Generic Reduced-Complexity MMSE Channel Estimation for OFDM and MC-CDMA

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***Abstract*— The performance of a decision-directed channel estimation scheme is analyzed in the context of both OFDM and MC-CDMA systems. A difficulty associated with the employment of the Least Squares (LS) approach to the problem of *a posteriori* channel estimation in the context of a MC-CDMA system is described and a suitable MMSE-based estimator is proposed instead. It is demonstrated that the computational complexity associated with the MMSE estimation method proposed is relatively high in compararison to that of the conventional LS technique. Thus a low-complexity version of the MMSE estimator is proposed. The MC-CDMA system using a low-complexity MMSE estimator proposed is shown to outperform the corresponding OFDM-based scheme.**

I. Introduction

The ever-increasing demand for high data rates in wireless networks requires the efficient utilisation of the limited bandwidth available, while supporting a high grade of mobility in diverse propagation environments. Orthogonal Frequency Devision Multiplexing (OFDM) and Multi-Carrier Code Devision Multiple Access (MC-CDMA) techniques [1] are capable of satisfying these requirements, since they are capable of coping with highly time-variant wireless channel characteristics. However, as pointed out in [2], the capacity and the achievable integrity of communication systems is highly dependent on the system's knowledge concerning the channel conditions encountered. Thus, the provision of an accurate and robust channel estimation strategy is a crucial factor in achieving a high performance.

The family of well-documented *decision directed* channel estimation (DDCE) methods [1], [3]–[5] provides a suitable solution for the problem of channel estimation in OFDM-based systems. The major benefit of the DDCE scheme is that in contrast to purely *pilot assisted* channel estimation methods [6], [7] both the pilot symbols as well as all the information symbols are utilised for channel estimation [1]. The simple philosophy of this method is that in the absence of transmission errors we can benefit from the availability of 100% pilot information by using the detected subcarrier symbols as an *a posteriori* reference signal. The employment of this method allows us to reduce the number of pilot symbols required.

A basic component of the DDCE schemes proposed in the literature is an *a posteriori* Least Squares (LS) temporal estimator of the OFDM-subcarrier-related Frequency-Domain Channel Transfer Function (FD-CTF) coefficients [1], [5]. The accuracy of the resultant temporal estimates is typically enhanced using one- or two-dimensional interpolation exploiting both the time- and the frequency-domain correlation between the desired FD-CTF coefficients. The LS-based temporal FD-CTF estimator was shown to be suitable for QPSK-modulated OFDM systems [1], [5], where the energy of the transmitted subcarrier-related information symbols is constant. However as it will be pointed out in Section III-B.1 of this contribution, the LS method cannot be employed in MC-CDMA systems, where – in contrast to OFDM systems – the energy of the transmitted subcarrier-related information symbols fluctuates as a function of both the modulated sequence and that of the choise of the potentially non-constant-modulus modulation scheme itself. Thus we propose an MMSE-based DDCE method, which is an appropriate solution for both OFDM and MC-CDMA systems.

The rest of this paper is structured as follows. The system model and the channel model considered are described in Section II. The difficulty of employing the Least Squares (LS) approach to the problem of estimating the OFDM-subcarrier-related FD-CTF coefficients is described in Section III-B.1. The alternative MMSE FD-CTF estimator circumventing the problem outlined in Section III-B.1 is analyzed in Section III-B.2. Our discourse evolves further by proposing a MMSE Channel Impulse Response (CIR) estimator exploiting the frequency-domain correlation of the FD-CTF coefficients in Section III-C.1 and a reduced-complexity version of the CIR MMSE estimator considered is proposed in Section III-C.2. The computational complexity of both methods is compared in Section III-D. Finally, the achievable performance of the estimation methods proposed is studied in Section IV.

