Generic MMSE Decision Directed Channel Estimation for OFDM and MC-CDMA

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***Abstract*— In this letter we propose a Minimum Mean Square Error (MMSE) Channel Impulse Response (CIR) estimator-aided Decision Directed Channel Estimation (DDCE) method, which is suitable for employment in both OFDM and MC-CDMA systems. Firstly, we outline the difficulty associated with the employment of the Least Squares (LS) approach to the problem of *a posteriori* Frequency Domain Channel Transfer Function (FDCTF) estimation in the context of a MC-CDMA system and derive a suitable MMSE-based CIR estimator, which circumvents the LS method's deficiencies. Secondly, we demonstrate that the computational complexity associated with the conventional MMSE estimation method is relatively high compared to that of the LS method. Thus a reduced-complexity version of the MMSE estimator is proposed, which has a complexity similar to that exhibited by the LS-aided estimator. The MC-CDMA system using the reduced-complexity MMSE estimator proposed is shown to outperform the corresponding OFDM-based scheme.**

I. Introduction

The family of well-documented *decision directed* channel estimation (DDCE) methods [1]–[4] provides a suitable solution for the problem of channel estimation in OFDM-based systems. The major benefit of the DDCE scheme is that in contrast to purely *pilot assisted* channel estimation methods [5]–[8] both the pilot symbols as well as all the information symbols are utilised for channel estimation [4]. The simple philosophy of this method is that in the absence of transmission errors we can benefit from the availability of 100% pilot information by using the detected subcarrier symbols as an *a posteriori* reference signal. The employment of this method allows us to reduce the number of pilot symbols required.

A basic component of the DDCE schemes proposed in the literature is an *a posteriori* Least Squares (LS) temporal estimator of the OFDM-subcarrier-related Frequency-Domain Channel Transfer Function (FD-CTF) coefficients [1], [3], [4]. The accuracy of the resultant temporal estimates is typically enhanced using either one- or two-dimensional interpolation exploiting both the time- and the frequency-domain correlation between the adjacent FD-CTF coefficients. The LS-based temporal FD-CTF estimator was shown to be suitable for QPSK-modulated OFDM systems [3], [4], where the energy of the transmitted subcarrier-related symbols is constant. However, as it will be pointed out in Section III-B.1, the LS method cannot be employed in MC-CDMA systems, where – in contrast to OFDM systems – the energy of the transmitted subcarrier-related symbols fluctuates as a function of both the modulated sequence and that of the choice of the potentially non-constant-modulus modulation scheme itself. Thus the novel contribution of this letter is that *we propose a Minimum Mean Square Error (MMSE) Channel Impulse Response (CIR) estimator-aided DDCE method, which constitutes an appropriate solution for both OFDM and MC-CDMA systems. Furhermore, we demonstrate that the proposed MMSE technique has a reduced complexity, which is similar to that of the LS-based DDCE technique.*

Acknowledgements: The financial support of the mobile VCE and of the EU under the auspices of the Newcom and Phoenix projects is gratefully acknowledged.

The rest of this paper is structured as follows. The system model and the channel model considered are described in Section II. The difficulty of employing the Least Squares (LS) approach to the problem of estimating the OFDM-subcarrier-related FD-CTF coefficients is described in Section III-B.1. The alternative MMSE FD-CTF estimator circumventing the problem outlined in Section III-B.1 is analyzed in Section III-B.2. Our discourse evolves further by proposing a MMSE CIR estimator exploiting the frequency-domain correlation of the FD-CTF coefficients in Section III-C.1 and a reduced-complexity version of the CIR MMSE estimator considered is proposed in Section III-C.2. Finally, the achievable performance of the estimation methods proposed is studied in Section IV.

