Performance of Fractionally Spread Multicarrier CDMA in AWGN as Well as Slow and Fast Nakagami-m Fading Channels

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***Abstract*—In multicarrier code-division multiple-access (MC-CDMA), the total system bandwidth is divided into a number of subbands, where each subband may use direct-sequence (DS) spreading and each subband signal is transmitted using a subcarrier frequency. In this paper, we divide the symbol duration into a number of fractional subsymbol durations also referred to here as fractions, in a manner analogous to subbands in MC-CDMA systems. In the proposed MC-CDMA scheme, the data streams are spread at both the symbol-fraction level and at the chip level by the transmitter, and hence the proposed scheme is referred to as the fractionally spread MC-CDMA arrangement, or FS MC-CDMA. Furthermore, the FS MC-CDMA signal is additionally spread in the frequency (F)-domain using a spreading code with the aid of a number of subcarriers. In comparison to conventional MC-CDMA schemes, which are suitable for communications over frequency-selective fading channels, our study demonstrates that the proposed FS MC-CDMA is capable of efficiently exploiting both the frequency-selective and the time-selective characteristics of wireless channels.**

***Index Terms*—Broadband communications, code-division multiple access (CDMA), fractionally spreading, frequency-domain spreading, multicarrier modulation, Nakagami fading, time-domain spreading.**

I. Introduction

IN MULTICARRIER code-division multiple-access (MC-CDMA) communication systems [1]–[15], the total bandwidth available is divided into a number of subbands, where each subband may employ direct-sequence (DS) spreading and each subband signal is transmitted with the aid of a subcarrier. In MC-CDMA systems [4], [6], [9], serial-to-parallel (S-P) conversion is invoked at the transmitter, in order to decrease the transmitted symbol rate for the sake of mitigating the effects of inter-symbol interference (ISI). Frequency diversity in MC-CDMA systems is usually achieved by repeating the transmitted signal in the frequency (F)-domain with the aid of several subcarriers [5], [6], [10]. Alternatively, in MC-CDMA systems the F-domain repetition can be replaced by F-domain spreading [16] using a spreading code. One of the advantages of using F-domain spreading instead of F-domain repetition in MC-CDMA systems is that frequency diversity can be achieved without reducing the maximum number of users supported by the system [16]. Classic studies of MC-CDMA have shown that the various MC-CDMA schemes proposed in the literature [3]–[15] are capable of supporting multiple users communicating over frequency-selective fading channels.

However, the future generations of broadband multiple-access communication systems [17] are expected to have a bandwidth on the order of tens or even hundreds of Megahertz. Broadband wireless mobile channels are typically time-varying and the received signals may experience both frequency-selective and time-selective fading [18], [19]. Since conventional MC-CDMA schemes have usually been designed without considering the time-selectivity of the wireless channels, they may not be sufficiently efficient, when communicating over wireless channels exhibiting both frequency-selective and time-selective fading. In the context of fading channels existing both frequency-selective and time-selective fading, signalling with the aid of single-carrier DS-CDMA has been investigated based on the time-frequency representation techniques [19]–[22]. Specifically, in these references the reception of DS-CDMA signals has been investigated, while simultaneously achieving both frequency diversity with the advent of the frequency-selectivity of the fading as well as time diversity as a result of the time-selectivity of the fading. Both frequency diversity and time diversity are achieved by utilizing a two-dimensional (2-D) RAKE receiver [19]. It has been shown in [19] and [20] that by exploiting both the frequency-selectivity and the time-selectivity of fast-fading wireless channels, the diversity gain can be substantially enhanced in comparison to using solely frequency-selectivity. However, when the transmitted signals experience pure frequency-selective fading, where no significant time (T)-domain fading is experiences, or when experience time-selective fading having an insufficiently high number of resolvable components in the T-domain, then using a 2-D RAKE receiver will inevitably combine noise, rather than useful signal energy.

In this paper, instead of using a 2-D RAKE receiver, firstly, we resort to multicarrier transmission of the DS-CDMA signals for the sake of achieving frequency diversity, as considered in [5], [6]. Secondly, in a manner analogous to the approach used in MC DS-CDMA schemes [5]—which divide the available bandwidth into a number of subbands—we divide the symbol-duration into a number of subsymbol-durations referred to here as fractions, for the sake of achieving time diversity, when communicating over fast-fading channels. In our proposed MC-CDMA scheme, the serial-to-parallel (S-P) converted data substreams are DS spread at both the T-domain fraction level and at the chip level, as we will see in our forthcoming discourse. Hence, the proposed scheme is referred to as fractionally spread

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MC-CDMA and we use the abbreviation of FS MC-CDMA for simplicity of description. Furthermore, as in [16], in our FS MC-CDMA scheme the doubly DS spread signal is further spread also in the frequency (F)-domain with the aid of a number of subcarriers, as it will be highlighted during our further discourse. The BER performance of the FS MC-CDMA system is investigated, when communicating over additive white Gaussian noise (AWGN) channels as well as over both frequency-selective slow and fast Nakagami-m fading channels [9], [23], [24]. Our results demonstrate that the proposed FS MC-CDMA is capable of efficiently exploiting both the frequency-selective and the time-selective characteristics of the wireless channels encountered for enhancing the achievable BER performance. Furthermore, the proposed FS MC-CDMA has the potential of flexibly achieving the best balance between the attainable spreading gain and diversity gain. For example, when the fading channels encountered are both frequency-selective and time-selective, FS MC-CDMA is likely to achieve the highest combined frequency- and time-diversity gain but correspondingly the lowest spreading gain. On the other hand, when the fading channels are neither frequency-selective nor time-selective, then FS MC-CDMA is unable to achieve diversity gain. However, instead of no diversity gain, FS MC-CDMA achieves its highest spreading gain. Furthermore, when the fading channels encountered are frequency-selective (or time-selective) but not time-selective (or frequency-selective), then FS MC-CDMA is expected to achieve a diversity gain, which is lower than that achieved in the former frequency- and time-selective scenario, but higher than that achieved in the latter frequency- and time-nonselective scenario. Correspondingly, the spreading gain achieved by FS MC-CDMA is higher than that achieved in the frequency- and time-selective fading channels, but lower than that achieved in the frequency- and time-nonselective fading channels. Owing to the above self-flexible properties, the proposed FS MC-CDMA scheme is beneficial for communications over wireless channels exhibiting frequency-selective fading and/or time-selective fading.

The remainder of this contribution is organized as follows. Section II describes the FS MC-CDMA system model in the context of its transmitter and receiver models. Section III derives the corresponding BER expressions. In Section IV, we provide our numerical results and provide further discussions. Finally, in Section V we present our conclusions.

II. System Model

A. Transmitted Signals

The transmitter diagram of the kth user is shown in Fig. 1 for the proposed FS MC-CDMA system. In this scheme, the original binary data stream having a bit duration of T_b is S-P converted to U parallel substreams, which are expressed as $\{b_{k1}, b_{k2}, \ldots, b_{kU}\}$. The new bit duration after S-P conversion, which is also often referred to as the symbol duration is given by $T_s = UT_b$. As shown in Fig. 1, after S-P conversion each of the substreams is spread using two time (T)-domain spreading codes, namely $a_k(t)$ and $c_k(t)$. More explicitly, the first T-domain spreading code $a_k(t)$ is applied at the fraction level and it is expressed as $a_k(t) = \sum_{i=-\infty}^{\infty} a_{ki} P_{T_D}(t - iT_D)$,

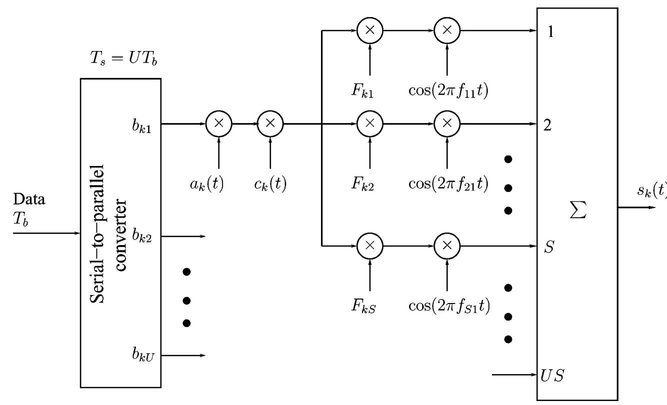

Fig. 1. Transmitter model of fractionally spread MC-CDMA.