II. System Model

The discrete baseband model of the OFDM/MC-CDMA system can be described as in [8]

$$y[n,k] = H[n,k]x[n,k] + w[n,k], \tag{1}$$

for $k = 0,\ldots,K-1$ and all n, where $y[n,k]$, $x[n,k]$ and $w[n,k]$ are the received symbol, the transmitted symbol and the Gaussian noise sample respectively, corresponding to the kth subcarrier of the nth OFDM block. Furthermore, $H[n,k]$ is the complex channel transfer function (CTF) coefficient associated with kth subcarrier and time instance n. Note that in the case of an M-QAM modulated OFDM system, $x[n,k]$ corresponds to the M-QAM symbol accommodated by the kth subcarrier, while in a MC-CDMA system, such as a Walsh-Hadamard Transform (WHT) assisted OFDM scheme using G-chip WH spreading code and hence capable of supporting G users [1] we have

$$x[n,k] = \sum_{p=0}^{G-1} c[k,p]s[n,p], \tag{2}$$

where $c[k,p]$ is the kth chip of the pth spreading code, while $s[n,p]$ is the M-QAM symbol spread by the pth code. Each of the G spreading codes is constituted by G chips.

As it was pointed out in [5], in OFDM/MC-CDMA systems using a sufficiently long cyclic prefix and adequate synchronisation, the

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discrete CTF can be expressed as

$$H[n,k] \triangleq H(nT, k\Delta f) = \sum_{l=0}^{K_0-1} W_K^{kl} h[n,l], \quad (3)$$

for $k = 0, 1, \cdots, K-1$, where $h[n,l] \triangleq h(nT, lT/K)$ is the sample-spaced CIR (SS-CIR) having significant non-zero tap values only at sample-spaced raster positions and $W_K = \exp(-\jmath 2\pi/K)$. Note that in realistic channel conditions associated with non-sample-spaced path-delays the receiver will encounter dispersed received signal components in several neighbouring samples owing to the convolution of the transmitted signal with the system's impulse response, which we refer to as "leakage" [1]. This phenomenon is usually unavoidable and therefore the resultant SS-CIR $h[n,l]$ will be constituted of numerous correlated non-zero taps. Although the results of the present contribution are applicable in the contest of estimating the fractionally-spaced CIR, we opted for concentrating our attention on the case of SS-CIRs for the sake of notational simplicity. Therefore, we assume hereafter a SS-CIR constituted of K_0 uncorrelated taps $h[n,l]$.

III. Channel Estimation

A. Decision Directed Channel Estimator

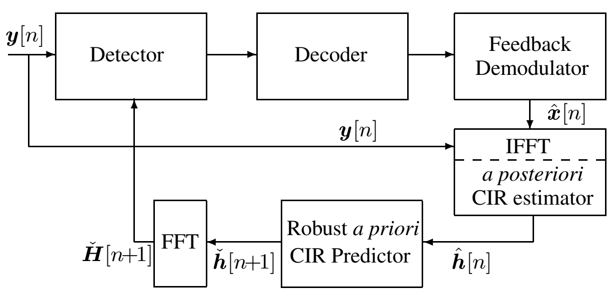


Fig. 1. Schematic of the channel estimator constituted by an *a posteriori* decision-directed CIR estimator, based on frequency-domain modulated symbol estimates, followed by an *a priori* CIR predictor.

The schematic of the channel estimation method considered is depicted in Figure 1. Our channel estimator is constituted by what we refer to as an *a posteriori* decision-directed CIR estimator followed by an *a priori* CIR predictor [1]. As seen in Figure 1, the task of the CIR estimator is to estimate the SS-CIR taps of Equation (3). The Inverse Fast Fourier Transform (IFFT) based transformation from the subcarrier-related frequency domain to the SS-CIR-related time domain is invoked in order to exploit the frequency-domain correlation of the subcarrier-related CTF coefficients as well as to reduce the computational complexity associated with the CTF prediction process, because the SS-CIR typically has a lower number of $K_0 \ll K$ taps, which have to be predicted, than the K number of FD-CTF coefficients. Hence the overall channel estimation complexity is reduced, even when the complexity of the FD-CTF to CIR transformation and its inverse are taken into account.

As seen in Figure 1, the *a posteriori* SS-CIR estimator inputs are the frequency-domain signal $\boldsymbol{y}[n]$ and the decision-based estimate $\hat{\boldsymbol{x}}[n]$. The transformation from the frequency to time domain is performed within the CIR estimator of Figure 1 and its output is an *a posteriori* estimate $\hat{x}[n,k]$ of the SS-CIR taps of Equation (3), which is fed into the low-rank time-domain SS-CIR tap predictor of Figure 1 for the sake of producing an *a priori* estimate $\check{h}[n+1,l]$, $l = 0, 1, \cdots, K_0-1$ of the next SS-CIR on a SS-CIR tap-by-tap basis [1]. Finally, the predicted SS-CIR is converted to the FD-CTF with the aid of the FFT block of Figure 1. The resultant FD-CTF is employed by the receiver for the sake of detecting and decoding of the next OFDM symbol. Note, that this principle requires the transmission of a pilot-based channel sounding sequence, such as for example pilot-assisted OFDM block, during the initialisation stage.