II. System Model

The discrete baseband model of the OFDM/MC-CDMA system can be described as in [9]

$$y[n,k] = H[n,k]x[n,k] + w[n,k], \tag{1}$$

for $k = 0, \ldots, K-1$ and all n, where $y[n,k], x[n,k]$ and $w[n,k]$ are the received symbol, the transmitted symbol and the Gaussian noise sample respectively, corresponding to the kth subcarrier of the nth OFDM symbol. Furthermore, $H[n,k]$ is the complex channel transfer function (CTF) coefficient associated with kth subcarrier and time instant n. Note that in the case of an M-QAM modulated OFDM system, $x[n,k]$ corresponds to the M-QAM symbol accommodated by the kth subcarrier, while in a MC-CDMA system, such as a Walsh-Hadamard Transform (WHT) assisted OFDM scheme using G-chip WH spreading code and hence capable of supporting

G users [4] we have

$$x[n,k] = \sum_{p=0}^{G-1} c[k,p] s[n,p], \quad (2)$$

where $c[k,p]$ is the kth chip of the pth spreading code, while $s[n,p]$ is the M-QAM symbol spread by the pth G-chip code. Each of the G spreading codes is constituted by G chips.

As it was pointed out in [3], in OFDM/MC-CDMA systems using a sufficiently long cyclic prefix and adequate synchronisation, the discrete CTF can be expressed as

$$H[n,k] = H(nT, k\Delta f) = \sum_{l=0}^{K_0-1} W_K^{kl} h[n,l], \quad (3)$$

for $k = 0, 1, \cdots, K-1$, where $h[n,l] = h(nT, lT/K)$ is the sample-spaced CIR (SS-CIR) having significant non-zero tap values only at sample-spaced raster positions and $W_K = \exp(-\jmath 2\pi/K)$. Note that in realistic channel conditions associated with non-sample-spaced path-delays the receiver will encounter dispersed received signal components in several neighbouring samples owing to the convolution of the transmitted signal with the system's impulse response, which we refer to as "leakage" [4]. This phenomenon is usually unavoidable and therefore the resultant SS-CIR $h[n,l]$ will be constituted of numerous correlated non-zero taps. Although the results of this contribution are applicable also in the context of estimating the fractionally-spaced CIR, we opted for concentrating our attention on the case of SS-CIRs for the sake of notational simplicity. Therefore, we assume hereafter a SS-CIR constituted of K_0 uncorrelated taps $h[n,l]$.

III. Channel Estimation

A. *Decision Directed Channel Estimator*

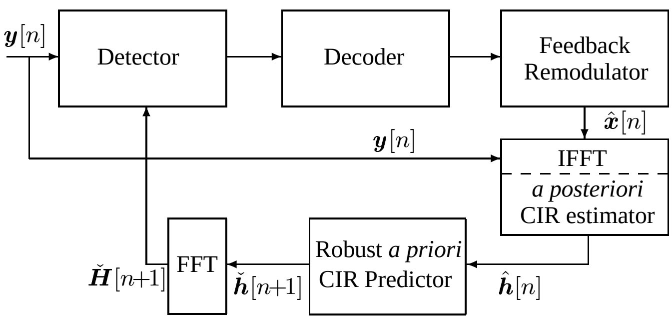

Fig. 1. Schematic of the channel estimator constituted by an *a posteriori* decision-directed CIR estimator, based on frequency-domain modulated symbol estimates, followed by an *a priori* CIR predictor.

The schematic of the channel estimation method considered is depicted in Figure 1. Our channel estimator is constituted by what we refer to as an *a posteriori* decision-directed CIR estimator, followed by an *a priori* CIR predictor [4]. As seen in Figure 1, the *a posteriori* SS-CIR estimator inputs are the frequency-domain signal $\boldsymbol{y}[n]$ and the decision-based transmitted symbol estimate $\hat{\boldsymbol{x}}[n]$. The transformation from the frequency to time domain is performed within the CIR estimator of Figure 1 and its output is an *a posteriori* estimate $\hat{x}[n,k]$ of the SS-CIR taps of Equation (3), which is fed into the low-rank time-domain SS-CIR tap predictor of Figure 1 for the sake of producing an *a priori* estimate $\check{h}[n+1,l],\ l = 0, 1, \cdots, K_0 - 1$ of the next SS-CIR on a SS-CIR tap-by-tap basis [4]. Finally, the predicted SS-CIR is converted to the FD-CTF with the aid of the FFT block of Figure 1. The resultant FD-CTF is employed by the receiver for the sake of detecting and decoding the next OFDM symbol.

