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Magneto-electric network models in electromagnetism

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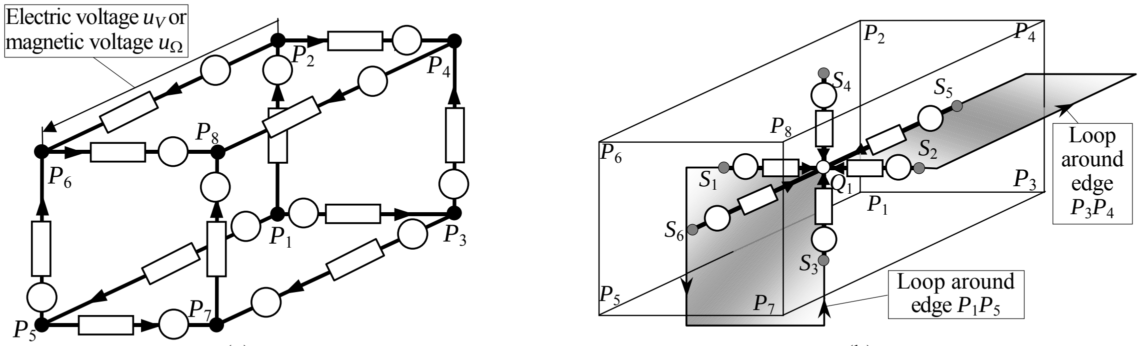
Abstract **– Network models of an electromagnetic field containing both eddy and displacement currents are presented. The models consist of magnetic and electric networks coupled via sources. The analogy between the finite element method and the loop and nodal formulations of electric circuits is emphasised. The models include networks containing branches associated with element edges (edge networks) or facets (facet networks). Methods of determining *mmf* sources of magnetic networks from loop and branch currents in electric circuits, as well as *emf* sources in electric networks on the basis of the rate of change of loop and branch fluxes in electric networks, have been carefully considered.**

I. Introduction

One of the oldest techniques for electromagnetic field analysis and computation relies on magnetic and/or electric field equivalent circuits. Historically such circuits tended to be simple with few degrees of freedom due to limitations of available computing power; notwithstanding, these methods are still helpful in providing efficient estimates of global parameters and are used for teaching purposes as they are well based physically and avoid complicated mathematical descriptions. Dramatic increases in computer speed and available memory have removed many restrictions and contemporary network equivalents are often based on finite element formulations and are very detailed and accurate. It has been shown before [1–3] that finite element equations are equivalent to loop or nodal descriptions of appropriate magnetic or electric networks. Thus models stemming from the finite element approach may be viewed as network models. The number of branches in such networks is consistent with the number of edges or facets in the discretised mesh. Hence the models are fully multi-node and multi-branch, which explains why they are called the networks. This contribution builds on previous publications and, in particular, addresses the coupling between magnetic networks and electric networks when both conduction and displacement currents may exist.

