.(15), which was found to be 0.666, 0.596 and 0.582 for the rectangular, half-sine and raised cosine chip-waveforms, respectively. Therefore, following above definition of the energy containment bandwidth, the required excess bandwidths are summarized in Table 1 for both of the above mentioned energy containment factors, along with the corresponding interference factors for the three time-limited chip-waveforms considered.

Waveform	Energy containment bandwidth $\geq 99\%$	Interference factor $E[\phi(\beta,\tau)]$
Rectangular	$\beta = 12.22$	0.666
Half-Sine	$\beta = 1.36$	0.596
Raised Cosine	$\beta = 1.82$	0.482

Table 1: The required excess bandwidth β and the corresponding interference factors $E[\phi(\beta,\tau)]$ for various time-domain chip-pulse shapes.

we will consider two different band-limited waveforms, both of them is limited in frequency-domain. The first is the well-known frequency-domain raised cosine Nyquist spectrum [5], whose energy spectral density function $Q^2(f)$ have a raised cosine shape, where β represents the Nyquist *roll-off* factor, quantifying the excess Nyquist bandwidth. We will refer to this Nyquist signalling pulse as a Band-limited Raised Cosine (BRC) signalling waveform, which is characterized by the spectral domain representation given by [7], and the interference factor of the pulse shape for the BRC waveform can be expressed as [3]:

$$E[\phi(\beta,\tau)] \;=\; 1-\frac{\beta}{4}. \qquad (29)$$

Another band-limited chip-waveform is the so-called optimum waveform [3] which was proposed by Cho and Lehnert [3] for the BER performance analysis of band-limited DS-CDMA systems communicating over an AWGN channels, and the interference factor of the optimum waveform is given by:

$$E[\phi(\beta,\tau)] \;=\; 1-\frac{\beta}{2}. \qquad (30)$$

4. NUMERICAL RESULTS

In this section we will investigate the achievable BER performance of band-limited DS-CDMA. As mentioned before, the normalized bandwidth can be expressed as $G(1+\beta) = 2WT_d$. Hence upon stipulating a specific $G(1+\beta)$ value, the system's bandwidth as well as the bit rate has been fixed. The parameters employed in our investigations were $L_p = 10, L_r = 4, \eta = 0.2$, and $K = 10$. In Figure 1 we investigated the BER performance of band-limited DS-CDMA in conjunction with the tree different time-limited and two band-limited chip-waveforms considered, when the bandwidth occupied was fixed to $G(1+\beta) = 1000$. From this figure we infer that a frequency-domain BRC waveform based DS-CDMA system employing $\beta = 0.22$ is capable of achieving a similar performance to that of using the optimum waveform [3] associated with $\beta = 0.22$, when communicating over a Rayleigh fading channel. Furthermore, the time-domain raised-cosine waveform aided CDMA system of Figure 1 was also capable of approaching the BER of both the frequency-domain BRC and optimum chip-waveform, although at $E_b/N_0 = 30$dB it exhibited an approximately factor two higher residual BER. Furthermore, Figure 2 portrays the BER performance of the band-limited DS-CDMA system investigated as a function of the number of users K supported. Finally, Figure 3 comparatively studied the performance of the BRC waveform and the optimum chip-waveform of [3] as a function of the Nakagami fading parameter m. From this figure we may conclude that the optimum waveform is capable of achieving a better performance than that of the BRC waveform, when we have $\beta \rightarrow 1$, although at $E_b/N_0 = 10$dB the associated performance difference is not dramatic.

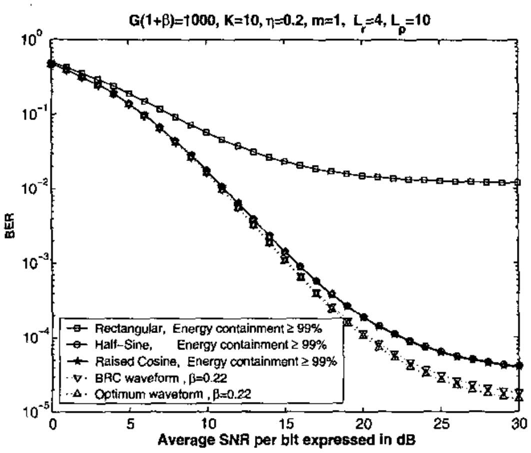

Figure 1: BER performance comparison of band-limited DS-CDMA as a function of E_b/N_0.

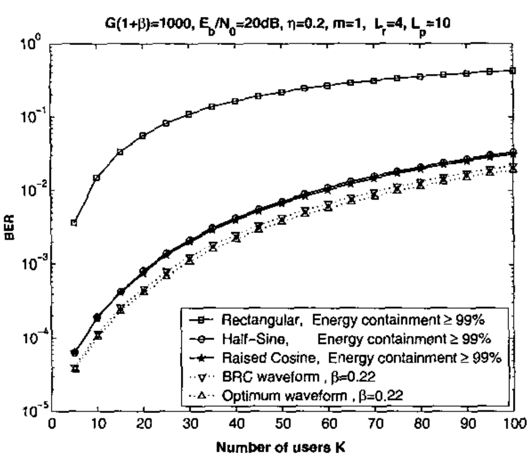

Figure 2: BER performance comparison of band-limited DS-CDMA as a function of the number of users K.

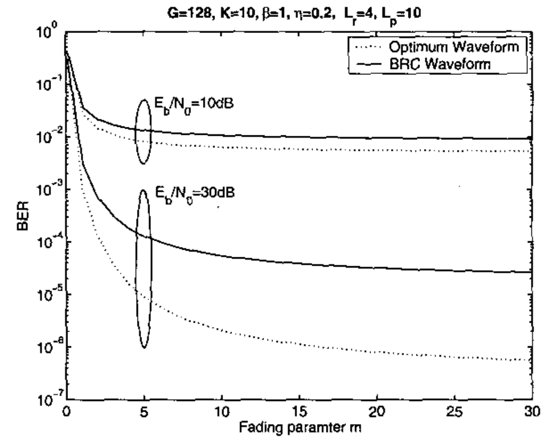

Figure 3: BER performance comparison of band-limited DS-CDMA as a function of the Nakagami fading parameter m in conjunction with the BRC and the optimum [3] waveform, when we have $\beta = 1$.

5. CONCLUSION

In this treatise, we have investigated the achievable performance of band-limited DS-CDMA in conjunction with three different time-limited and two band-limited chip-waveforms. In the context of the time-limited waveforms the raised cosine waveform based DS-CDMA system achieved the best performance. By contrast, when we considered band-limited waveforms, we investigated the BER performance of both the optimum [3] and BRC waveform based DS-CDMA systems. Both of these band-limited waveform based DS-CDMA schemes exhibited a better BER performance than that of the time-limited waveforms. When aiming for an energy containment in excess of 99%, the raised cosine waveform based DS-CDMA scheme was capable of achieving a similar performance to that of the optimum waveform based DS-CDMA arrangement.

6. REFERENCES

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