B. *A Posteriori* FD-CTF Estimation

In order to emphasize the major difference between the OFDM and MC-CDMA systems in the context of the associated channel estimation scheme, first we would like to analyze the performance of the temporal estimator of the subcarrier-related FD-CTF coefficients $H[n,k]$ based on the *a posteriori* decision-aided estimates of the transmitted subcarrier-related samples $x[n,k]$ of Equation (1). In Section III-B.1 we will show that the LS approach typically employed in DDCE-aided OFDM systems [1], [5] is not applicable in the case of MC-CDMA systems. In Section III-C.1 we propose an MMSE estimator, which renders the DDCE philosophy discussed in [1], [5] suitable for MC-CDMA systems. However, the estimator introduced in Section III-C.1 exhibits a computational complexity, which is significantly higher than the computational complexity of the conventional LS-based estimator of [1], [5]. Thus a reduced-complexity approximation of the MMSE estimator of Section III-C.1 is proposed in Section III-C.2.

1) Least Squares CTF estimator: Following Equation (1), the Least Squares (LS) approach [9] to the problem of estimating the discrete-abscissa FD-CTF coefficients $H[n,k]$, based on the knowledge of the decision-aided estimates $\hat{x}[n,k]$ of the transmitted frequency-domain samples $x[n,k]$ of Equation (1) can be expressed as

$$\tilde{H}[n,k] = \frac{y[n,k]}{\hat{x}[n,k]} = H[n,k] \cdot \frac{x[n,k]}{\hat{x}[n,k]} + \frac{w[n,k]}{\hat{x}[n,k]}, \quad (4)$$

where $H[n,k]$ represents the Rayleigh-distributed FD-CTF coefficients having a variance of σ_H^2, while $x[n,k]$ denotes the transmitted subcarrier-related samples having zero mean and a variance of σ_x^2. The distribution of the samples $x[n,k]$ is dependent on the particular modulation scheme employed by the system. For instance, in a MC-CDMA system using an arbitrary modulation scheme, the samples $x[n,k]$ are complex-Gaussian distributed, having a Rayleigh-distributed amplitude $|x[n,k]|$ and uniformly-distributed phase $\theta[n,k]$. By contrast, in a M-PSK-modulated OFDM system the samples $x[n,k]$ are uniformly distributed within the set of M-PSK symbols having a constant amplitude $|x[n,k]| = \sigma_x$ and a discrete-uniform distributed phase $\theta[n,k] = 2\pi\frac{m}{M}, m = 0, 1, \cdots, M-1$. Finally, the noise samples $w[n,k]$ are independent identically distributed (i.i.d.) complex-Gaussian variables having a zero mean and a variance of σ_w^2.

Under the assumption of carrying out error-free decisions we have $\hat{x}[n,k] = x[n,k]$ and Equation (4) may be simplified to

$$\tilde{H}[n,k] = \frac{y[n,k]}{\hat{x}[n,k]} = H[n,k] + \frac{w[n,k]}{\hat{x}[n,k]}. \quad (5)$$

The Mean Square Error (MSE) associated with the LS FD-CTF estimator of (5) is given by

$$MSE_{\text{LS}} = E\left\{\left|H[n,k] - \tilde{H}[n,k]\right|^2\right\} = E\left\{\left|\frac{w[n,k]}{x[n,k]}\right|^2\right\}. \quad (6)$$

The less ambiguous measure of the estimator's performance is the Normalized MSE (NMSE), which is defined as the MSE normalized by the variance of the parameter being estimated. The NMSE corresponding to the estimator of Equation (5) is given by

$$NMSE_{\mathrm{LS}} = \frac{1}{\sigma_H^2} E\left\{ \left| \frac{w[n,k]}{x[n,k]} \right|^2 \right\}. \qquad (7)$$

The AWGN samples $w[n,k]$ are known to be i.i.d. complex-Gaussian and hence the MSE of Equation (6) is determined by the statistical distribution of the transmitted subcarrier-related samples $x[n,k]$. The NMSE encountered assumes its minimum value, when $|x[n,k]|^2 = \sigma_x^2$ is constant, as in the case of an M-PSK-modulated OFDM system. Thus, we have