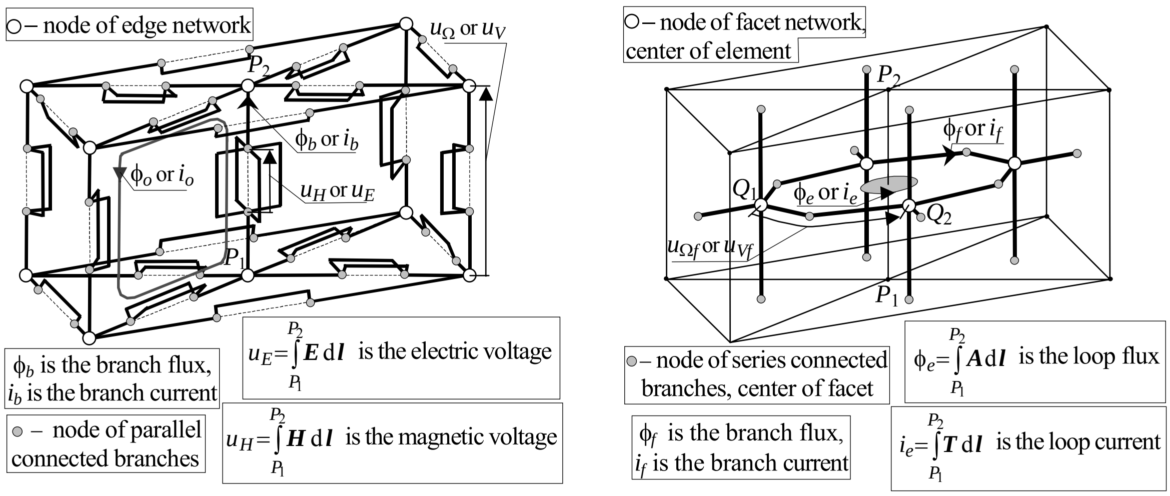
II. Edge and Facet Models

It has been shown [1] that it is helpful to introduce two types of models: ‘edge networks’ (EN) where branches are associated with edges of the elements, and ‘facet networks’ (FN) with branches connecting the centres of the relevant facets with the centre of the element volume. Figure 1 illustrates both types of networks for a hexahedron. Fragments of networks divided into prisms with a triangular base are depicted in Fig. 2 and refer to four elements. The facet model shows one loop around the edge P_1P_2, whereas the edge model includes one complete branch associated with that edge. Table 1 summarises the branch equations for both models. The parameters of the edge model (permeance $\boldsymbol{\Lambda}$, conductance $\boldsymbol{G}$, capacitance $\boldsymbol{C}$) may be established from the interpolation functions of the edge element, while the parameters of the facet model (reluctance $\boldsymbol{R}_\mu$ and impedance $\boldsymbol{Z}$) result from the interpolation functions of the facet element [1]. It should be noted that in models established using edge or facet elements there exist inter-branch couplings. For example, the flux in the ith branch of the edge (permeance) element model depends on the voltage across the permeance of the jth branch, whereas the magnetic voltage of the branch q in the facet (reluctance) model is linked to the flux in the branch p. Thus, when considering equations of Table 1, care must be taken as the matrices of branch parameters are not diagonal and matrix inversion may be very cumbersome. Such matrix inversion is normally avoided by applying a nodal method to the edge models and a loop method to the facet models. From the equations in Table 1, the nodal equations for the edge network follow:



(a) (b)

Fig.1. Models of hexahedron: (a) edge model, (b) faced model



(a) (b)

Fig.2. Models of region with 9-edge prisms (4 prisms): (a) EN; (b) (FN)

Table 1. Branch equations and substitutions for edge and facet models

Type of network	Branch equation	Substitution	Comments
Edge – magnetic	$\phi_b = \Lambda(\boldsymbol{u}_\Omega+\Theta_b)$	$\boldsymbol{u}_\Omega=\boldsymbol{k}_n\boldsymbol{\Omega}$	$\boldsymbol{\Omega}$, $\boldsymbol{V}$ are the vectors of nodal potentials; Λ, $\boldsymbol{G}$, $\boldsymbol{C}$ are the matrices of branch permeances, conductances, capacitances; $\boldsymbol{\Theta}_b$, $\boldsymbol{e}_b$ are the vectors of branch *mmfs* and *emfs*; $\boldsymbol{k}_n$ is the transposed nodal incidence matrix for EN, see Figs 1a, 2a
Edge – electric	$\boldsymbol{i}_b=(\boldsymbol{G}+\mathrm{p}\boldsymbol{C})(\boldsymbol{u}_V+\boldsymbol{e}_b)$	$\boldsymbol{u}_V=\boldsymbol{k}_n\boldsymbol{V}$	
Facet – magnetic	$\boldsymbol{u}_{\Omega f}=\boldsymbol{R}_\mu\phi_f-\boldsymbol{\Theta}_f$	$\phi_f=\boldsymbol{k}_e\phi_e$	ϕ_e, $\boldsymbol{i}_e$ are the vectors of loop fluxes and currents; $\boldsymbol{R}_\mu$, $\boldsymbol{Z}$ are the matrices of branch reluctances and impedances, $\boldsymbol{e}_f$ $\boldsymbol{\Theta}_f$, are the vectors of branch *mmfs* and *emfs*; $\boldsymbol{k}_e$ is the loop (mesh) matrix for EN and the transposed loop matrix for FN, see also Figs 1a, 2a
Facet – electric	$\boldsymbol{u}_{Vf}=\boldsymbol{Z}\boldsymbol{i}_f-\boldsymbol{e}_f$	$\boldsymbol{i}_f=\boldsymbol{k}_e\boldsymbol{i}_e$	