B. A Posteriori *FD-CTF Estimation*

In order to emphasize the major difference between the OFDM and MC-CDMA systems in the context of the associated channel estimation scheme, first we analyze the performance of the temporal estimator of the subcarrier-related FD-CTF coefficients $H[n,k]$ based on the *a posteriori* decision-aided estimates of the transmitted subcarrier-related samples $x[n,k]$ of Equation (1).

1) Least Squares CTF estimator: Following Equation (1), the Least Squares (LS) approach [10] to the problem of estimating the discrete FD-CTF coefficients $H[n,k]$, based on the knowledge of the decision-aided estimates $\hat{x}[n,k]$ of the transmitted frequency-domain samples $x[n,k]$ of Equation (1) can be expressed as

$$\tilde{H}[n,k] = \frac{y[n,k]}{\hat{x}[n,k]} = H[n,k] \cdot \frac{x[n,k]}{\hat{x}[n,k]} + \frac{w[n,k]}{\hat{x}[n,k]}, \quad (4)$$

where $H[n,k]$ represents the Rayleigh-distributed FD-CTF coefficients having a variance of σ_H^2, while $x[n,k]$ denotes the transmitted subcarrier-related samples having zero mean and a variance of σ_x^2. The distribution of the samples $x[n,k]$ is dependent on the particular modulation scheme employed by the system. For instance, in a MC-CDMA system using an arbitrary modulation scheme, the samples $x[n,k]$ are complex-Gaussian distributed, having a Rayleigh-distributed magnitude $|x[n,k]|^2$ and uniformly-distributed phase $\theta[n,k]$. By contrast, in an M-PSK-modulated OFDM system the samples $x[n,k]$ are uniformly distributed within the set of M-PSK symbols having a constant amplitude $|x[n,k]| = \sigma_x$ and a discrete-uniform distributed phase $\theta[n,k] = 2\pi\frac{m}{M}, m = 0, 1, \cdots, M-1$. Finally, the noise samples $w[n,k]$ are independent identically distributed (i.i.d.) complex-Gaussian variables having a zero mean and a variance of σ_w^2.

Under the assumption of carrying out error-free decisions we have $\hat{x}[n,k] = x[n,k]$ and thus the Normalized Mean Square Error (NMSE) associated with the LS FD-CTF estimator of (4) is given by

$$NMSE_{\mathrm{LS}} = \frac{1}{\sigma_H^2} E\left\{ \left| \frac{w[n,k]}{x[n,k]} \right|^2 \right\}. \quad (5)$$

The AWGN samples $w[n,k]$ are known to be i.i.d. complex-Gaussian and hence the NMSE of Equation (5) is determined by the statistical distribution of the transmitted subcarrier-related samples $x[n,k]$. The NMSE encountered assumes its minimum value, when $|x[n,k]|^2 = \sigma_x^2$ is constant, as in the case of an M-PSK-modulated OFDM system. Thus, we have

$$NMSE_{\mathrm{LS,min}} = \frac{1}{\sigma_H^2 \sigma_x^2} E\{w[n,k]\} = \frac{\sigma_w^2}{\sigma_H^2 \sigma_x^2} = \frac{1}{\gamma}, \quad (6)$$

where $\gamma = \frac{\sigma_H^2 \sigma_x^2}{\sigma_w^2}$ is the average SNR level. On the other hand, the NMSE value will increase substantially, if the energy of the transmitted samples $x[n,k]$ varies as in the case of M-ary Quadrature Amplitude Modulation (M-QAM)-based OFDM or MC-CDMA. In fact, in the case of strictly Gaussian-distributed samples $x[n,k]$, which corresponds to encountering a MC-CDMA system having a sufficiently long spreading code, the NMSE value of Equation (5) does not exist, since the variance of the resultant Cauchy distributed variable associated with the ratio of two Gaussian-distributed variables $x[n,k]$ and $w[n,k]$ of Equation (5) cannot be defined [11]. The NMSE of the LS estimator of Equation (4) derived for QPSK, 16-, 64- and 256-QAM-modulated OFDM, as well as QPSK-modulated MC-CDMA is depicted in Figure 2(a). The solid line in Figure 2(a) corresponds to the lower NMSE bound described by Equation (6).