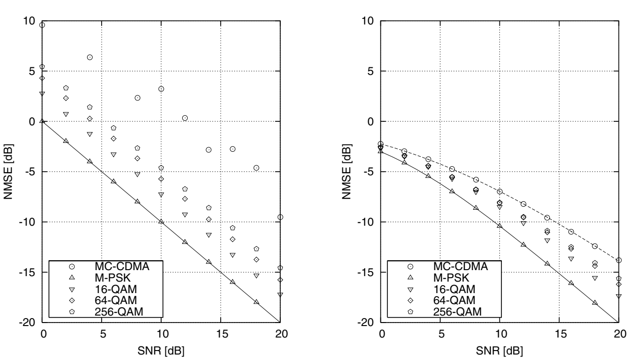
$$NMSE_{\mathrm{LS,min}} = \frac{1}{\sigma_H^2 \sigma_x^2} E\{w[n,k]\} = \frac{\sigma_w^2}{\sigma_H^2 \sigma_x^2} = \frac{1}{\gamma}, \qquad (8)$$

where

$$\gamma = \frac{1}{\sigma_w^2} E\{H[n,k]x[n,k]\} = \frac{\sigma_H^2 \sigma_x^2}{\sigma_w^2} \qquad (9)$$

is the average SNR level. On the other hand, the NMSE value will increase substantially, if the energy of the transmitted samples $x[n,k]$ varies as in the case of M-ary Quadrature Amplitude Modulation (M-QAM)-based OFDM or MC-CDMA. In fact, in the case of strictly Gaussian-distributed samples $x[n,k]$, which corresponds to encountering a MC-CDMA system having a sufficiently long spreading code, the NMSE value of Equation (7) does not exist, since the variance of the resultant Cauchy distributed variable associated with the ratio of two Gaussian-distributed variables $x[n,k]$ and $w[n,k]$ of Equation (7) cannot be defined [10]. The NMSE of the LS estimator of Equation (5) derived for QPSK, 16-, 64- and 256-QAM-modulated OFDM, as well as QPSK-modulated MC-CDMA is depicted in Figure 2(a). The solid line in Figure 2(a) corresponds to the lower NMSE bound described by Equation (8).

The performance degradation of the LS estimator of Equation (5) was imposed by the energy-fluctuation of the near-Gaussian distributed subcarrier-related samples $x[n,k]$, which renders the LS estimator inapplicable for employment in MC-CDMA systems. Therefore, for the sake of mitigating this performance degradation we would like to turn our attention to the MMSE estimation approach.



(a) (b)

Fig. 2. **NMSE** associated with (a) **Least Squares (LS)** and (b) **Minimum Mean Square Error (MMSE)** estimators of the uncorrelated Rayleigh-distributed subcarrier-related CTF coefficients $H[n,k]$ of Equation (1) corresponding to the various statistical distributions of the transmitted subcarrier-related samples $x[n,k]$. The markers on the plot correspond to the simulated cases of M-PSK, 16-, 64- and 256-QAM modulated OFDM as well as M-QAM modulated MC-CDMA, while the lines correspond to the analytically calculated performance recorded for the cases of M-PSK OFDM (solid) and MC-CDMA (dashed), which represent the lower and the upper NMSE bounds, respectively. Note that the upper bound for the LS estimator in conjunction with MC-CDMA does not exist.

2) MMSE CTF Estimator: In order to derive a FD-CTF estimator, which is suitable for employment in a MC-CDMA system, where the energy-fluctuation of the subcarrier-related samples $x[n,k]$ is near-Gaussian, we turn to the MMSE approach. Following the Bayesian linear model theory of [9], the MMSE estimator of the FD-CTF coefficients $H[n,k]$ of the scalar linear model described by Equation (1), where the parameters $H[n,k]$ are assumed to be complex-Gaussian distributed with a zero mean and a variance of σ_H^2, is given by [9]:

$$\tilde{H}_{\mathrm{MMSE}}[n,k] = \frac{x^*[n,k]y[n,k]}{|x[n,k]|^2 + \frac{\sigma_w^2}{\sigma_H^2}}. \qquad (10)$$