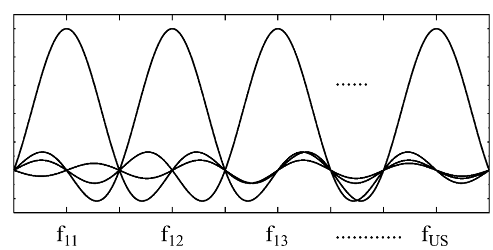

Fig. 2. Subcarrier frequency arrangement in the FS MC-CDMA system.

where $T_D = T_s/N_1$ represents the fraction's time-duration, a_{ki} assumes the binary values of $+1$ or -1 with equal probability, while $P_{T_D}(t)$ represents the rectangular pulses of duration T_D. Hence, $a_k(t)$ consists of a sequence of mutually independent rectangular pulses of duration T_D and amplitude of $+1$ or -1, both having an equal probability. The second T-domain spreading code $c_k(t)$ is at the chip level and is defined as $c_k(t) = \sum_{i=-\infty}^{\infty} c_{ki} P_{T_c}(t - iT_c)$, where $\{c_{ki}\}$ is again a random sequence with $c_{ki} \in \{+1, -1\}$ and $P_{T_c}(t)$ is the rectangular chip waveform defined over the time interval $[0, T_c)$.

Let the total number of subcarrier frequencies, namely US, be arranged according to Fig. 2, where S is defined as the length of the F-domain spreading codes to be invoked additionally. We assume that the spacing between two adjacent subcarrier frequencies is $2/T_c$ and there exists no overlap among the main spectral lobes of the subcarriers. As shown in Fig. 1, after T-domain spreading the uth substream, where $u = 1, 2, \ldots, U$, is further spread in the frequency (F)-domain using an S-chip F-domain spreading code $\{F_{k1}, F_{k2}, \ldots, F_{kS}\}$ associated with the S number of subcarrier frequencies of $\{f_{1u}, f_{2u}, \ldots, f_{Su}\}$. Finally, the US number of subcarrier-modulated substreams are superimposed on each other in order to form the transmitted signal, which can be expressed as

$$s_k(t) = \sqrt{\frac{2E_b}{T_s S}} \sum_{u=1}^{U} \sum_{s=1}^{S} b_{ku}(t) a_k(t) c_k(t) F_{ks} \times \cos\left(2\pi f_{su} t + \phi_{su}^{(k)}\right) \quad (1)$$

where E_b represents the energy per bit, $b_{ku}(t)$ denotes the uth binary data's waveform after the S-P conversion, while $\phi_{su}^{(k)}$

represents a random phase due to carrier modulation. Assuming $N_2 = T_D/T_c$ being an integer, then the total T-domain spreading factor is $N = T_s/T_c = T_s/T_D \times T_D/T_c = N_1 N_2$.

B. Channel Model

In order to gain an insight into the characteristics of the proposed FS MC-CDMA system, in this contribution three different channel models are considered, namely both AWGN channels, as well as frequency-selective slow and fast Nakagami-m fading channels. The AWGN and fast fading channel models represent two extreme cases corresponding to the best-case nonfading environments and the worst-case communication environments exhibiting both frequency-selective and time-selective fading. The reason for considering Nakagami-m fading is that the Nakagami-m distribution constitutes a versatile, generic class of distributions. As shown in [24]–[28], the parameter m involved in the Nakagami-m distribution characterizes the severity of the fading. Specifically, $m = 1$ represents Rayleigh fading, $m \rightarrow \infty$ corresponds to the conventional Gaussian scenario, and $m = 1/2$ describes the so-called one-sided Gaussian fading, i.e., the worst-case fading condition. The Rician and lognormal distributions can also be closely approximated by the Nakagami distribution in conjunction with values of $m > 1$.

We assume that there are K asynchronous FS MC-CDMA users in the system. The average power received from each user at the base station is also assumed to be identical. Consequently, when K signals obeying the form of (1) are transmitted over AWGN channels, the signal received by the base station can be expressed as

$$\begin{aligned} r(t) = &\sqrt{\frac{2E_b}{T_s S}} \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{u=1}^{U} \sum_{s=1}^{S} b_{ku}(t-\tau_k) \\ &\times a_{kn} P_{T_D}(t - nT_D - \tau_k) c_k(t-\tau_k) F_{ks} \\ &\times \cos\left(2\pi f_{su} t + \psi_{su}^{(k)}\right) + n(t) \end{aligned} \tag{2}$$

where $\psi_{su}^{(k)} = \phi_{su}^{(k)} - 2\pi f_{su}\tau_k$, which is assumed to be an i.i.d. random variable having a uniform distribution in $[0, 2\pi)$, while $n(t)$ represents the AWGN noise having zero mean and a double-sided power spectrum density of $\mathcal{N}_0/2$.

In the context of rapidly fluctuating, high-Doppler frequency-selective Nakagami-m fading channels, we assume that the *delay-spread* of the channel, denoted by T_m is lower than the chip-duration T_c, i.e., we have $T_m < T_c$. In practice the condition of $T_m < T_c$ can be achieved by employing high chip-duration spreading sequences for each of the subcarriers, but assigning an increased number of subcarriers. The required frequency diversity again is achieved by transmitting the same data on several subcarriers experiencing independent fading. Since we assume that $T_m < T_c$, the number of resolvable paths associated with each subcarrier is therefore one, i.e., each subcarrier signal experiences flat fading [5], [7], [8]. As shown in Fig. 1, the FS MC-CDMA transmitter usually employs S-P conversion and U data bits are transmitted in parallel within each symbol-duration. Hence, the symbol-duration is $T_s = UT_b$. For rapidly time-varying wireless channels, which may be encountered by high-velocity mobile terminals or fast moving large-bodied objects in the vicinity of the mobile terminal, the fading amplitude may change significantly within a given symbol-duration, resulting in high-Doppler time-selective fading. The results of [19], [20], and [22] have shown that the time-selective characteristics of the wireless channels can be exploited for significantly increasing the achievable diversity gain of single-carrier DS-CDMA systems. Again, in this paper, in a manner analogous to MC DS-CDMA [5]—which divides the total available bandwidth into a number of subbands so that each subcarrier signal experiences flat fading—we divide the symbol-duration of T_s into N_1 number of sub-symbol-duration referred to as fractions. We assume that the fading amplitude within each fraction of T_D is a constant, while the received signal experiences independent fading during each fraction. The above assumption implies that the *coherence time* [18] $(\Delta t)_c$ of the channel is higher but close to the fraction-duration of T_D, i.e., that we have $(\Delta t)_c \gtrapprox T_D$. Note that, in practice, if the fractions are subject to correlated fading and if achieving the time-diversity is also one of the design objectives, then interleaving over time may be employed after the fractionally spreading stage of Fig. 1, in order to guarantee the independent fading of the subcarrier signals in each fraction. However, when achieving the time-diversity is not a design objective, for example since sufficient diversity has been provided by the frequency-diversity, the above-mentioned interleaving is then unnecessary. In this case, as can be shown in our forthcoming analysis, instead of achiveing pure time-diversity, the detector is capable of automatically reaching a tradeoff between the time-diversity gain and the spreading gain, depending on the correlation among the fractions. Based on the above assumptions, the asynchronous signal received by the base station can be expressed as