Consequently, the performance degradation of the LS estimator of Equation (4) is a result of the energy-fluctuations associated with the near-Gaussian distributed subcarrier-related samples $x[n,k]$, which renders the LS estimator inapplicable for employment in MC-CDMA systems. Consequently, for the sake of mitigating the resultant performance degradation we would like to turn our attention to the MMSE estimation approach.

2) MMSE CTF Estimator: Following the Bayesian theory of [10], the MMSE estimator of the FD-CTF coefficients $H[n,k]$ of the linear scalar model described by Equation (1), where the parameters $H[n,k]$ are assumed to be complex-Gaussian distributed with a zero mean and a variance of σ_H^2, is given by [10]

$$\tilde{H}_{\mathrm{MMSE}}[n,k] \;=\; \frac{x^*[n,k]y[n,k]}{|x[n,k]|^2 + \frac{\sigma_w^2}{\sigma_H^2}}. \tag{7}$$

The corresponding NMSE can be expressed as [10]

$$NMSE_{\mathrm{MMSE}} \;=\; \frac{\sigma_w^2}{\sigma_H^2 |x[n,k]|^2 + \sigma_w^2}. \tag{8}$$

As we have seen previously in the context of Equation (8), the NMSE is determined by the statistical distribution of the transmitted subcarrier-related samples $x[n,k]$ and assumes its minimum value, when the energy of these samples $|x[n,k]|^2 = \sigma_x^2$ is constant. On the other hand, in contrast to the NMSE of the LS estimator of Equation (4), the NMSE of the MMSE estimator of Equation (7) is upper-bounded, which is evidenced by Figure 2(b). The NMSE assumes its maximum value, when the samples $x[n,k]$ are complex-Gaussian distributed, as in the case of a MC-CDMA system having a sufficiently high spreading factor. Explicitly, the maximum NMSE may be expressed as follows:

$$\begin{aligned} NMSE_{H,\max}(\gamma) \;&=\; \underset{x\in N(0,\sigma_x^2)}{E}\{NMSE(\gamma,x)\} \\ =\int_0^\infty \frac{1}{\gamma r+1}\; e^{-r}\mathrm{d}r \;&=\; \frac{1}{\gamma}\; e^{\frac{1}{\gamma}}\mathrm{Ei}\left(\frac{1}{\gamma}\right), \end{aligned} \tag{9}$$

where we define the *exponential integral* function as $\mathrm{Ei}(x) = \int_x^\infty \frac{e^{-t}}{t}\,\mathrm{d}t$.

In the following section we employ the following vectorial notation $\boldsymbol{v}[n] = (v[n,1],\cdots,v[n,K])^{\mathrm{T}}$.

C. A Posteriori *SS-CIR Estimator*

1) MMSE SS-CIR Estimator: We would like to commence our portrayal of the proposed channel estimation philosophy by briefly recalling the DDCE-aided OFDM scheme advocated in [4, Chapter 15], [3].