The corresponding NMSE can be expressed as [9]

$$NMSE_{\mathrm{MMSE}} = \frac{\sigma_w^2}{\sigma_H^2 |x[n,k]|^2 + \sigma_w^2}, \qquad (11)$$

where γ is the average SNR level defined by Equation (9). As we have seen previously in the context of Equation (11), the NMSE is determined by the statistical distribution of the transmitted subcarrier-related samples $x[n,k]$ and assumes its minimum value, when the energy of these samples $|x[n,k]|^2 = \sigma_x^2$ is constant. On the other hand, in contrast to the NMSE of the LS estimator of Equation (4), the NMSE of the MMSE estimator of Equation (10) is upper-bounded, which is evidenced by Figure 2(b). The NMSE assumes its maximum value, when the samples $x[n,k]$ are complex-Gaussian distributed, as in the case of a MC-CDMA system having a sufficiently high spreading factor.

C. A Posteriori *SS-CIR Estimator*

1) MMSE SS-CIR Estimator: We would like to commence our portrayal of the proposed channel estimation philosophy rendering the DDCE OFDM scheme of [1], [5] also applicable to employment in MC-CDMA with the derivation of the *a posteriori* MMSE SS-CIR estimator of Figure 1.

By substituting the FD-CTF of Equation (3) into (1) we arrive at

$$\boldsymbol{y}[n] = \mathrm{diag}\,(x[n,k])\,\boldsymbol{W}\boldsymbol{h}[n] + \boldsymbol{w}[n], \qquad (12)$$

where we define the $(K \times K)$-dimensional matrix $\mathrm{diag}\,(v[k])$ as a diagonal matrix having the corresponding elements of the vector $v[k]$ on the main diagonal, as well as the $(K \times K_0)$-dimensional Fourier Transform matrix $\boldsymbol{W}$, which corresponds to the Fourier transform of the zero-padded SS-CIR vector $\boldsymbol{h}[n]$ and is defined by $W_{kl} \triangleq W_K^{kl}$ for $k = 0, 1, \cdots, K-1$ and $l = 0, 1, \cdots, K_0 - 1$.

As before, the SS-CIR taps $h[l]$ are assumed to be uncorrelated complex-Gaussian distributed variables having a zero mean and a covariance matrix of $\boldsymbol{C}_h = \mathrm{diag}\left(\sigma_l^2\right)$.

The MMSE estimator of the SS-CIR taps $h[n,l]$ of the linear vector model described by Equation (12) is given by [9]

$$\hat{\boldsymbol{h}} = \left(\mathrm{diag}\left(\frac{1}{\sigma_l^2}\right) + \frac{1}{\sigma_w^2} \boldsymbol{W}^{\mathrm{H}} \mathrm{diag}\left(|\hat{x}[k]|^2\right) \boldsymbol{W} \right)^{-1} \times \frac{1}{\sigma_w^2} \boldsymbol{W}^{\mathrm{H}} \mathrm{diag}\,(\hat{x}^*[k])\,\boldsymbol{y}, \qquad (13)$$

where we omit the time-domain OFDM-block-spaced index n for the sake of notational simplicity.

Based on Equation (11) the corresponding NMSE associated with the lth MMSE SS-CIR tap estimate $\hat{h}[l]$ can be expressed as follows:

$$NMSE_{\mathrm{MMSE},l} = \frac{\sigma_l^2}{\sigma_H^2} \frac{\sigma_w^2}{\sigma_w^2 + K\sigma_l^2 \sigma_x^2} = \frac{1}{\gamma} \frac{1}{\frac{\sigma_w^2}{\sigma_x^2 \sigma_l^2} + K}. \qquad (14)$$

The overall NMSE corresponding to the MMSE SS-CIR estimator of Equation (13) may be found by summing all the lth NMSE contributions in Equation (14) over the K_0 taps of the CIR encountered, which can be expressed as

$$NMSE_{\mathrm{MMSE}} = \frac{1}{\gamma}\sum_{l=0}^{K_0-1}\frac{1}{\frac{\sigma_w^2}{\sigma_x^2\sigma_l^2}+K} \approx \frac{1}{\gamma}\frac{L}{K}, \tag{15}$$

where, as before, K is the number of OFDM subcarriers and γ is the average SNR value, while L is the number of non-zero SS-CIR taps encountered.