$$\begin{aligned} r(t) = &\sqrt{\frac{2E_b}{T_s S}} \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{u=1}^{U} \sum_{s=1}^{S} \alpha_{uns}^{(k)} b_{ku} \\ &\times (t-\tau_k) a_{kn} P_{T_D}(t - nT_D - \tau_k) \\ &\times c_k(t-\tau_k) F_{ks} \cos\left(2\pi f_{su} t + \psi_{uns}^{(k)}\right) + n(t) \end{aligned} \tag{3}$$

where $n(t)$ is the same random variable as in (2), while we have $\psi_{uns}^{(k)} = \phi_{su}^{(k)} + \varphi_{uns}^{(k)} - 2\pi f_{su}\tau_k$ and $\varphi_{uns}^{(k)}$ is the channel-induced phase contribution. Furthermore, in (3) $\alpha_{uns}^{(k)}$ is an amplitude fading parameter associated with the kth user, with the nth fraction as well as with the subcarrier indexed by the values of u and s. The fading amplitude $\alpha_{uns}^{(k)}$ is a random variable obeying the Nakagami-m distribution having a PDF given by [27]

$$p_{\alpha_{uns}^{(k)}}(r) = \frac{2r^{2m-1}}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \exp\left(-\frac{mr^2}{\Omega}\right) \tag{4}$$

where $\Gamma(\cdot)$ is the gamma function [18], $\Omega = E[(\alpha_{uns}^{(k)})^2]$ and m is the Nakagami-m fading parameter, as we discussed previously in this section. Note that, in the context of the frequency-selective slow fading channel model, the received signal can also be expressed as in (3) by assuming that $\alpha_{uns}^{(k)}$ and $\psi_{uns}^{(k)}$ are independent of the fraction index of n.

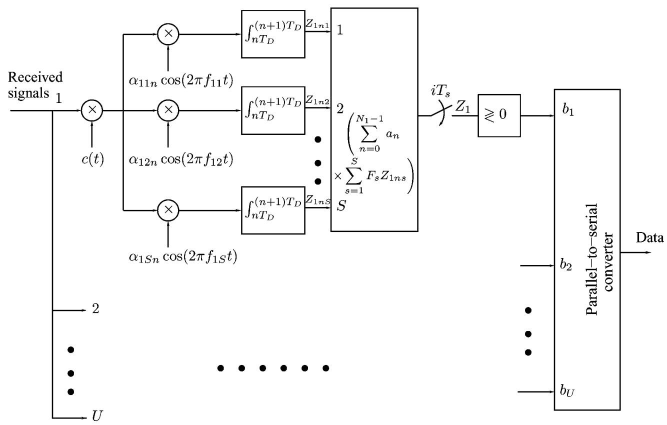

Fig. 3. Receiver schematic diagram of fractionally spread MC-CDMA.

C. Receiver Model

Let the first user be the user-of-interest and let us ignore the subscript as well as the superscript associated with the reference user in Fig. 3 in our forthcoming discussions. The transmitter of Fig. 1 and (1) transmits each data bit on S number of subcarriers using N_1 fractions. At the receiver side, these $N_1 S$ number of signals conveying the same data bit are combined based on the maximum ratio combining (MRC) principle, when assuming rapidly fading channels. Therefore, we have to estimate both the fading amplitude α_{uns} and the phase ψ_{uns} associated with each of the subcarrier signals within each fraction. In this contribution we assume that these channel parameters are perfectly estimated using, for example, the approaches in [31] and [32]. Note that when AWGN channels are considered, the fading amplitudes of $\{\alpha_{uns}\}$ are set to units, while the phases due to carrier modulation and channel delay are assumed to be perfect estimates. By contrast, for the slowly fading channels considered, both the fading amplitudes as well as the phases are independent of the fraction index of n. The FS MC-CDMA receiver's schematic diagram is shown in Fig. 3, which is suitable for receiving the FS MC-CDMA signals in all three types of channel models considered. In Fig. 3 each subcarrier signal is first despread using the T-domain spreading code $c(t)$ of the reference user associated with each fraction. Then, the subcarrier signals conveying the same data bit are despread using the F-domain spreading code $\{F_1, F_2, \ldots, F_S\}$ and combined using a MRC scheme with the aid of the channel's fading envelope estimates $\{\alpha_{un1}, \alpha_{un2}, \ldots, \alpha_{unS}\}_{u=1}^{U}$. Finally, the N_1 number of signals corresponding to N_1 fractions of the same symbol are despread using the T-domain spreading code $a(t)$, yielding the decision variable $Z_u, u = 1, \ldots, U$ acquired for the uth binary bit. The process of generating the decision variable Z_u for the first symbol can be summarized using the following equations:

$$Z_u = \sum_{n=0}^{N_1-1} a_n Z_{un}, \quad u = 1, \ldots, U \tag{5}$$

$$Z_{un} = \sum_{s=1}^{S} F_s Z_{uns} \tag{6}$$

$$Z_{uns} = \alpha_{uns} \times \int_{nT_D}^{(n+1)T_D} r(t)c(t)\cos(2\pi f_{su} t)\, dt \tag{7}$$

where we assumed that $\tau_1 = 0$ and $\psi_{uns} = 0$, representing perfect synchronization with the subcarrier signal of the fraction that is being considered.

Based on the decision variable $Z_u, u = 1, \ldots, U$, the current data bit of the uth substream is decided to be 0 or 1, depending on whether Z_u is higher than zero. Finally, the U number of parallel data substreams are parallel-to-serial (P-S) converted, in order to output the serial data bits. Let us now analyze the statistics of the decision variables and the achievable BER with the aid of these statistics.

III. Bit Error Rate Analysis

In this section we derive the bit error rate (BER) expression for the proposed FS MC-CDMA system, when communicating over the AWGN, and over the slow or fast frequency-selective Nakagami-m fading channels. Considered both the AWGN and the frequency-selective slow fading channel models can be readily derived from the frequency-selective fast fading channel model by setting the fading amplitudes and the phases to appropriate values. Therefore, our analysis is essentially carried out for the frequency-selective fast fading channel model. For the AWGN and for the frequency-selective slow fading channel models we will only provide the final results. Let us first derive the statistics of the decision variables $\{Z_u\}_{u=1}^{U}$.

A. Decision Variable Statistics

In MC DS-CDMA systems, when the main spectral lobes of the subcarrier signals do not overlap, the interference imposed on a 'specific' subcarrier is mainly contributed by the same subcarrier of the interfering users [29]. The interference imposed by the other subcarriers using subcarrier frequencies different from the 'specific' subcarrier's can be ignored. Hence, in our forthcoming discourse we only consider the multiuser interference (MUI) imposed by the specific interfering subcarriers, which have the same subcarrier frequencies, as the reference subcarrier considered. Upon substituting $r(t)$ of (3) into (7), it can be shown that the variable of Z_{uns} can be expressed as

$$Z_{uns} = \sqrt{\frac{E_b T_s}{2S}} \left(D_{uns} + \sum_{k=2}^{K} I_{uns}^{(k)} + \mathcal{N}_{uns} \right) \tag{8}$$

where $\mathcal{N}_{uns}$ is contributed by $n(t)$ of (3), which is a Gaussian random variable having zero mean and a variance of $(S\alpha_{uns}^2 \mathcal{N}_0 / 2N_1 E_b)$. In (8) D_{uns} represents the desired output matched to the subcarrier signal at index us, to the nth fraction of the considered symbol as well as to the reference user of $k = 1$, hence D_{uns} can be expressed as