By substituting the FD-CTF of Equation (3) into (1) we arrive at

$$y[n,k] = \sum_{l=0}^{K_0-1} W_K^{kl} h[n,l]x[n,k] + w[n,k]. \tag{10}$$

The SS-CIR taps $h[l]$ are assumed to be uncorrelated complex-Gaussian distributed variables having a zero mean and a covariance matrix $\boldsymbol{C}_h = \mathrm{diag}\left(\sigma_l^2\right)$. The MMSE estimator of the SS-CIR taps $h[n,l]$ of the linear vector model described by Equation (10) is given by [10]

$$\begin{aligned} \hat{\boldsymbol{h}} \;&=\; \left(\sigma_w^2\boldsymbol{I} + \mathrm{diag}\left(\sigma_l^2\right)\boldsymbol{W}^{\mathrm{H}}\mathrm{diag}\left(|\hat{x}[k]|^2\right)\boldsymbol{W}\right)^{-1} \\ &\times \quad \mathrm{diag}\left(\sigma_l^2\right)\boldsymbol{W}^{\mathrm{H}}\mathrm{diag}\left(\hat{x}^*[k]\right)\boldsymbol{y}, \end{aligned} \tag{11}$$

where we omit the time-domain OFDM-symbol-related index n for the sake of notational simplicity.

2) Reduced-Complexity SS-CIR Estimator: As it is seen from Equation (11), the direct MMSE approach to the problem of estimating the SS-CIR taps $h[n,l]$ involves an inversion of a time-variant matrix, which imposes a relatively high computational complexity [4]. In order to reduce the associated computational complexity, we introduce a two-step reduced-complexity SS-CIR estimator invoking a method, which bypasses the computationally intensive matrix inversion operation encountered in Equation (11).

Following the Bayesian theory of [10], the MMSE CTF estimates $\tilde{H}_{\mathrm{MMSE}}[n,k]$ of Equation (7) may be modelled as complex Gaussian-distributed variables having a mean identical to that of $H[n,k]$, which represents the actual FD-CTF coefficients encountered and a variance of $\sigma_v^2 = \sigma_H^2 NMSE_{\max}$, where σ_H^2 is the average channel output power and $NMSE_{\max}$ is the average NMSE quantified in Equation (9). Thus we can write

$$\tilde{H}_{\mathrm{MMSE}}[n,k] = H[n,k] + v[n,k], \tag{12}$$

where $v[n,k]$ represents the i.i.d. complex-Gaussian noise samples having a zero mean and a variance of σ_v^2. Furthermore, by substituting (3) into (12) we arrive at

$$\tilde{H}_{\mathrm{MMSE}}[n,k] = \sum_{l=0}^{K_0-1} W_K^{kl} h[n,k] + v[n,k], \tag{13}$$

where $W_K = e^{-\jmath 2\pi\frac{1}{K}}$. Furthermore, the MMSE estimator of the SS-CIR taps $h[n,l]$ of the linear vector model described by Equation (13) is given by [10]

$$\hat{\boldsymbol{h}} = (\boldsymbol{C}_h^{-1} + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{C}_v^{-1}\boldsymbol{W})^{-1}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{C}_v^{-1}\tilde{\boldsymbol{H}}_{\mathrm{MMSE}}, \tag{14}$$

where we define $\boldsymbol{C}_h$ and $\boldsymbol{C}_v$ as the covariance matrices of the SS-CIR vector $\boldsymbol{h}$ and the scalar-MMSE FD-CTF estimator's noise vector $\boldsymbol{v}$, respectively. The elements of the noise vector $\boldsymbol{v}$ are assumed to be complex-Gaussian i.i.d. samples and therefore we have $\boldsymbol{C}_v = \sigma_v^2\boldsymbol{I}$. On the other hand, as follows from the assumption of having uncorrelated SS-CIR taps, the