2) Low Complexity SS-CIR Estimator: As it is seen from Equation (13), the direct MMSE approach to the problem of estimating the SS-CIR taps $h[n,l]$ involves a time-variant matrix inversion, which introduces a relatively high computational complexity [1]. In order to reduce the associated computation complexity, we introduce a two-step low-complexity SS-CIR estimator invoking an approach, which bypasses the computationally intensive matrix inversion operation encountered in Equation (13). We will show that the method proposed first employs a scalar MMSE estimator of the subcarrier-related FD-CTF coefficients $H[n,k]$ of Equation (10), followed by employing a simplified MMSE SS-CIR estimator, which exploits the average MSE expression associated with the scalar MMSE FD-CTF estimator of the first processing step.

Following the Bayesian estimation theory of [9] the MMSE CTF estimates $\tilde{H}_{\mathrm{MMSE}}[n,k]$ of Equation (10) may be modelled as complex Gaussian-distributed variables having a mean identical to that of $H[n,k]$, which represents the actual FD-CTF coefficients encountered and a variance of $\sigma_v^2 = \sigma_H^2 NMSE_{\max}$, where σ_H^2 is the average channel output power and $NMSE_{\max}$ is the average NMSE. Thus we can write

$$\tilde{H}_{\mathrm{MMSE}}[n,k] = H[n,k] + v[n,k], \tag{16}$$

where $v[n,k]$ represents the i.i.d. complex-Gaussian noise samples having a zero mean and a variance of σ_v^2.

By substituting (3) into (16) we arrive at

$$\tilde{\boldsymbol{H}}_{\mathrm{MMSE}}[n] = \boldsymbol{W}\boldsymbol{h}[n] + \boldsymbol{v}[n], \tag{17}$$

where the $(K\times K_0)$-dimensional matrix $\boldsymbol{W}$ corresponds to the Fourier transform of the zero-padded SS-CIR vector $\boldsymbol{h}[n]$ and is defined by $W_{kl} \triangleq W_K^{kl}$ for $k = 0, 1, \cdots, K-1$ and $l = 0, 1, \cdots, K_0 - 1$.

The MMSE estimator of the FS-CIR taps $\alpha_l[n]$ of the linear vector model described by Equation (17) is given by [9]

$$\hat{\boldsymbol{h}} = (\boldsymbol{C}_h^{-1} + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{C}_v^{-1}\boldsymbol{W})^{-1}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{C}_v^{-1}\tilde{\boldsymbol{H}}_{\mathrm{MMSE}}, \tag{18}$$

where we omit the time-domain OFDM-block-spaced index n for the sake of notational simplicity and define $\boldsymbol{C}_h$ and $\boldsymbol{C}_v$ as the covariance matrices of the SS-CIR vector $\boldsymbol{h}$ and the scalar-MMSE FD-CTF estimator's noise vector $\boldsymbol{v}$, respectively. The elements of the noise vector $\boldsymbol{v}$ are assumed to be complex-Gaussian i.i.d. samples and therefore we have $\boldsymbol{C}_v = \sigma_v^2\boldsymbol{I}$. On the other hand, as follows from the assumption of having uncorrelated SS-CIR taps, the SS-CIR taps' covariance matrix is a diagonal matrix $\boldsymbol{C}_h = \mathrm{diag}\left(\sigma_l^2\right)$, where we have $\sigma_l^2 \triangleq E\left\{|h[n,l]|^2\right\}$. Substituting $\boldsymbol{C}_h$ and $\boldsymbol{C}_v$ into Equation (18) yields

$$\hat{\boldsymbol{h}} = \mathrm{diag}\left(\frac{\sigma_l^2}{\sigma_v^2 + K\sigma_l^2}\right)\boldsymbol{W}^{\mathrm{H}}\tilde{\boldsymbol{H}}_{\mathrm{MMSE}}. \tag{19}$$

Finally, upon substituting Equation (10) into Equation (19) we arrive at a scalar expression for the Reduced-Complexity (RC) *a posteriori* MMSE SS-CIR estimator in the form of:

$$\hat{h}[n,l] = \frac{\sigma_l^2}{\sigma_v^2 + K\sigma_l^2}\sum_{k=0}^{K-1} W_K^{kl}\frac{\hat{x}^*[n,k]y[n,k]}{|\hat{x}[n,k]|^2 + \frac{\sigma_w^2}{\sigma_H^2}}. \tag{20}$$