$$D_{uns} = \left(\frac{F_s a_n \alpha_{uns}^2}{N_1} \right) b_u. \tag{9}$$

Furthermore, $I_{uns}^{(k)}$ in (8) represents the MUI imposed by the kth interfering user, which can be expressed as

$$I_{uns}^{(k)} = \frac{F_{ks}\alpha_{uns}}{T_s}\left[a_{kn_{-1}}\alpha_{un_{-1}s}^{(k)}\cos\left(\psi_{un_{-1}s}^{(k)}\right)b_{kun_{-1}}R_k(\tau_k) + a_{kn_0}\alpha_{un_0s}^{(k)}\cos\left(\psi_{un_0s}^{(k)}\right)b_{kun_0}\hat{R}_k(\tau_k)\right] \quad (10)$$

where $a_{kn_{-1}}$, $\alpha_{un_{-1}s}^{(k)}$, $\psi_{un_{-1}s}^{(k)}$, and $b_{kun_{-1}}$ represent the fraction-related value of the first T-domain spreading code, the fading amplitude, the phase and the transmitted data bit, respectively, associated with the kth user's signal in the context of the $(n-1)$th fraction. By contrast, a_{kn_0}, $\alpha_{un_0s}^{(k)}$, $\psi_{un_0s}^{(k)}$, and b_{kun_0} represent the fraction-related value of the first T-domain spreading code, the fading amplitude, the phase and the transmitted data bit, respectively, associated with the kth user's signal in the context of the nth fraction. The phase variables $\psi_{un_{-1}s}^{(k)}$ and $\psi_{un_0s}^{(k)}$ in (10) are assumed to be i.i.d. random variables obeying a uniform distribution in $[0, 2\pi)$. Furthermore, $R_k(\tau_k)$ and $\hat{R}_k(\tau_k)$ in (10) represent the partial cross-correlation functions between the kth user's spreading sequence $c_k(t)$ and the reference user's spreading sequence $c(t)$, which are defined as

$$R_k(\tau_k) = \int_{nT_D}^{nT_D+\tau_k} c_k(t-\tau_k)c(t)\,dt \quad (11)$$

$$\hat{R}_k(\tau_k) = \int_{nT_D+\tau_k}^{(n+1)T_D} c_k(t-\tau_k)c(t)\,dt. \quad (12)$$

As shown in (6), the variable Z_{un} is the output after the F-domain despreading. After ignoring the common factor of $\sqrt{(E_bT_s)/(2S)}$ in (8), Z_{un} can be expressed as

$$Z_{un} = D_{un} + \sum_{k=2}^{K} I_{un}^{(k)} + \mathcal{N}_{un} \quad (13)$$

where $\mathcal{N}_{un} = \sum_{s=1}^{S} F_s\mathcal{N}_{uns}$, which is a Gaussian random variable having zero mean and a variance of $\sum_{s=1}^{S}(S\alpha_{uns}^2\mathcal{N}_0/2N_1E_b)$, while D_{un} and $I_{un}^{(k)}$ can be expressed as

$$D_{un} = \left(\sum_{s=1}^{S}\frac{a_n\alpha_{uns}^2}{N_1}\right)b_u \quad (14)$$

$$I_{un}^{(k)} = \frac{S}{T_s}\left[\beta_{1k}(n_{-1})a_{kn_{-1}}b_{kun_{-1}}R_k(\tau_k) + \beta_{1k}(n_0)a_{kn_0}b_{kun_0}\hat{R}_k(\tau_k)\right] \quad (15)$$

where, by definition, $\beta_{1k}(n_{-1})$ and $\beta_{1k}(n_0)$ are given by

$$\beta_{1k}(n_{-1}) = \frac{1}{S}\sum_{s=1}^{S}F_sF_{ks}\alpha_{uns}\alpha_{un_{-1}s}^{(k)}\cos\left(\psi_{un_{-1}s}^{(k)}\right) \quad (16)$$

$$\beta_{1k}(n_0) = \frac{1}{S}\sum_{s=1}^{S}F_sF_{ks}\alpha_{uns}\alpha_{un_0s}^{(k)}\cos\left(\psi_{un_0s}^{(k)}\right) \quad (17)$$

which are variables related to the cross-correlation of the F-domain spreading codes between user k and the reference user of $k = 1$.

Finally, the decision variable Z_u of Fig. 3 is obtained by despreading $\{Z_{un}\}_{n=0}^{N_1-1}$ using the T-domain spreading sequence $\{a_n\}$, which was shown in (5). The decision variable Z_u can be expressed as

$$Z_u = D_u + \sum_{k=2}^{K} I_u^{(k)} + \mathcal{N}_u \quad (18)$$

where $\mathcal{N}_u = \sum_{n=0}^{N_1-1}\sum_{s=1}^{S} a_nF_s\mathcal{N}_{uns}$, which is a Gaussian random variable having zero mean and a variance of $\sum_{n=0}^{N_1-1}\sum_{s=1}^{S}(S\alpha_{uns}^2\mathcal{N}_0/2N_1E_b)$, while D_u represents the desired output after the T-domain despreading, F-domain despreading and T-domain fractionally despreading portrayed in Fig. 3, which can be expressed as

$$D_u = \left(\sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\frac{\alpha_{uns}^2}{N_1}\right)b_u. \quad (19)$$

Finally, $I_u^{(k)}$ in (18) represents the resultant MUI imposed by the kth interfering user, after the T-domain despreading, F-domain despreading and T-domain fractionally despreading. $I_u^{(k)}$ can be expressed as

$$I_u^{(k)} = \frac{N_1S}{T_s}\left[\rho_{1k}(n_{-1})b_{kun_{-1}}R_k(\tau_k) + \rho_{1k}(n_0)b_{kun_0}\hat{R}_k(\tau_k)\right] \quad (20)$$

where, by definition, $\rho_{1k}(n_{-1})$ and $\rho_{1k}(n_0)$ are given by

$$\rho_{1k}(n_{-1}) = \frac{1}{N_1}\sum_{n=0}^{N_1-1}a_na_{kn_{-1}}\beta_{1k}(n_{-1}) \quad (21)$$

$$\rho_{1k}(n_0) = \frac{1}{N_1}\sum_{n=0}^{N_1-1}a_na_{kn_0}\beta_{1k}(n_0) \quad (22)$$

which are variables related to the cross-correlation of the fractional T-domain spreading codes and to the cross-correlation of the F-domain spreading codes between user k and the reference user associated with $k = 1$.

For AWGN channels we have $\alpha_{uns}^{(k)} = 1$ and $\phi_{uns}^{(k)} = \phi_{su}^{(k)}$, which are independent of the index of n, hence the decision variable Z_u for $u = 1, 2, \ldots, U$ can also be represented by (18) with the terms at the right-hand side of (18) given as follows:

$$D_u = Sb_u \quad (23)$$

$$I_u^{(k)} = \frac{N_1S\cos\left(\psi_{su}^{(k)}\right)}{T_s} \times \left[\underbrace{\frac{1}{N_1}\sum_{n=0}^{N_1-1}a_na_{kn_{-1}}}_{\text{T-domain correlation}} \times \underbrace{\frac{1}{S}\sum_{s=1}^{S}F_sF_{ks}}_{\text{F-domain correlation}}\right.$$

$$\times\, b_{kun_{-1}} \underbrace{R_k(\tau_k)}_{\text{T-domain correlation}} + \underbrace{\frac{1}{N_1}\sum_{n=0}^{N_1-1} a_n a_{kn_0}}_{\text{T-domain correlation}}$$