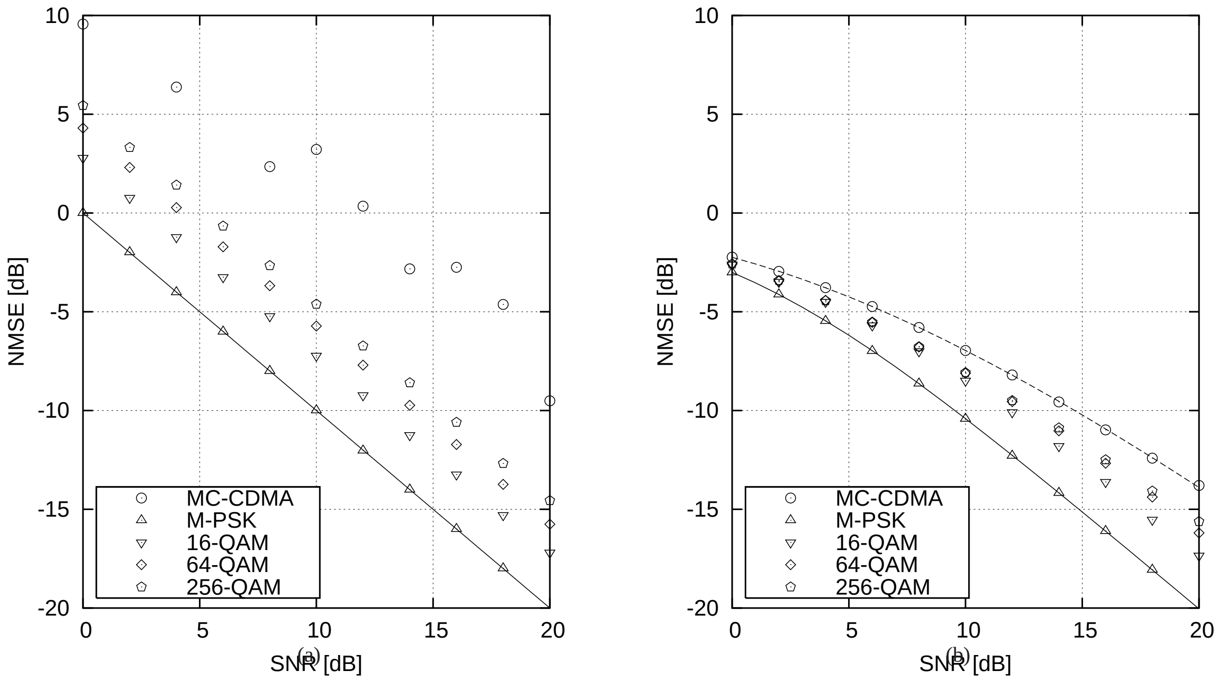

Fig. 2. **NMSE** associated with (a) **Least Squares (LS)** and (b) **Minimum Mean Square Error (MMSE)** estimators of the uncorrelated Rayleigh-distributed subcarrier-related CTF coefficients $H[n,k]$ of Equation (1) corresponding to the various statistical distributions of the transmitted subcarrier-related samples $x[n,k]$. The markers on the plot correspond to the simulated cases of M-PSK, 16-, 64- and 256-QAM modulated OFDM as well as M-QAM modulated MC-CDMA, while the lines correspond to the analytically calculated performance recorded for the cases of M-PSK OFDM (solid) and MC-CDMA (dashed), which represent the lower and the upper NMSE bounds, respectively. Note that the upper bound for the LS estimator in conjunction with MC-CDMA does not exist.

SS-CIR taps' covariance matrix is a diagonal matrix $\boldsymbol{C}_h = \mathrm{diag}\left(\sigma_l^2\right)$, where $\sigma_l^2 = E\left\{|h[n,l]|^2\right\}$. Substituting $\boldsymbol{C}_h$ and $\boldsymbol{C}_v$ into Equation (14) yields

$$\begin{aligned}\hat{\boldsymbol{h}} &= \left(\mathrm{diag}\left(\frac{1}{\sigma_l^2}\right) + \frac{1}{\sigma_v^2}\boldsymbol{W}^{\mathtt{H}}\boldsymbol{W}\right)^{-1} W^{\mathtt{H}}\frac{1}{\sigma_v^2}\tilde{\boldsymbol{H}}_{\mathrm{MMSE}} \\ &= \mathrm{diag}\left(\frac{\sigma_l^2}{\sigma_v^2 + K\sigma_l^2}\right)\boldsymbol{W}^{\mathtt{H}}\tilde{\boldsymbol{H}}_{\mathrm{MMSE}}, \qquad (15)\end{aligned}$$

where we have exploited the fact that $[\boldsymbol{W}^{\mathtt{H}}\boldsymbol{W}]_{l,l'} = \sum_{k=0}^{K-1} e^{-\jmath 2\pi\frac{k(l-l')}{K}} = K\delta[l-l']$ and therefore $\boldsymbol{W}^{\mathtt{H}}\boldsymbol{W} = K\boldsymbol{I}$, where $\boldsymbol{I}$ is a $(K_0 \times K_0)$-dimensional identity matrix.

