REDUCED-COMPLEXITY MAXIMUM-LIKELIHOOD DETECTION IN MULTIPLE-ANTENNA-AIDED MULTICARRIER SYSTEMS

Jos Akhtman and Lajos Hanzo *
University of Southampton
Southampton, SO17 1BJ, UK
{yja02r,lh}@soton.ecs.ac.uk

Abstract In this contribution we explore a family of novel Optimized Hierarchy Reduced Search Algorithm (OHRSA)-aided space-time processing methods, which may be regarded as an advanced extension of the Complex Sphere Decoder (CSD) method. The algorithm proposed extends the potential application range of the CSD method, as well as reduces the associated computational complexity.

1. Introduction

Multi-Carrier (MC) modulation techniques [1, 2] have found their way into several wireless broadband communications standards as well as into local area networks. Owing to their advantageous properties they also constitute strong contenders for the next-generation cellular mobile communications standards.

The relevant information-theoretical analysis predicts [3] that substantial capacity gains are achievable in wireless communication systems employing a Multiple Input Multiple Output (MIMO) architecture using multiple antennas. Specifically, provided that the fading processes corresponding to different transmit-receive antenna pairs may be assumed to be independently Rayleigh distributed, the associated attainable ca-

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pacity was shown to linearly increase with the smaller of the numbers of the transmit and receive antennas. Additionally, the employment of a MIMO architecture allows for the efficient exploitation of the spatial diversity available in a wireless MIMO environment, thus an improvement of the system's transmission integrity, as well as a further increase in the system's capacity becomes possible.

Hence, in this contribution we propose a novel Optimized Hierarchy Reduced Search Algorithm (OHRSA)-aided space-time processing method, which may be regarded as an advanced extension of the Complex Sphere Decoder (CSD) method, portrayed in [4]. The algorithm proposed extends the potential application range of the CSD methods of [5] and [4], as well as reduces the associated computational complexity. Moreover, the OHRSA-aided SDM detector proposed combines the near-optimum performance of the ML SDM detector with the low-complexity of the linear MMSE SDM detector, which renders it an attractive design alternative for practical systems.

The rest of this paper is structured as follows. The system model as well as the principles of ML space-time detection are briefly outlined in Section 2.1. A novel recursive ML detection technique is derived in Section 2.2. The corresponding bitwise real-valued system model is derived in Section 2.3. Furthermore, the search optimization rules are described in Section 2.4. The achievable performance of the proposed technique is quantified using extensive computer simulations and the corresponding results are provided in Section 3, before offering our conclusions in Section 4.

2. Optimized Hierarchy Reduced Search Algorithm

2.1 Maximum Likelihood Detection

The subcarrier-related MIMO-OFDM system model considered is given by [1]

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{y}, \mathbf{w}$ and $\mathbf{x}$ denote the n_{r}-dimensional received signal and AWGN sample vectors as well as the m_{t}-dimensional transmitted signal vector, respectively. Furthermore, $\mathbf{H}$ represents a $(n_{\mathrm{r}} \times m_{\mathrm{t}})$-dimensional matrix of subcarrier-related CTF coefficients. Note that for the sake of brevity we omit the OFDM subcarrier and symbol indices k and n. As outlined in [1], the ML SDM detector provides an m_{t}-antenna-based estimated signal vector candidate $\hat{\mathbf{s}}$, which maximizes the objective function defined as the conditional *a posteriori* probability $\mathtt{P}\{\check{\mathbf{s}}|\mathbf{y},\mathbf{H}\}$ over the set $\mathcal{M}^{m_{\mathrm{t}}}$

of legitimate solutions. More explicitly, we have

$$\hat{\mathbf{s}} = \arg \max_{\check{\mathbf{s}} \in \mathcal{M}^{m_t}} \mathsf{P}\left\{\check{\mathbf{s}}|\mathbf{y},\mathbf{H}\right\}, \quad (2)$$

where $\mathcal{M}^{m_t}$ is the set of **all possible** m_t-dimensional candidate symbol vectors of the m_t-antenna-based transmitted signal vector $\mathbf{s}$.