$$\left. \times \underbrace{\frac{1}{S}\sum_{s=1}^{S} F_s F_{ks}}_{\text{F-domain correlation}} \times b_{kun_0} \underbrace{\hat{R}_k(\tau_k)}_{\text{T-domain correlation}} \right] \tag{24}$$

and $\mathcal{N}_u = \sum_{n=0}^{N_1-1}\sum_{s=1}^{S} a_n F_s \mathcal{N}_{uns}$, which is a Gaussian random variable having zero mean and a variance of $(S^2\mathcal{N}_0/2E_b)$. The MUI expression of (24) implies that the multiuser interference is suppressed by the T-domain despreading represented by $R_k(\tau_k)$ and $\hat{R}_k(\tau_k)$, by the F-domain despreading represented as $(1/S)\sum_{s=1}^{S} F_s F_{ks}$, as well as by the T-domain fractional despreading indicated by $(1/N_1)\sum_{n=0}^{N_1-1} a_n a_{kn_{-1}}$ and $(1/N_1)\sum_{n=0}^{N_1-1} a_n a_{kn_0}$. Therefore, the FS MC-CDMA scheme is, in fact, equivalent to the conventional single-carrier DS-CDMA scheme employing solely T-domain spreading and also to the conventional MC-CDMA arrangement using solely F-domain spreading. Hence they are capable of achieving a similar BER performance, provided that the resultant spreading-code dependent cross-correlation factors invoked in these three types of CDMA schemes are similar.

In order to obtain the probability density function (PDF) of the decision variable Z_u of (18) for deriving the bit error probability, we approximate the MUI by additive Gaussian noise, since it is constituted by the sum of numerous independent random variables. Based on the Gaussian approximation [30], the decision variable Z_u of (18) can be approximated as a Gaussian random variable having a mean given by

$$\mathbf{E}[Z_u] = D_u = \left(\sum_{n=0}^{N_1-1}\sum_{s=1}^{S} \frac{\alpha_{uns}^2}{N_1}\right) b_u \tag{25}$$

and a variance given by

$$\mathbf{Var}[Z_u] = (K-1)\mathbf{Var}\left[I_u^{(k)}\right] + \mathbf{Var}[\mathcal{N}_u] \tag{26}$$

where $\mathbf{Var}[\mathcal{N}_u]$ was previously given in the context of (18), while $\mathbf{Var}[I_u^{(k)}]$ is derived in the Appendix and expressed as

$$\mathbf{Var}\left[I_u^{(k)}\right] = \frac{\Omega}{3N_1^2 N_2}\sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\alpha_{uns}^2 \tag{27}$$

where we have $\Omega = E[(\alpha_{uns}^{(k)})^2]$. Upon substituting $\mathbf{Var}[\mathcal{N}_u]$ and $\mathbf{Var}[I_u^{(k)}]$ of (27) into (26), we obtain the variance of the decision variable Z_u, which is expressed as

$$\mathbf{Var}[Z_u] = \left[\frac{(K-1)}{3N_1^2 N_2} + \frac{S}{N_1}\left(\frac{2\Omega E_b}{\mathcal{N}_0}\right)^{-1}\right]\Omega \times \sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\alpha_{uns}^2. \tag{28}$$

Having obtained the statistics of the decision variables, below we derive the BER expression of the FS MC-CDMA system, when communicating over both AWGN, as well as over frequency-selective slow and fast Nakagami-m fading channels.

B. Bit Error Rate

With the aid of (25) and (28), the signal to interference plus noise ratio (SINR) for the given channel parameters of $\{\alpha_{uns}^2\}$ can be expressed as [18]

$$\begin{aligned}\text{SINR}\left(\{\alpha_{uns}^2\}\right) &= \frac{\mathbf{E}^2[Z_u]}{2\mathbf{Var}[Z_u]} \\ &= \left[\frac{2(K-1)}{3N_2} + SN_1\left(\frac{\Omega E_b}{\mathcal{N}_0}\right)^{-1}\right]^{-1} \\ &\quad \times \sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\frac{\alpha_{uns}^2}{\Omega}.\end{aligned} \tag{29}$$

In the context of AWGN channels we have $\alpha_{uns}^2 = 1$ and correspondingly also $\Omega = 1$. Upon substituting these results into (29), the SINR achievable over AWGN channels can be expressed as

$$\text{SINR} = \left[\frac{2(K-1)}{3N_1N_2S} + \left(\frac{E_b}{\mathcal{N}_0}\right)^{-1}\right]^{-1} \tag{30}$$

which, in fact, is identical to the SINR expression of a classic single-carrier DS-CDMA system having the T-domain spreading factor of N_1N_2S. Furthermore, by studying (29) and (30) in further detail, it can be argued that the proposed FS MC-CDMA system constitutes a self-flexible system without requiring external reconfiguration. More explicitly, it can be shown that when the time-domain fluctuation of the communication channel is sufficiently rapid, for the fading envelope of each signal of the consecutive fractions to become independent, the proposed FS MC-CDMA system is capable of achieving the highest attainable diversity order of N_1S, as suggested by the factor $\sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\alpha_{uns}^2/\Omega$ in (29). However, in exchange for achieving the highest attainable diversity order, the spreading factor is at its minimum, which is given by the T-domain spreading factor $N_2 = T_D/T_c$, as shown in (29). By contrast, when the communication channel is non-fading, then the FS MC-CDMA system has no diversity gain and achieves its Gaussian performance, but correspondingly reaches its highest possible spreading factor value of N_1N_2S, as seen by the first term in the bracket of (30). The above analysis implies furthermore that when the rate of channel fading is not sufficiently fast for satisfying the independent fading condition of each fraction-signal, the diversity order will be lower than the maximum achievable diversity order of N_1S, while the corresponding spreading factor will be higher than the minimum given by N_2 and it is in the range of (N_2, N_1N_2S).

Specifically, let us assume for example that we encounter frequency-selective slow fading, such that the fading amplitudes $\{\alpha_{uns}\}$ are independent of the index of n, i.e., we have $\alpha_{uns} =$

α_{su}. Then, the expression of SINR of (29) can be simplified to

$$\text{SINR}\left(\{\alpha_{su}^2\}\right) = \left[\frac{2(K-1)}{3N_1N_2} + S\left(\frac{\Omega E_b}{\mathcal{N}_0}\right)^{-1}\right]^{-1} \times \sum_{s=1}^{S} \frac{\alpha_{su}^2}{\Omega} \quad (31)$$

which indicates that the diversity order obeys $S < SN_1$, while the spreading factor N_1N_2 is in the range of (N_2, N_1N_2S).

With the aid of (29), (30), and (31), let us now derive the corresponding BER expressions. The BER of the FS MC-CDMA system communicating over AWGN channels can be expressed as

$$P_b = Q\left(\sqrt{2\cdot \text{SINR}}\right) = Q\left(\left[\frac{K-1}{3N_1N_2S} + \left(\frac{2E_b}{\mathcal{N}_0}\right)^{-1}\right]^{-1/2}\right) \quad (32)$$

where $Q(x)$ represents the Gaussian Q-function, which can either be represented in the form of $Q(x) = (1/\sqrt{2\pi})\int_x^\infty \exp(-(t^2/2))\,dt$, or alternatively, as $Q(x) = (1/\pi)\int_0^{\pi/2} \exp(-(x^2/2\sin^2\theta))\,d\theta,\ x \geq 0$, as shown in [26] and [28].