Finally, upon substituting Equation (7) into Equation (15) we arrive at a scalar expression for the Reduced-Complexity (RC) *a posteriori* MMSE SS-CIR estimator in the form of

$$\hat{h}[n,l] = \frac{\sigma_l^2}{\sigma_v^2 + K\sigma_l^2}\sum_{k=0}^{K-1} W_K^{kl}\frac{\hat{x}^*[n,k]y[n,k]}{|\hat{x}[n,k]|^2 + \frac{\sigma_w^2}{\sigma_H^2}}. \qquad (16)$$

Observe that the RC-MMSE estimator of Equation (16) does not require any matrix inversion and therefore the associated complexity is similar to that exhibited by the conventional LS estimator.

In order to complete the DDCE scheme of Figure 1 we employ the *a priori* CIR predictor as derived in [4]. For the sake of brevity we leave the description of the *a priori* CIR predictor employed out of the scope of this contribution and the interested reader may refer to [4] for further details.

IV. Simulation Results

Our simulations were performed in the base-band frequency domain and the system configuration characterised in Table I is reminiscent to that used in [3]. In the OFDM mode, the system utilizes 128 QPSK-modulated orthogonal subcarriers. In the MC-CDMA mode we employ a full set of 128-chip Walsh-Hadamard (WH) codes for frequency-domain spreading of the QPSK-modulated bits over the 128 orthogonal subcarriers. All the 128 WH spreading codes are assigned to a single user and hence the data-rate is similar in both the OFDM and the MC-CDMA modes. For forward error correction (FEC) we use $\frac{1}{2}$-rate turbo coding [13] employing two constraint-length $K = 3$ Recursive Systematic Convolutional (RSC) component codes and a 124-bit WCDMA code interleaver [12]. The octally represented generator polynomials of (7,5) were used. We employed the seven-path Rayleigh-fading COST-207 Bad Urban (BU) channel model characterised in [14], using the delay spread of $\tau_{max} = 8\mu$s and the normalized Doppler frequency of $F_D = 0.01$.

TABLE I

System parameters.

Parameter	OFDM	MC-CDMA
Channel bandwidth	800 kHz	
Number of carriers K	128	
Symbol duration T	160 μs	
Max. delay spread τ_{max}	40 μs	
No. of CIR taps	3	
Channel interleaver	WCDMA [12] 248 bit	–
Modulation	QPSK	
Spreading scheme	–	WH
FEC component codes code interleaver	Turbo code [13] , rate 1/2 RSC, K=3(7,5) WCDMA (124 bit)	