The corresponding NMSE associated with the lth LC-MMSE SS-CIR tap estimate $\hat{h}[l]$ is given by [9]

$$NMSE_{\mathrm{LCMMSE},l} = \frac{\sigma_v^2}{\sigma_H^2}\frac{\sigma_l^2}{\sigma_v^2 + K\sigma_l^2} = \frac{\sigma_v^2}{\sigma_H^2}\frac{1}{\frac{\sigma_v^2}{\sigma_l^2} + K}, \tag{21}$$

where $\sigma_v^2 = \sigma_H^2 NMSE_{H,\max}$ is the variance of the noise samples $v[k]$ in Equation (16), while $NMSE_{H,\max}$ is the maximum NMSE of the scalar MMSE FD-CTF estimator of Equation (10). The overall NMSE corresponding to the MMSE SS-CIR estimator of Equation (20) can be found by summing all of the lth contributions quantified by Equation (21) over the K_0 taps of the CIR encountered, which can be expressed as

$$NMSE_{\mathrm{LCMMSE}} = \frac{\sigma_v^2}{\sigma_H^2}\sum_{l=0}^{K_0-1}\frac{1}{\frac{\sigma_v^2}{\sigma_l^2} + K} \approx \frac{\sigma_v^2}{\sigma_H^2}\frac{L}{K}, \tag{22}$$

where, as before, K is the number of OFDM subcarriers and γ is the average SNR value, while L is the number of non-zero SS-CIR taps encountered. The resultant NMSE described by Equation (22) represents the lower-bound of the NMSE exhibited by the RC-MMSE SS-CIR estimator in conjunction with complex-Gaussian distributed transmitted samples $x[n,k]$ typically encountered in a MC-CDMA system having a high spreading factor.

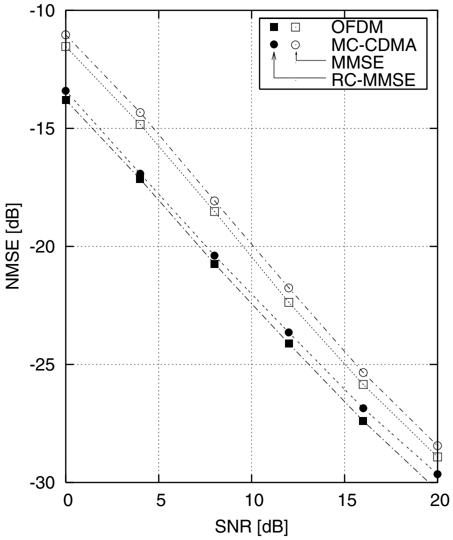
D. Complexity Study

As it was shown in Section III-B, the LS approach to the problem of DDCE-aided OFDM schemes [1] is not suitable in the case of MC-CDMA systems. The MMSE approach of Section III-C.1 constitutes an appropriate solution, however it exhibits a relatively high computational complexity imposed by the evaluation and inversion of the $(K_0 \times K_0)$-dimensional matrix $\left(\boldsymbol{A} + \boldsymbol{W}^{\mathrm{H}}\mathrm{diag}\left(|x[k]|^2\right)\boldsymbol{W}\right)$ in Equation (13). More explicitly, the MMSE SS-CIR estimator of Equation (13) has a computational complexity, which is of the order of $O(K^2K_0 + KK_0^2 + K_0^3)$, where K is the number of OFDM subcarriers and K_0 is the number of SS-CIR taps encountered. By contrast, the reduced-complexity SS-CIR estimator of Equation (20), which avoids the matrix inversion operation, has a complexity of the order of $O(K + K\log_2 K + K_0)$, which is similar to the complexity associated with the conventional LS estimator employed in [1]. It can be seen that the difference between the proposed estimation methods expressed in terms of the associated computational complexity is substantial.