$$\boldsymbol{k}_n^T\Lambda\boldsymbol{k}_n\Omega=-\boldsymbol{k}_n^T\Lambda\Theta_b, \quad \boldsymbol{k}_n^T(\boldsymbol{G}+\mathrm{p}\boldsymbol{C})\boldsymbol{k}_n\boldsymbol{V}=-\boldsymbol{k}_n^T(\boldsymbol{G}+\mathrm{p}\boldsymbol{C})\boldsymbol{e}_b. \tag{1a,b}$$

The loop equations, on the other hand, may be written as

$$\boldsymbol{k}_e^T\boldsymbol{R}_\mu\boldsymbol{k}_e\phi_e=\boldsymbol{k}_e^T\Theta_f, \quad \boldsymbol{k}_e^T\boldsymbol{Z}\boldsymbol{k}_e\boldsymbol{i}_e=\boldsymbol{k}_e^T\boldsymbol{e}_f. \tag{2a,b}$$

The nodal equations (1) are equivalent to the nodal finite element formulation using scalar potentials Ω and V, whereas equations (2) refer to the edge element formulation based on vector potentials $\boldsymbol{A}$ and $\boldsymbol{T}$. The vector $\boldsymbol{\phi}_e$ of loop fluxes of a facet network equals the vector of edge values of potential $\boldsymbol{A}$, while the vector of loop currents $\boldsymbol{i}_e$ is the same as the vector of edge values of potential $\boldsymbol{T}$.

The parameters of the edge and facet models may also be obtained in an approximate way [4, 6, 7], in which case no coupling between branches can be established, thus no mutual reluctances, permeances, conductances or capacitances are available. Only magneto-electric couplings are preserved, resulting from the dependence of *mmf* on current and *emf* on time derivative of magnetic flux.

III. Magnetomotive and Electromotive Forces

Branch sources in FN are established from loop quantities in EN, and – by symmetry – branch sources in EN are found from loop quantities in FN. Branch *mmf*s $\boldsymbol{\Theta}_b$ in EN correspond to loop currents $\boldsymbol{i}_e$ in FN, e.g. the *mmf* in branch P_1P_2 of the magnetic network of Fig. 2a is equal to the loop current of the electric network of Fig. 2b in the loop around the edge P_1P_2. Branch *emf*s $\boldsymbol{e}_b$ in EN are found as time derivatives of loop fluxes $\boldsymbol{\phi}_e$ in FN, hence the sources in (1) may be expressed as

$$\boldsymbol{\Theta}_b = \boldsymbol{i}_e , \quad \boldsymbol{e}_b = -\mathrm{d}\boldsymbol{\phi}_e / \mathrm{d}t . \tag{3a,b}$$

The branch *mmf*s $\boldsymbol{\Theta}_f$ in FN are represented by the loop currents $\boldsymbol{i}_o$ of the edge network, e.g. the *mmf* in branch Q_1Q_2 of the facet network of Fig. 2b is equal to the loop current i_o of the edge network depicted in Fig. 2a. The time derivative of the flux ϕ_o in the loop shown in Fig. 2a is equal (with the negative sign) to the *emf* in the branch Q_1Q_2 of the electric FN (Fig. 2b), thus the sources in (2) may be described as

$$\boldsymbol{\Theta}_f = \boldsymbol{i}_o , \; \boldsymbol{e}_f = -\mathrm{d}\boldsymbol{\phi}_o / \mathrm{d}t . \tag{4a,b}$$