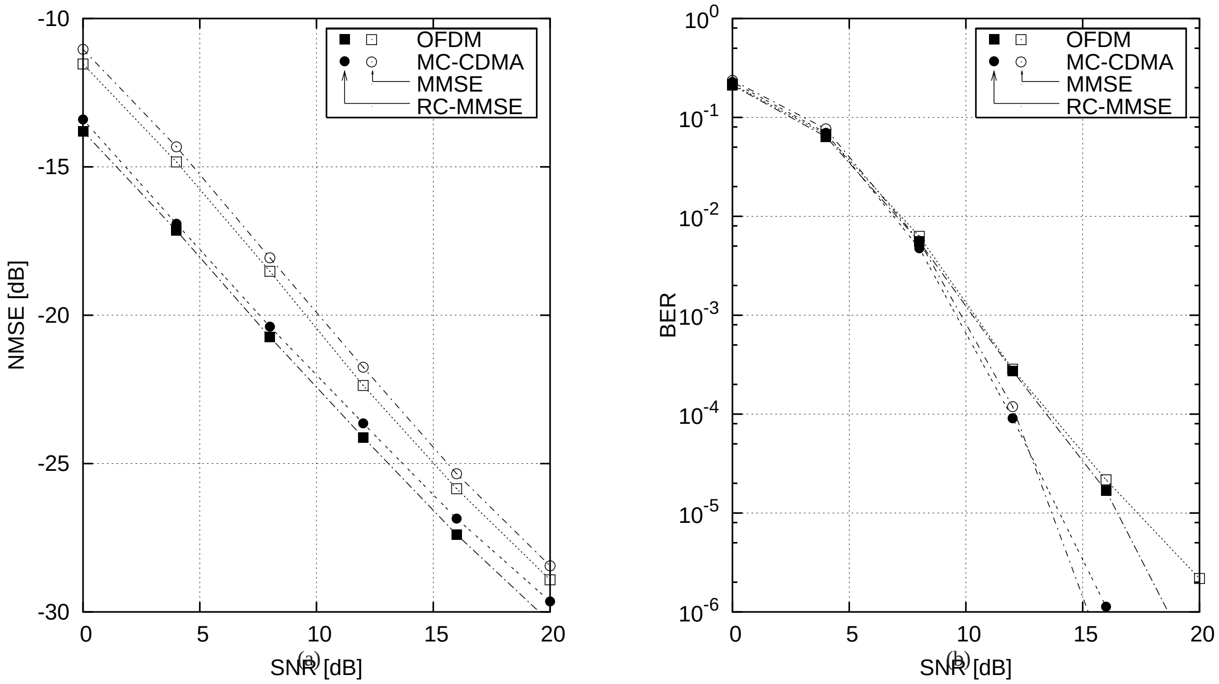

Fig. 3. (a) **Normalised Mean Square Error (NMSE)** and (b) **Bit Error Rate (BER)** exhibited by the channel estimator which follows the philosophy of Figure 1 and employs the Minimum Mean Square Error (MMSE) and the Reduced-Complexity MMSE *a posteriori* SS-CIR estimators of Equations (11) and (14), respectively. The *a priori* prediction is performed using the robust SS-CIR predictor [4] assuming matching propagation conditions described by the COST-207 BU channel model having a normalised Doppler frequency of $f_D = 0.01$. The turbo-coded QPSK-modulated OFDM and MC-CDMA modes are identified using the and $\circ$ markers, respectively.

Figure 3 demonstrates (a) the Normalized Mean Square Error (NMSE) performance and (b) the achievable turbo-coded Bit Error Rate (BER) of the DDCE scheme of Figure 1 using both the full-complexity MMSE and the RC-MMSE SS-CIR estimators described in Sections III-C.1 and III-C.2, respectively, in the context of both the above-mentioned OFDM and MC-CDMA systems. The simulations were carried out over the period of 10,000 QPSK-modulated $K = 128$-subcarrier OFDM/MC-CDMA symbols. It can be seen in Figure 3(a), that the RC-MMSE method performs at least as well as its MMSE counterpart in the context of both the OFDM and MC-CDMA systems considered. Moreover, as it becomes evident from Figure 3(b), the MMSE/RC-MMSE SS-CIR operating in the context of the MC-CDMA system outperforms its OFDM counterpart.

V. Conclusions

In this paper we have proposed a reduced-complexity MMSE SS-CIR estimator, which is suitable for employment in both DDCE-aided OFDM and MC-CDMA systems. As it was shown in Section IV, the performance of the reduced-complexity MMSE method proposed is at least as good as that of its high complexity counterpart, while its complexity does not exceed that of the conventional LS-based estimator. Furthermore, it was shown that the turbo-coded MC-CDMA system using the proposed MMSE DDCE method outperforms the corresponding OFDM-based system.

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