In the context of the frequency-selective fast Nakagami-m fading channels, the BER conditioned on the fading amplitudes $\{\alpha_{uns}\}$ can be written as

$$P_b(\gamma) = Q\left(\sqrt{2\cdot \text{SINR}\left(\{\alpha_{uns}^2\}\right)}\right) = Q\left(\sqrt{2\cdot \sum_{n=0}^{N_1-1}\sum_{s=1}^{S} \gamma_{ns}}\right) \quad (33)$$

where we have

$$\gamma_{ns} = \gamma_c \cdot \frac{\alpha_{uns}^2}{\Omega} \quad (34)$$

$$\gamma_c = \left[\frac{2(K-1)}{3N_2} + \left(\frac{\Omega E_b}{SN_1\mathcal{N}_0}\right)^{-1}\right]^{-1}. \quad (35)$$

The unconditional BER of the FS MC-CDMA systems communicating over frequency-selective fast Nakagami-m fading channels, can be obtained by averaging (33) with respect to the PDFs of γ_{ns} with the aid of (4). The corresponding BER result can be obtained from (50) of [9] with mL replaced by mSN_1, which can be written as

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{m\sin^2\theta}{\gamma_c + m\sin^2\theta}\right)^{mSN_1} d\theta \quad (36)$$

explicitly showing that the diversity order achieved is SN_1. Furthermore, it can be shown that the average BER of (36) can also be expressed as [9], [28]

$$P_b = \sqrt{\frac{\gamma_c}{\gamma_c+m}}\,\frac{(1+\gamma_c/m)^{-mN_1S}\,\Gamma(mN_1S+1/2)}{2\sqrt{\pi}\Gamma(mN_1S+1)} \times {}_2F_1\left(1, mN_1S+\frac{1}{2}; mN_1S+1; \frac{m}{m+\gamma_c}\right) \quad (37)$$

where ${}_2F_1(a,b;c;z)$ is the hypergeometric function defined in [25] as ${}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty}((a)_k(b)_k z^k)/((c)_k k!)$ and $(a)_k = a(a+1)\dots(a+k-1), (a)_0 = 1$.

Upon using a similar approaches to that used deriving the BER expression of (37) derived for frequency-selective fast Nakagami-m fading channels, the BER expression valid for communicating over frequency-selective slow Nakagami-m fading channels can be expressed as

$$P_b = \sqrt{\frac{\gamma_c}{\gamma_c+m}}\,\frac{(1+\gamma_c/m)^{-mS}\Gamma(mS+1/2)}{2\sqrt{\pi}\Gamma(mS+1)} \times {}_2F_1\left(1, mS+\frac{1}{2}; mS+1; \frac{m}{m+\gamma_c}\right) \quad (38)$$

where, in harmony with the SINR expression of (31) derived for frequency-selective slow Nakagami-m fading channels, γ_c is given by

$$\gamma_c = \left[\frac{2(K-1)}{3N_1N_2} + \left(\frac{\Omega E_b}{S\mathcal{N}_0}\right)^{-1}\right]^{-1}. \quad (39)$$

It can also be shown [9] that the limit of (36) with respect to $m \to \infty$ will converge to (32), which quantifies the BER in the context of AWGN channels. This characteristic implies that when the channel quality improves and the fading envelope becomes near-constant, the FS MC-CDMA will automatically leverage the diversity gain into spreading gain.

Above we have derived the BER expressions for the proposed FS MC-CDMA system, when communicating over both AWGN, as well as over frequency-selective slow or fast Nakagami-m fading channels. Let us now illustrate its performance in quantitative terms.

IV. Performance Results and Discussion

In this section we evaluate the achievable performance of FS MC-CDMA based on the expressions derived in Section III. In our evaluation we assumed a FS MC-CDMA system using $U = 4$ bits per symbol. Furthermore, for convenience, the parameters shown in the figures are also repeated here as follows:

- $E_b/\mathcal{N}_0$ signal-to-noise ratio (SNR) per bit;
- N_1 number of fractions per symbol;
- N_2 number of chips per fraction;
- S F-domain spreading factor;
- m fading parameter of the Nakagami-m fading channels;
- K number of users.

Based on the above parameters, it can be shown that, for transmitting at a bit rate of $R_b = 1/T_b$ bits/second per user, the bit rate after the S-P conversion becomes $R_b' = 1/UT_b$

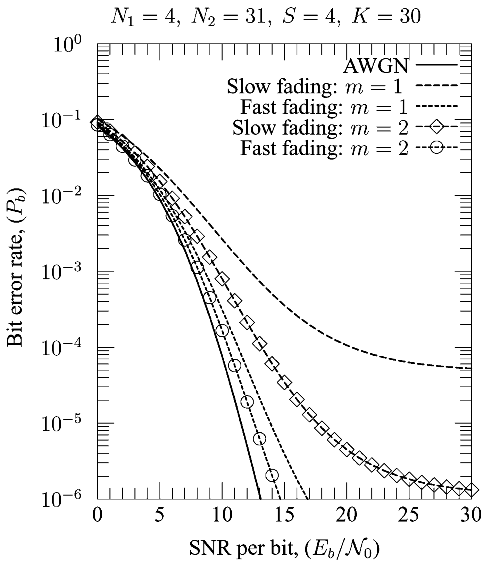

Fig. 4. BER versus $E_b/\mathcal{N}_0$ performance of the FS-MC-CDMA systems, when communicating over both non-fading AWGN, as well as over slow frequency-selective and fast frequency-selective Rayleigh $(m = 1)$ or Nakagami-m $(m = 2)$ fading channels.

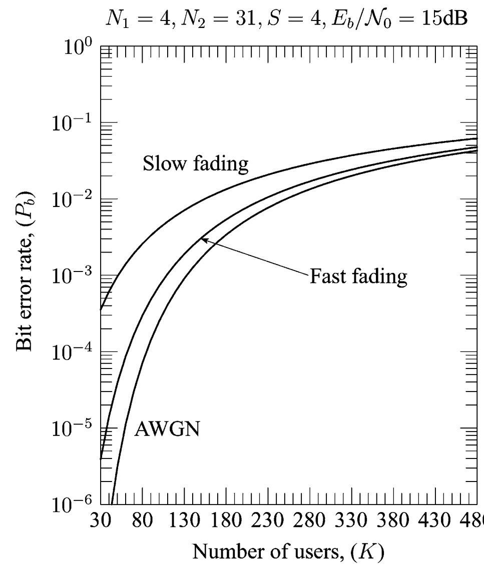

Fig. 5. BER performance versus the number of users K for the FS-MC-CDMA systems, when communicating over both non-fading AWGN, as well as over slow frequency-selective and fast frequency-selective Rayleigh $(m = 1)$ fading channels.

bits/second. Hence, the total system bandwidth required is about $U \times SN_1N_2R'_b = SN_1N_2R_b$ Hz.

In Fig. 4, we show the corresponding comparison of the BER performance of the FS MC-CDMA system, when communicating over both the nonfading AWGN, as well as over the frequency-selective slow fading and fast fading channels, assuming both Rayleigh $(m = 1)$ and Nakagami-m $(m = 2)$ fading models. The curves in the figure were plotted against the average SNR per bit of $E_b/\mathcal{N}_0$ for the parameters of $N_1 = 4, N_2 = 31, S = 4$, and $K = 30$. From the results of Fig. 4 we observe that for a given SNR per bit value, the frequency-selective fast fading channel model achieves a lower BER, than the frequency-selective slow fading channel model, regardless of $m = 1$ or $m = 2$. Furthermore, for $m = 1$ or $m = 2$, the BER performance curve of the frequency-selective fast fading channel model is only about 4 dB or 2 dB away from the BER performance curve of the AWGN channel, at the BER of 10^{-6}, respectively. By contrast, at the same BER of 10^{-6}, the BER performance of the frequency-selective slow fading channel is significantly worse than that over AWGN channels, regardless of $m = 1$ and $m = 2$. For $m = 1$ or $m = 2$ we observe the formation of an error floor for the frequency-selective slow fading channel model at the SNR per bit values of about 20 dB or 25 dB, respectively. The reason for the performance trends of Fig. 4 is that in the context of the frequency-selective fast fading channel model the total diversity order is $SN_1 = 16$. By contrast, in the context of the frequency-selective slow fading channel model, the diversity order is only $S = 4$.