When using the loop method it is not necessary to know the branch sources, instead the loop sources are needed. For example, when dealing with equations (2), the branch values of $\boldsymbol{\Theta}_f$ and $\boldsymbol{e}_f$ are not required and we can concentrate on deriving the loop sources $\boldsymbol{\Theta}_m$ and $\boldsymbol{e}_m$, where $\boldsymbol{\Theta}_m = \boldsymbol{k}_e^T \boldsymbol{\Theta}_f$ and $\boldsymbol{e}_m = \boldsymbol{k}_e^T \boldsymbol{e}_f$. The loop *mmf* is equivalent to the current passing through the loop of the magnetic network, thus loop *mmf*s $\boldsymbol{\Theta}_m$ in the facet network correspond to the branch currents $\boldsymbol{i}_b$ in the edge network, e.g. the *mmf* in the loop shown in Fig. 2b (the loop embracing the edge P_1P_2) is equal to the current of the branch P_1P_2 of the electric network of Fig 2a. The loop *emf*s may be found by taking time derivatives of branch fluxes in the magnetic network passing through the loops of the electric network, e.g. loop *emf*s $\boldsymbol{e}_m$ in the electric facet network may be established from the fluxes associated with branches of the magnetic edge network, $\boldsymbol{e}_m = -\mathrm{d}\boldsymbol{\phi}_b/\mathrm{d}t$. Thus, when solving (2), we are allowed to use the following identities:

$$\boldsymbol{k}_e^T \boldsymbol{\Theta}_f = \boldsymbol{\Theta}_m = \boldsymbol{i}_b , \quad \boldsymbol{k}_e^T \boldsymbol{e}_f = \boldsymbol{e}_m = -\mathrm{d}\boldsymbol{\phi}_b / \mathrm{d}t \tag{5a,b}$$

In order to determine fluxes $\boldsymbol{\phi}_b$, $\boldsymbol{\phi}_o$ and currents $\boldsymbol{i}_b$, $\boldsymbol{i}_o$ associated with edge networks, it is not essential to solve the network equations; instead we may use the solutions for the facet network and apply a transposition matrix $\boldsymbol{N}$. The elements of this matrix are given by the product of the interpolating functions of the relevant facet and edge elements [1]. Employing the matrix $\boldsymbol{N}$ yields

$$\boldsymbol{\phi}_o = \boldsymbol{N}\boldsymbol{\phi}_e , \quad \boldsymbol{i}_o = \boldsymbol{N}\boldsymbol{i}_e , \tag{6a,b}$$

$$\boldsymbol{\phi}_b = \boldsymbol{N}^T \boldsymbol{\phi}_f , \quad \boldsymbol{i}_b = \boldsymbol{N}^T \boldsymbol{i}_f . \tag{7a,b}$$

The matrix $\boldsymbol{N}$ may also be used do establish currents $\boldsymbol{i}_e$ and fluxes ϕ_e, related to the loops of the facet network, from currents $\boldsymbol{i}_o$ and fluxes ϕ_o in the loops of the edge network:

$$\phi_e = \boldsymbol{N}^T \phi_o\,, \qquad \boldsymbol{i}_e = \boldsymbol{N}^T \boldsymbol{i}_o\,. \tag{8a,b}$$

These relationships are illustrated in Fig. 3, where hexahedron elements are considered for which all entries in the matrix $\boldsymbol{N}$ are equal to 1/8.