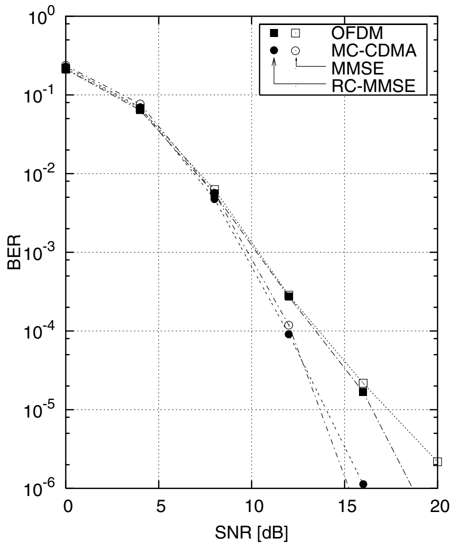
IV. Simulation Results

In this section, we present our simulation results for both the full-complexity MMSE and for the RC-MMSE SS-CIR estimation scheme advocated in the context of both OFDM and MC-CDMA systems communicating over seven-path dispersive Rayleigh fading channels.

Our simulations were performed in the base-band frequency domain and the system configuration characterised in Table I is to a large extent similar to that used in [5]. We assume having a total bandwidth of 800kHz. In the OFDM mode the system utilises 128 QPSK-modulated orthogonal subcarriers. In the MC-CDMA mode we employ a full set of 128-chip Walsh-Hadamard (WH) codes for frequency-domain spreading of the QPSK-modulated bits over the 128 orthogonal subcarriers. All the 128 WH spreading codes are assigned to a single user and hence the data-rate is similar in both the OFDM and the MC-CDMA modes. For forward error correction (FEC) we use $\frac{1}{2}$-rate turbo coding [11] employing two constraint-length $K = 3$ Recursive Systematic Convolutional (RSC) component codes and a 124-bit WCDMA code interleaver [12]. The octally represented generator polynomials of (7,5) were used.



(a)



(b)

Fig. 3. (a) **Normalised Mean Square Error (NMSE)** and (b) **Bit Error Rate (BER)** exhibited by channel estimator which follows the philosophy of Figure 1 and employs **Minimum Mean Square Error (MMSE)** and **Reduced-Complexity MMSE** *a posteriori* SS-CIR estimators of Equations (13) and (20) respectively. The *a priori* prediction is performed using the robust SS-CIR predictor [1] assuming matching propagation conditions described by the COST-207 BU channel model with normalised Doppler frequency of $F_D = 0.01$. The Turbo-coded QPSK-modulated OFDM and MC-CDMA modes are depicted using □ and ○ markers respectively.

We employed the seven-path Rayleigh-fading COST-207 Bad Urban (BU) channel model characterised in [13], using the delay spread of $\tau_{max} = 8\mu$s and the normalized Doppler frequency of $F_D = 0.01$.

Figure 3 demonstrates (a) the Normalized Mean Square Error (NMSE) performance and (b) the achievable turbo-coded Bit Error Rate (BER) of the DDCE scheme of Figure 1 using both the full-complexity MMSE and the RC-MMSE SS-CIR estimators described in Sections III-C.1 and III-C.2, respectively, in the context of both the above-mentioned OFDM and MC-CDMA systems. The simulations were carried out over the period of 10,000 QPSK-modulated $K = 128$-subcarrier OFDM/MC-CDMA symbols. It can be seen in Figure 3(a), that the RC-MMSE method performs at least as well as its MMSE counterpart in the context of both the OFDM and MC-CDMA systems considered. Moreover, as it becomes evident from Figure 3(b), the MMSE/RC-MMSE SS-CIR operating in the context of the MC-CDMA system outperforms its OFDM counterpart.

TABLE I

SYSTEM PARAMETERS.

Parameter	OFDM	MC-CDMA
Channel bandwidth	800 kHz	
Number of carriers K	128	
Symbol duration T	160 μs	
Max. delay spread τ_{max}	40 μs	
No. of CIR taps	3	
Channel interleaver	WCDMA [12] 248 bit	–
Modulation	QPSK	
Spreading scheme	–	WH
FEC component codes code interleaver	Turbo code [11] , rate 1/2 RSC, K=3(7,5) WCDMA (124 bit)	

V. CONCLUSIONS

In this paper we have proposed a low-complexity MMSE SS-CIR estimator, which is suitable for employment in DDCE-aided OFDM and MC-CDMA systems. As it was shown in Section IV, the performance of the low-complexity MMSE method proposed is at least as good as that of its high complexity counterpart, while its complexity does not exceed the complexity of the conventional LS-based estimator. Furthermore, it was shown that the turbo-coded MC-CDMA system using the proposed MMSE DDCE method outperforms the corresponding OFDM-based system.

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