Furthermore, it was shown in [1] that the probability maximization problem of Equation (2) is equivalent to the corresponding Euclidean distance minimization problem. Specifically, we have

$$\hat{\mathbf{s}} = \arg \min_{\check{\mathbf{s}} \in \mathcal{M}^{m_t}} \|\mathbf{y} - \mathbf{H}\check{\mathbf{s}}\|^2, \quad (3)$$

where the probability-based objective function of Equation (2) is substituted by the objective function determined by the Euclidean distance between the received signal vector $\mathbf{y}$ and the corresponding product of the channel matrix $\mathbf{H}$ with the *a priori* candidate of the transmitted signal vector $\check{\mathbf{s}} \in \mathcal{M}^{m_t}$.

2.2 Recursive ML Detection

Subsequently, our detection method relies on the observation, which may be summarized in the following lemma. For the sake of brevity in this contribution we omit the proof of Lemma 1.

LEMMA 1 *The ML solution of Equation* (2) *of a noisy linear problem described by Equation* (1) *is given by*

$$\hat{\mathbf{s}} = \arg \min_{\check{\mathbf{s}} \in \mathcal{M}^{m_t}} \left\{\|\mathbf{U}(\check{\mathbf{s}} - \hat{\mathbf{x}})\|^2\right\}, \quad (4)$$

where $\mathbf{U}$ *is an upper-triangular matrix having positive real-valued elements on the main diagonal and satisfying*

$$\mathbf{U}^{\mathrm{H}}\mathbf{U} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_w^2\mathbf{I}), \quad (5)$$

while

$$\hat{\mathbf{x}} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_w^2\mathbf{I})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y} \quad (6)$$

is the unconstrained MMSE estimate of the transmitted signal vector $\mathbf{s}$.

Observe that Lemma 1 imposes no constraints on the dimensions, or rank of the matrix $\mathbf{H}$ of the linear system described by Equation (1). This property is particularly important, since it enables us to apply our proposed detection technique to the scenario of *over-loaded* systems, where the number of transmit antenna elements exceeds that of the receive antenna elements.

Using Lemma 1, in particular the fact that the matrix $\mathbf{U}$ is an upper-triangular matrix, we may introduce an objective function $J(\check{\mathbf{s}})$ described as follows

$$\begin{aligned} J(\check{\mathbf{s}}) &= \|\mathbf{U}(\check{\mathbf{s}}-\hat{\mathbf{x}})\|^2 = (\check{\mathbf{s}}-\hat{\mathbf{x}})^{\mathrm{H}}\mathbf{U}^{\mathrm{H}}\mathbf{U}(\check{\mathbf{s}}-\hat{\mathbf{x}}) \\ &= \sum_{i=1}^{m_\mathrm{t}} \Big| \sum_{j=i}^{m_\mathrm{t}} u_{ij}(\check{s}_j - \hat{x}_j) \Big|^2 = \sum_{i=1}^{m_\mathrm{t}} \phi_i(\check{\mathbf{s}}_i), \end{aligned} \tag{7}$$

where $J(\check{\mathbf{s}})$ and $\phi_i(\check{\mathbf{s}}_i)$ are positive real-valued cost and sub-cost functions, respectively. Elaborating a little further, we have

$$\begin{aligned} \phi_i(\check{\mathbf{s}}_i) &= \Big| \sum_{j=i}^{m_\mathrm{t}} u_{ij}(\check{s}_j - \hat{x}_j) \Big|^2 \\ &= \Big| u_{ii}(\check{s}_i - \hat{x}_i) + \underbrace{\sum_{j=i+1}^{m_\mathrm{t}} u_{ij}(\check{s}_j - \hat{x}_j)}_{a_i} \Big|^2. \end{aligned} \tag{8}$$

Note that the term a_i is a complex-valued scalar, which is independent of the specific symbol value $\check{s}_i$ of the ith element of the *a priori* candidate signal vector $\check{\mathbf{s}}$.