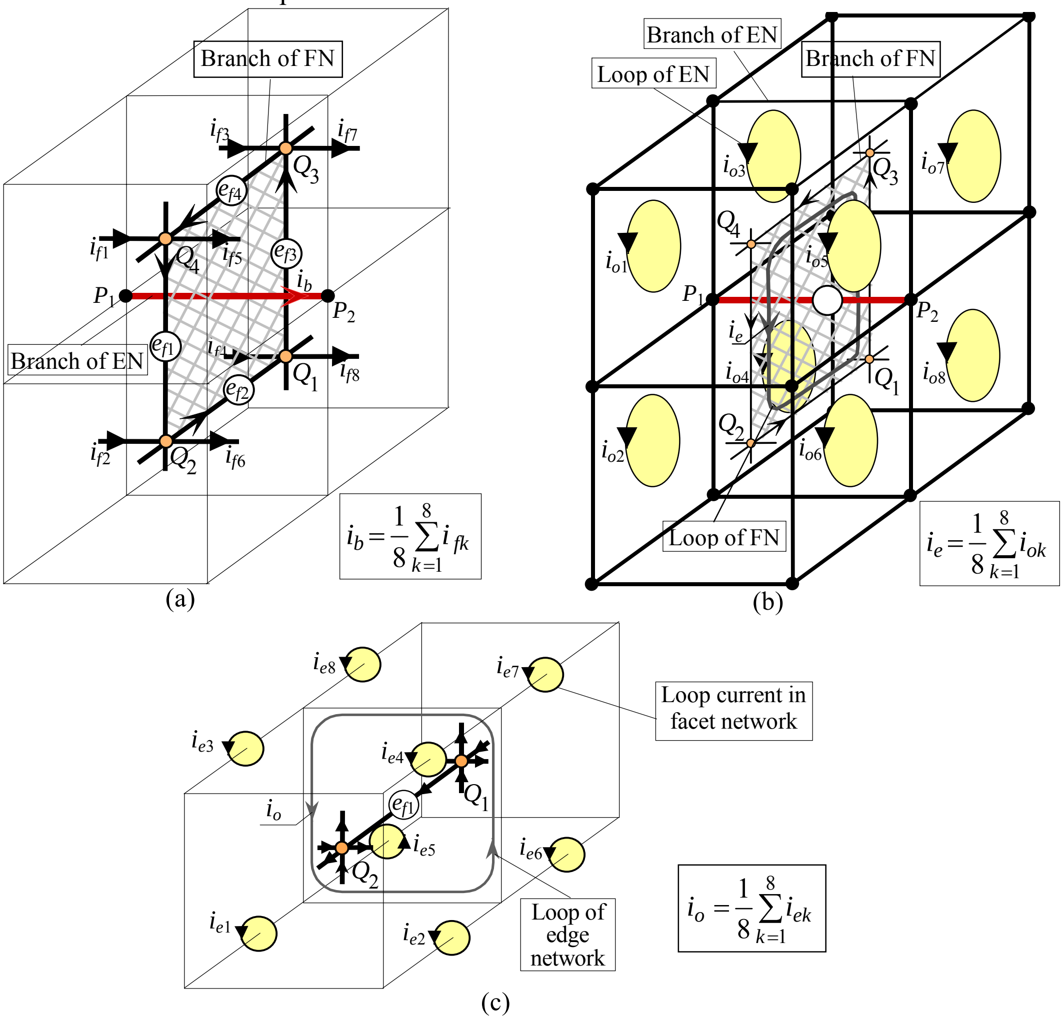


Fig.3. Transformation of currents: (a) FN into EN branch, (b) EN into FN loop, (c) FN into EN loop

From the above discussion it may be concluded that – due to a better representation of sources – the field description using loop quantities is more universal. This deduction is consistent with an observation that the loop approach establishes correspondence with vector potentials, and it is generally agreed that formulations in terms of vector potentials are more powerful than those using scalar potentials.

IV. Coupled Electro-Magnetic Networks

Models of the electromagnetic field are provided by the coupled, via sources, magnetic and electric networks. It has already been noted that – due to the couplings between branches (mutual permeances, conductances and capacitances in EN, and mutual reluctances and impedances in FN) – it is more convenient to analyse edge networks using nodal approach, whereas facet networks are better handled using loop methods. Thus it follows (refer also to our previous comments about establishing *mmf*s and *emf*s) that a system containing an electromagnetic field may be described using the following coupled network models: (a) magnetic and electric facet network (FM-FE) – Fig. 4, (b) magnetic facet network and electric edge network (FM-EE) - Fig 5a, or (c) magnetic edge network and electric facet network (EM-FE) – Fig. 5b.