Fig. 5 demonstrates the comparison of the BER performance versus the number of users K for the FS MC-CDMA system, when communicating over both non-fading AWGN, as well as over frequency-selective slow Rayleigh $(m = 1)$ fading and fast Rayleigh $(m = 1)$ fading channels. The curves in Fig. 5 were plotted versus the number of users K for the

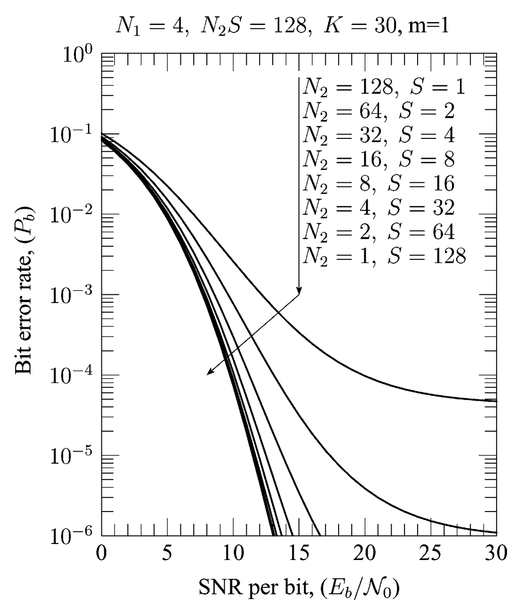

Fig. 6. BER versus $E_b/\mathcal{N}_0$ performance of the FS-MC-CDMA systems, when the frequency-selectivity of the fast Rayleigh $(m = 1)$ fading channels and hence the frequency diversity increase. In this figure $N_2 = 128, S = 1$ represents the frequency non-selective fading, while $N_2 = 1, S = 128$ corresponds to the scenario of strongest frequency-selective fading.

parameters of $N_1 = 4, N_2 = 31, S = 4$, and $E_b/\mathcal{N}_0 = 15$ dB. The results of Fig. 5 also show that the achievable BER performance is better in frequency-selective fast fading environments, than in the frequency-selective slow fading environments. This is a consequence of the higher diversity order achievable over the frequency-selective fast fading channels, than over the frequency-selective slow fading channels.

In Figs. 6 and 7 we illustrate the effect of the frequency-selectivity of fast Rayleigh fading channels on the BER performance of the FS MC-CDMA system. The results of Fig. 6

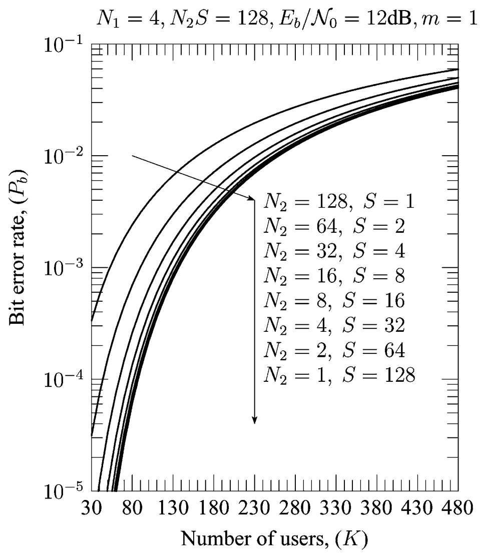

Fig. 7. BER performance versus the number of users of K for the FS-MC-CDMA systems, when the frequency-selectivity of the fast Rayleigh $(m = 1)$ fading channels and hence the frequency diversity increase.

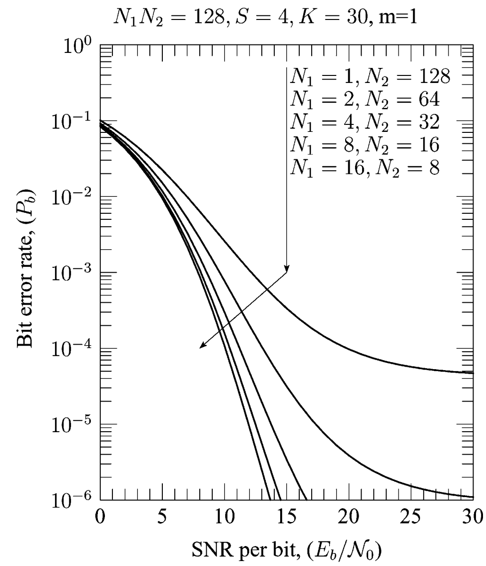

Fig. 8. BER versus $E_b/\mathcal{N}_0$ performance of the FS-MC-CDMA systems, when the rate of fading of the frequency-selective Rayleigh $(m = 1)$ fading channels varies from slow to rapid. $N_1 = 1, N_2 = 128$ represents the time nonselective fading, while $N_1 = 16, N_2 = 8$ corresponds to the rapidest fading scenario.

were computed against the SNR per bit value of $E_b/\mathcal{N}_0$, while the results of Fig. 7 were computed versus the number of active users K. All the results were computed by using the equations derived in Section III. In our computations we assumed having $N_1 = 4$ fractions per symbol, which implies encountering a fast fading channel having a constant coherence time. By contrast, the frequency-selectivity or the coherence bandwidth was a variable, which may span a range from low to high values. For a given system bandwidth, owing to this wide-range coherence bandwidth, the number of independent subcarrier signals may also span a range from low values corresponding to $S = 1$ to high values associated with $S = 128$, for example. When combining these independently faded subcarrier signals at the receiver, a scenario associated with a high spreading factor and low diversity order encountered, when communicating over low frequency-selective fading channels will be converted to the low spreading factor and high diversity order scenario, when communicating over strongly frequency-selective fading channels. Furthermore, in our computations we assumed that $N_2S = 128$ was a constant for the sake of guaranteeing a constant system bandwidth. From the results of Figs. 6 and 7 we can see that for the given parameters shown at the top of the figures, the BER performance becomes better, when the frequency-selectivity of the channels becomes more dominant. However, upon comparing the results of Fig. 6 with the BER performance of the AWGN scenario characterized in Fig. 4 a frequency diversity order of $S = 5$ has the potential of achieving a BER performance close to recorded for AWGN channels.

In contrast to Figs. 6 and 7 which demonstrate the effect of the frequency-selectivity of wireless channels on the BER performance, in Fig. 8 we evaluated the effect of the time-selectivity of wireless channels on the BER performance of the FS MC-CDMA systems. As shown in Fig. 8, in our computations we assumed that the number of independently faded fractions within a symbol duration was $N_1 = 1, 2, 4, 8, 16$, where $N_1 = 1$ implies slow fading conditions, rendering the fading amplitude near-constant over a symbol duration. By contrast, the scenario corresponding to $N_1 = 16$ represents the fastest fading conditions, since there exist $N_1 = 16$ fractions within a symbol duration, which experience independent fading. The results of Fig. 8 were evaluated using the equations derived in Section III versus the SNR per bit of $E_b/\mathcal{N}_0$ for the parameters of $S = 4, K = 30, m = 1$. In order that the system bandwidth and the transmitted data rate remain constants for the sake of fair comparisons, we assumed that the product of two T-domain spreading factors was a constant, namely $N_1N_2 = 128$. From the results of Fig. 8 we observe that for a given frequency-selectivity associated with $S = 4$, the BER performance of the FS MC-CDMA system is enhanced, when the grade of time-selectivity of the wireless channels increases, or when the fading-rate of the wireless channel becomes faster. This is because, when the time-selectivity becomes more dominant, the FS MC-CDMA receiver is capable of achieving a higher diversity order by combining the independently faded fractions. Furthermore, from the results of Fig. 8 we can see that the BER performance associated with $N_1 = 2$ or 3 is significantly better than that associated with $N_1 = 1$. Therefore, in the FS MC-CDMA system even a relatively modest grade of time-selectivity (or Doppler spread) encountered in practise can be leveraged into substantial diversity gains.