Furthermore, let $J_i(\check{\mathbf{s}}_i)$ be a Cumulative Sub-Cost (CSC) function recursively defined as

$$J_{m_\mathrm{t}}(\check{s}_{m_\mathrm{t}}) = \phi_{m_\mathrm{t}}(\check{s}_{m_\mathrm{t}}) = |u_{m_\mathrm{t}m_\mathrm{t}}(\check{s}_{m_\mathrm{t}} - \hat{x}_{m_\mathrm{t}})|^2 \tag{9a}$$

$$J_i(\check{\mathbf{s}}_i) = J_{i+1}(\check{\mathbf{s}}_{i+1}) + \phi_i(\check{\mathbf{s}}_i), \quad m_\mathrm{t}–1, \cdots, 1, \tag{9b}$$

where we define the candidate subvector as $\check{\mathbf{s}}_i = [\check{s}_i, \cdots, \check{s}_{m_\mathrm{t}}]$. Clearly, $J_i(\check{\mathbf{s}}_i)$ exhibits the following essential property

$$J(\check{\mathbf{s}}) = J_1(\check{\mathbf{s}}_1) > J_2(\check{\mathbf{s}}_2) > \cdots > J_{m_\mathrm{t}}(\check{s}_{m_\mathrm{t}}) > 0 \tag{10}$$

for all possible realizations of $\hat{\mathbf{x}} \in \mathbb{C}^{m_\mathrm{t}}$ and $\check{\mathbf{s}} \in \mathcal{M}^{m_\mathrm{t}}$, where the space $\mathbb{C}^{m_\mathrm{t}}$ contains all possible unconstrained MMSE estimates $\hat{\mathbf{x}}$ of the transmitted signal vector $\mathbf{s}$.

2.3 Bitwise System Model

It is evident that each phasor point c_m of an M-QAM constellation map may be represented as the inner product of a unique bit-based vector $\boldsymbol{d}_m = \{d_{ml} = -1, 1\}_{l=1,\cdots,b}$ and the corresponding *quantisation vector* $\boldsymbol{q}$. Specifically, we have

$$c_m = \boldsymbol{q}^{\mathrm{T}}\boldsymbol{d}_m. \tag{11}$$

For instance, the quantization vectors corresponding to the modulation schemes of QPSK and 16-QAM are $\boldsymbol{q} = \frac{1}{\sqrt{2}}[1, j]$ and $\boldsymbol{q} = \frac{1}{\sqrt{10}}[1, j, 2, 2j]$, respectively.

Furthermore, we define a $(bm_{\mathrm{t}} \times m_{\mathrm{t}})$-dimensional *quantization matrix* $\mathbf{Q} = \mathbf{I} \otimes \boldsymbol{q}$, where $\mathbf{I}$ is an $(m_{\mathrm{t}} \times m_{\mathrm{t}})$-dimensional identity matrix and $\boldsymbol{q}$ is the aforementioned *quantization vector*, while $\otimes$ denotes the *matrix direct product* [6]. Consequently the M-QAM-modulated signal vector $\mathbf{s}$ may be represented as

$$\mathbf{s} = \mathbf{Q}\mathbf{t}, \tag{12}$$

where $\mathbf{t} = [\boldsymbol{t}_1^{\mathrm{T}}, \cdots, \boldsymbol{t}_{m_{\mathrm{t}}}^{\mathrm{T}}]^{\mathrm{T}}$ is a $r = bm_{\mathrm{t}}$-dimensional column supervector comprising the bit-based vectors $\boldsymbol{t}_i$ associated with each transmitted signal vector component s_i. Substituting Equation (12) into the system model of Equation (1) yields

$$\mathbf{y} = \mathbf{H}\mathbf{Q}\mathbf{t} + \mathbf{w}. \tag{13}$$

Moreover, since $\mathbf{t}$ is a real-valued vector, we can elaborate a bit further and deduce a real-valued system model as follows

$$\tilde{\mathbf{y}} = \begin{bmatrix} \mathcal{R}\{\mathbf{y}\} \\ \mathcal{I}\{\mathbf{y}\} \end{bmatrix} = \begin{bmatrix} \mathcal{R}\{\mathbf{HQ}\} \\ \mathcal{I}\{\mathbf{HQ}\} \end{bmatrix} \mathbf{t} + \begin{bmatrix} \mathcal{R}\{\mathbf{w}\} \\ \mathcal{I}\{\mathbf{w}\} \end{bmatrix} = \tilde{\mathbf{H}}\mathbf{t} + \tilde{\mathbf{w}}, \tag{14}$$

where $\tilde{\mathbf{H}}$ is a real-valued $(2n_{\mathrm{r}} \times bm_{\mathrm{t}})$-dimensional bitwise channel matrix.