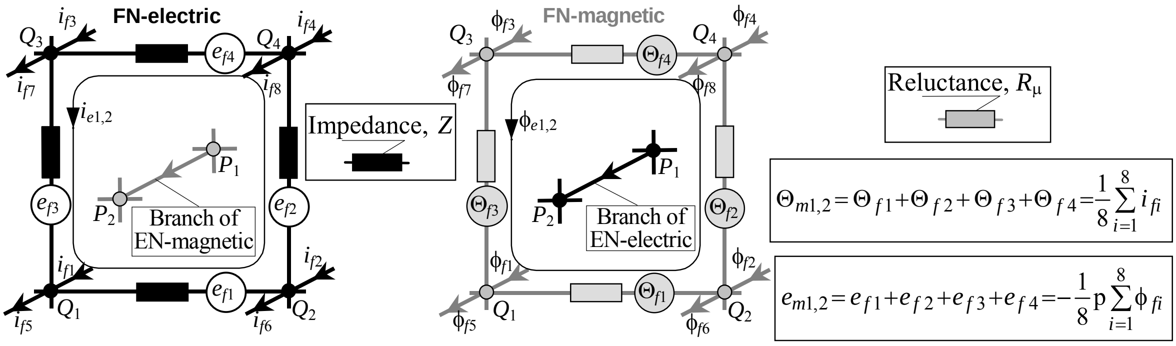
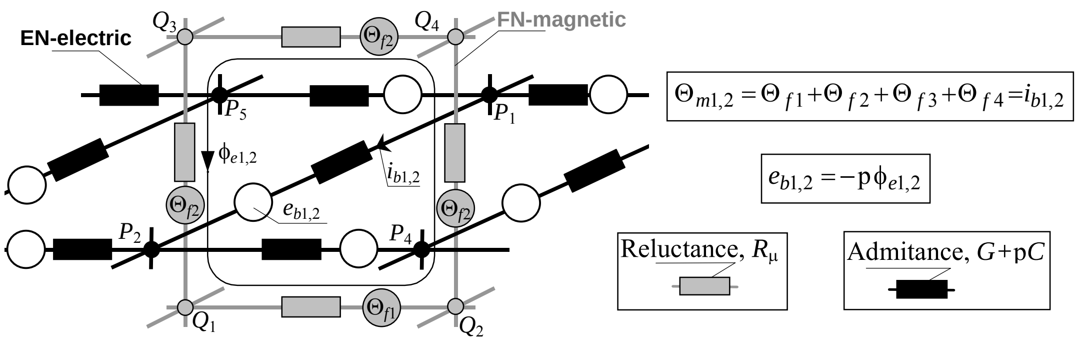
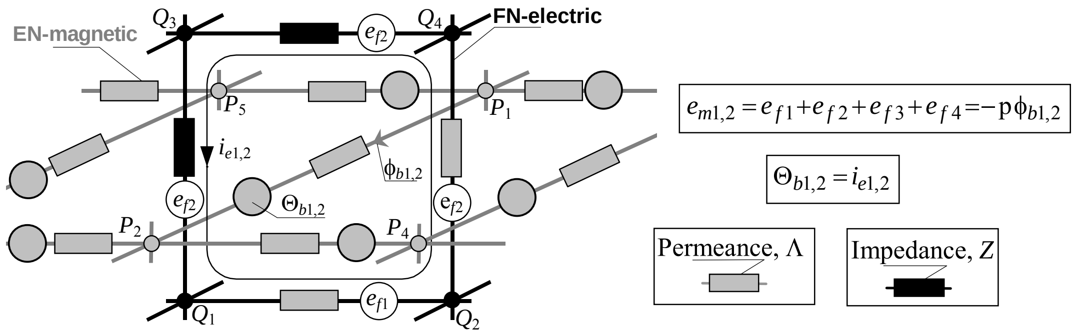


Fig. 4. Coupled facet networks (FM-FE), a network representation of the ***A-T*** method



(a)



(b)