Finally, Fig. 9 shows the BER performance of the FS MC-CDMA system with respect to the Nakagami fading parameter m, when communicating over frequency-selective slow or fast Nakagami-m fading channels. For the sake of comparison, the benchmark BER performance achieved in AWGN channels was also plotted in Fig. 9. The results of Fig. 9 illustrate that when the value of m increases, the BER performance of FS MC-CDMA over both fast and slow fading channels will approach the BER

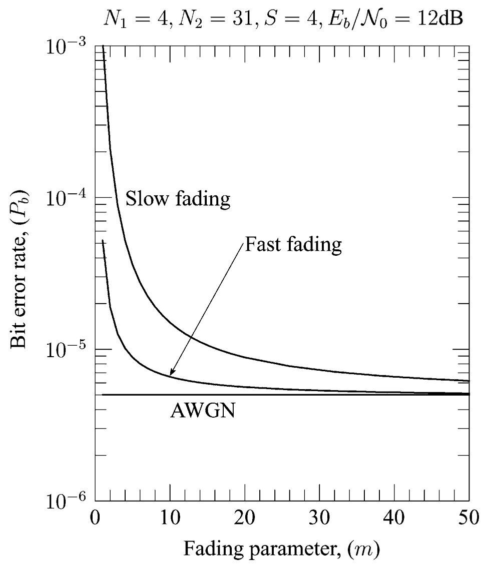

Fig. 9. BER performance versus the Nakagami fading parameter of m for the FS-MC-CDMA systems, when communicating over both the nonfading AWGN, as well as over frequency-selective slow fading and fast fading channels.

performance of AWGN channels. However, for any given fading parameter m, the FS MC-CDMA system communicating over fast fading channels is capable of achieving a lower BER, than over slow fading channels.

V. Conclusion

In this paper, we have proposed a novel fractionally spread multicarrier CDMA arrangement, i.e., the FS MC-CDMA scheme, which employs both T-domain spreading and F-domain spreading. Specifically, we have proposed the employment of concatenated T-domain spreading for improving the achievable performance, when communicating over wireless channels exhibiting both frequency-selective and time-selective fading. The BER performance of the FS MC-CDMA system has been investigated both analytically and numerically, when communicating over both AWGN, and frequency-selective slow Nakagami-m fading or frequency-selective fast (time-selective) Nakagami-m fading channels. Based on our results we infer the following conclusions. Firstly, FS MC-CDMA constitutes a self-flexible scheme without requiring external reconfiguration. It is capable of automatically converting the spreading gain to the diversity gain, when the frequency-selectivity and/or time-selectivity of the wireless channel becomes more dominant. Conversely, it has the ability to leverage the diversity gain into the spreading gain, when the frequency-selectivity and/or time-selectivity of the wireless channel becomes less prevalent. Secondly, the FS MC-CDMA scheme is capable of sufficiently exploiting the time-selectivity of the channel for improving the achievable performance. In the FS MC-CDMA system even a relatively modest time-selectivity encountered may be transformed to substantial diversity gains. Furthermore, our numerical results show that in FS MC-CDMA the BER performance attained, when communicating over fast fading channels is significantly better than that over slow fading channels. The BER performance achieved over fading channels approaches that achievable over AWGN channels, when the grade of frequency-selectivity and/or time-selectivity of the fading channel increases. Therefore, the FS MC-CDMA scheme is beneficial for employment over wireless channels exhibiting frequency-selective fading and/or time-selective fading.

Appendix
Derivation of $\mathbf{Var}[I_u^{(k)}]$

In this Appendix, we derive the variance of the MUI $I_u^{(k)}$, where $I_u^{(k)}$ is given by (20). Since random spreading codes are assumed for both T-domain spreading and for F-domain spreading, and since for a given value of τ_k, the partial correlation functions $R_k(\tau_k)$ and $\hat{R}_k(\tau_k)$ in (20) are also independent of each other, the variance of $I_u^{(k)}$ can be expressed as

$$\mathbf{Var}\left[I_u^{(k)}\right] = \frac{N_1^2 S^2}{T_s^2}\left[\mathbf{E}\left[\rho_{1k}^2(n_{-1})\right]\mathbf{E}\left[R_k^2(\tau_k)\right] + \mathbf{E}\left[\rho_{1k}^2(n_0)\right]\mathbf{E}\left[\hat{R}_k^2(\tau_k)\right]\right] \quad (40)$$

where upon extending $\beta_{1k}(n_{-1})$ in (21) with the aid of (16), we have

$$\mathbf{E}\left[\rho_{1k}^2(n_{-1})\right] = \mathbf{E}\left[\left(\frac{1}{N_1}\sum_{n=0}^{N_1-1} a_n a_{kn_{-1}} \frac{1}{S} \times \sum_{s=1}^{S} F_s F_{ks} \alpha_{uns}\alpha_{un_{-1}s}^{(k)} \cos\left(\psi_{un_{-1}s}^{(k)}\right)\right)^2\right]. \quad (41)$$

Since random spreading codes are assumed, after assembly average, the right-hand side of the above equation retains only the squared terms, while all the other terms are zero. Hence, (41) can be expressed as

$$\mathbf{E}\left[\rho_{1k}^2(n_{-1})\right] = \frac{\Omega}{2N_1^2 S^2}\sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\alpha_{uns}^2. \quad (42)$$

Similarly, we can derive the variance of $\mathbf{E}[\rho_{1k}^2(n_0)]$, and it can be shown that $\mathbf{E}[\rho_{1k}^2(n_0)] = \mathbf{E}[\rho_{1k}^2(n_{-1})]$. Upon using $\mathbf{E}[\rho_{1k}^2(n_{-1})]$ and $\mathbf{E}[\rho_{1k}^2(n_0)]$, (40) can be simplified to

$$\mathbf{Var}\left[I_u^{(k)}\right] = \frac{\Omega}{2T_s^2}\left[\mathbf{E}\left[R_k^2(\tau_k)\right] + \mathbf{E}\left[\hat{R}_k^2(\tau_k)\right]\right] \times \sum_{n=0}^{N_1-1}\sum_{s=1}^{S}\alpha_{uns}^2 \quad (43)$$

where $\mathbf{E}[R_k^2(\tau_k)]$ and $\mathbf{E}[\hat{R}_k^2(\tau_k)]$ represent the variance of the partial cross-correlation functions $R_k^2(\tau_k)$ and $\hat{R}_k^2(\tau_k)$. It can be readily shown that for random spreading sequences and for rectangular chip-waveforms, the expression of $\mathbf{E}[R_k^2(\tau_k)] + \mathbf{E}[\hat{R}_k^2(\tau_k)]$ is given by [9] and [30]

$$\mathbf{E}\left[R_k^2(\tau_k)\right] + \mathbf{E}\left[\hat{R}_k^2(\tau_k)\right] = \frac{2T_d^2}{3N_2} = \frac{2T_s^2}{3N_1^2 N_2}. \quad (44)$$

Upon substituting (44) into (43), finally, we obtain the variance of $\mathbf{Var}[I_u^{(k)}]$, which is expressed as

$$\mathbf{Var}\left[I_u^{(k)}\right] = \frac{\Omega}{3N_1^2 N_2} \sum_{n=0}^{N_1-1} \sum_{s=1}^{S} \alpha_{uns}^2. \qquad (45)$$

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