Substituting the system model of Equation (1) by the bitwise real-valued system model of Equation (14) and subsequently exploiting the properties of the corresponding objective function $J(\check{\mathbf{t}})$ of Equation (10) enables us to employ a highly efficient reduced-complexity search algorithm, which decreases the number of objective function evaluations of the minimization problem outlined in Equation (4) to a small fraction of the set $\mathcal{M}^{m_{\mathrm{t}}}$. This reduced-complexity search algorithm is outlined in the next section.

2.4 Search Strategy

Firstly, we commence the recursive search process with the evaluation of the CSC function value $J_r(\check{t}_r)$ of Equation (9a). Secondly, at each recursive step i of the search algorithm proposed we stipulate two legitimate hypotheses -1 and 1 concerning the value of the transmitted bitwise symbol t_i and subsequently calculate the conditioned sub-cost function $J_i(\check{\mathbf{t}}_i)$ of Equation (9b). Furthermore, for each tentatively assumed value of $\check{t}_i$ we execute a successive recursive search step $i-1$, which is conditioned on the hypotheses made in all preceding recursive steps $j = i, \cdots, r = bm_{\mathrm{t}}$. Upon each arrival at the index $i = 1$ of the recursive process, a complete candidate vector $\check{\mathbf{t}}$ is hypothesized and the corresponding value of the cost function $J(\check{\mathbf{t}})$ formulated in Equation (7) is evaluated.

Observe that the recursive hierarchical search procedure described above may be employed to perform an exhaustive search through all possible values of the transmitted signal vector $\check{\mathbf{t}}$ and the resultant search process is guaranteed to arrive at the ML solution $\hat{\mathbf{t}}$, which minimizes the value of the cost function $J(\check{\mathbf{t}})$ of Equation (7). Fortunately however, as opposed to other ML search schemes, the search process described above can be readily optimized, resulting in a dramatic reduction of the associated computational complexity. Specifically, the potential optimization complexity gain accrues from the fact that most of the hierarchical search branches can be discarded at an early stage of the recursive search process. The corresponding optimization rules proposed may be outlined as follows.

Rule 1 We reorder the system model of Equation (1) as suggested in [7]. Specifically, we apply the *best-first* detection strategy outlined in [1], which implies that the transmitted signal vector components are detected in the decreasing order of the associated channel quality. More specifically, the columns of the bitwise channel matrix $\tilde{\mathbf{H}}$ are sorted in the increasing order of their norm. Namely, we have

$$\|(\tilde{\mathbf{H}})_1\|^2 \leq \|(\tilde{\mathbf{H}})_2\|^2 \leq \cdots \leq \|(\tilde{\mathbf{H}})_r\|^2, \tag{15}$$

where $(\tilde{\mathbf{H}})_i$ denotes the ith column of the bitwise channel matrix $\tilde{\mathbf{H}}$.

Rule 2 At each recursive detection step $i = r, \cdots, 1$, the potential candidate values $d_m = \{-1, 1\}$ of the transmitted bitwise signal component t_i are considered in the increasing order of the corresponding value of the sub-cost function $\phi_i(\check{\mathbf{t}}_i) = \phi_i(d_m, \check{\mathbf{t}}_{i+1})$ of Equation (8), where we have

$$\phi_i(d_1, \check{\mathbf{t}}_{i+1}) < \phi_i(d_2, \check{\mathbf{t}}_{i+1}) \tag{16}$$

Rule 3 We define a *cut-off* value of the cost function $J_{\min} = \min\{J(\check{\mathbf{t}})\}$ as the minimum value of the total cost function obtained up to the present point of the search process. Consequently, at each arrival at step $i = 1$ of the recursive search process, the *cut-off* value of the cost function is updated as follows

$$J_{\min} = \min\{J_{\min}, J(\check{\mathbf{t}})\}. \tag{17}$$

Rule 4 Finally, at each recursive detection step i, only the high probability search branches corresponding to the highly likely bitwise symbol candidates d_m resulting in particularly low values of the CSC function obeying $J_i(d_m) < J_{\min}$ are pursued.