Fig. 5. Coupled facet and edge networks: (a) a network representation of the ***A***-*V* method (FM-EE)
(b) a network representation of the Ω-***T*** method (EM-FE)

The loop equations of the network model FM-FE correspond to the edge formulation using ***A-T***. The loop sources are established directly from the branch quantities. In the FM-EE and EM-FE models, the branches of the magnetic network pass through the loops of the electric network, while the branches of the electric network pass through the loops of the magnetic network, as shown in Fig. 6. The equations of the FM-EE, EM-FE models correspond to the ***A***-*V*, Ω-***T*** formulations of the finite element method, respectively. The loop sources in the facet networks are obtained from the branch quantities in the in the edge networks, whereas branch sources in the edge networks from the loop quantities in the facet networks. From the loop equations applied to the facet networks of the FM-EE and

EM-FE models, it is possible to derive edge formulation arising from the field description. It should be assumed that the nodes of the edge networks are equipotential, thus $\boldsymbol{E}=-\mathrm{d}\boldsymbol{A}/\mathrm{d}t$, $\boldsymbol{H}=\boldsymbol{T}$ and loop currents in the electric network represent edge values of vector $\boldsymbol{H}$, while time derivatives of the loop fluxes in the magnetic network correspond to the edge values of vector $\boldsymbol{E}$.

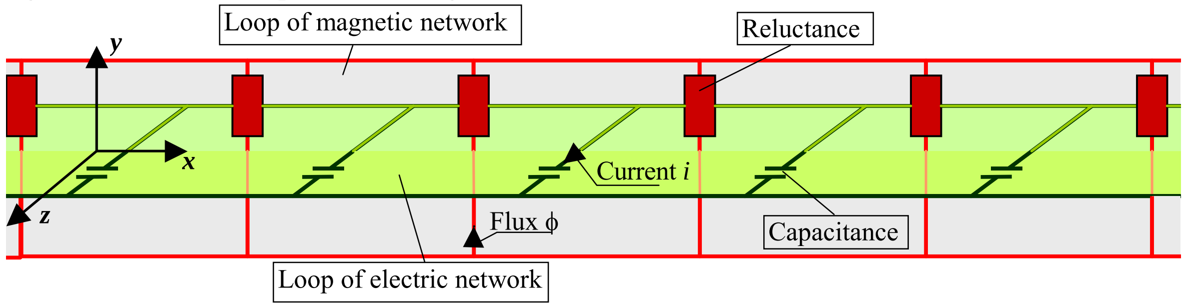


Fig. 6. Network model of a region with a plane wave, $\boldsymbol{E}=\mathbf{1}_x E_x(z)$, $\boldsymbol{H}=\mathbf{1}_y H_y(z)$

V. Conclusions

The proposed network models provide good physical insight, help understanding of complicated electromagnetic phenomena and aid explanation of methods of analysis of electromagnetic systems. The models are general and allow creation of networks of electromagnetic systems containing non-homogenous materials and multiply-connected conducting regions. It is possible, for example, to represent windings containing filament or thin conductors, as well as rod conductors (e.g. in cage rotors). For thin conductors the best suited model is the facet electric network, which is a circuit representation of the method using electric vector potential $\boldsymbol{T}$, but care must be taken to replace 'large' loops of the windings with 'small' loops around the edge of the elements [5]. The facet model can also be used to model cage windings of an induction machine – despite a common opinion that vector $\boldsymbol{T}$ is not appropriate for such systems – as well as conductors with holes all way through, i.e. multiply connected regions. The classical $\boldsymbol{T}$ formulation leads to loop equations around the element edges. Although the number of such loops is usually higher than the number of independent loops, in the multiply connected region it is not possible to set up a complete system of independent loops. It is therefore necessary to complement these equations by introducing additional loops embracing the 'holes' which provide the required extra equations. This conclusion – which may be considered obvious from the circuit theory point of view – is not easy to arrive at using the classical finite element formulation. It may be argued therefore that the presented analogies between the finite element formulation and the equivalent network models not only facilitate understanding of the methods of field analysis but also help to formulate efficient computational algorithms.

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