3. Simulation results

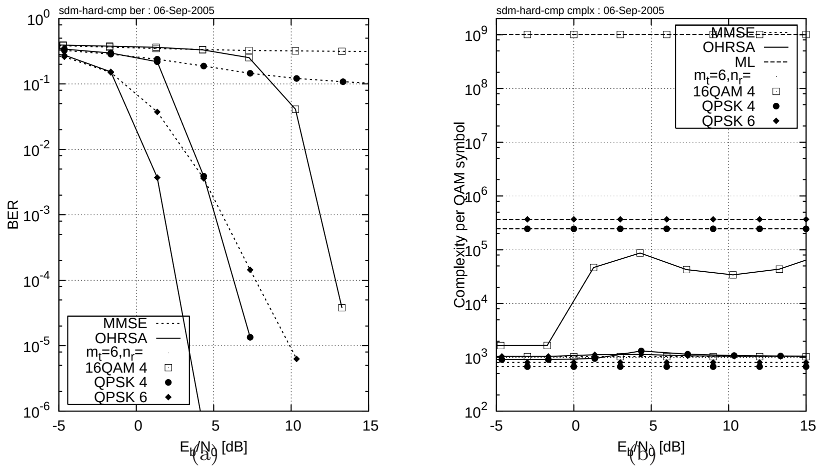

Figure 1. (a) **Bit Error Rate** and (b) the corresponding **complexity** exhibited by the rate-$\frac{1}{2}$ turbo-coded **SDM-OFDM** system employing the **OHRSA-ML** SDM detector. The abscissa represents the average E_b/N_0.

Our simulations were performed in the base-band frequency domain. The OFDM system considered utilises 128 QAM-modulated orthogonal subcarriers. For forward error correction (FEC) we use $\frac{1}{2}$-rate turbo coding [8] employing two constraint-length $K = 3$ Recursive Systematic Convolutional (RSC) component codes and the standard 124-bit WCDMA UMTS turbo code interleaver of [9]. The octally represented RCS generator polynomials of (7,5) were used. We assume a 9-tap CIR Rayleigh-fading multipath channel and stipulate the assumption of perfect channel knowledge, where the knowledge of the frequency-domain subcarrier-related coefficients $H[n, k]$ is deemed to be available in the receiver.

Figure 1(a) characterizes the achievable BER performance of the MIMO-OFDM system employing the OHRSA detector proposed, as well as that of the MMSE detector. We can see that as opposed to the MMSE detector, the proposed OHRSA detector performs equally well both in fully-loaded as well as in overloaded scenarios, where the number of transmit antennas exceeds that of the receive antennas and thus we have $m_{\mathrm{t}} > n_{\mathrm{r}}$. Furthermore, the OHRSA method exhibits an equally good performance, when employed in the overloaded 16-QAM-modulated MIMO-OFDM system.

On the other hand, Figure 1(b) illustrates the corresponding computational complexity quantified in terms of the total number of real addition and multiplication operations per detected QAM symbol. Observe that the complexity exhibited by the OHRSA method in the QPSK scenario is only slightly higher than that exhibited by the MMSE detector. Moreover, the complexity exhibited by the OHRSA detector in the overloaded 16-QAM scenario is more than four orders of magnitude lower than that exhibited by the exhaustive ML detector, while their BER performance is expected to be similar. We note however that the BER performance of the ML detector is not shown since it would impose a more than 10^4-times higher complexity.

4. Conclusion

We proposed a novel OHRSA-ML space-time detector, which may be regarded as an advanced extension of the CSD method. We demonstrated that the MIMO-OFDM system employing the OHRSA-ML detector proposed is capable of achieving the near optimum ML performance in the overloaded scenario, where the number of transmit antennas exceeds that of the receive antennas.

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