Genetically Enhanced TTCM Assisted MMSE Multi-User Detection for SDMA-OFDM

M. Jiang and L. Hanzo[1]

School of ECS, Univ. of Southampton, SO17 1BJ, UK.
Tel: +44-703-593 125, Fax: +44-703-593 045
Email: [1]lh@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

Abstract – **Space Division Multiple Access (SDMA) aided Orthogonal Frequency Division Multiplexing (OFDM) systems assisted by efficient Multi-User Detection (MUD) techniques have recently attracted intensive research interests. The Maximum Likelihood Detection (MLD) arrangement was found to attain the best performance, although this was achieved at the cost of a computational complexity, which increases exponentially both with the number of users and with the number of bits per symbol transmitted by higher-order modulation schemes. By contrast, the Minimum Mean-Square Error (MMSE) SDMA-MUD exhibits a lower complexity at the cost of a performance loss. Forward Error Correction (FEC) schemes such as Turbo Trellis Coded Modulation (TTCM) may be efficiently amalgamated with SDMA-OFDM systems for the sake of improving the achievable performance. Genetic Algorithm (GA) based multiuser detection techniques have been shown to provide a good performance in MUD-aided Code Division Multiple Access (CDMA) systems. In this contribution a GA-aided MMSE MUD is proposed for employment in a TTCM-assisted SDMA-OFDM system, which is capable of achieving a similar performance to that attained by its MLD-aided counterpart at a significantly lower complexity, especially at high user loads.**

1. INTRODUCTION

Space Division Multiple Access (SDMA) based Orthogonal Frequency Division Multiplexing (OFDM) [1] communication invoking Multi-User Detection (MUD) techniques has recently attracted intensive research interests. In SDMA Multi-Input-Multi-Output (MIMO) systems the transmitted signals of L simultaneous uplink mobile users - each equipped with a single transmit antenna - are received by the P different receiver antennas of the Base Station (BS). At the BS the individual users' signals are separated with the aid of their unique, user-specific spatial signature constituted by their channel transfer functions or, equivalently, Channel Impulse Responses (CIRs). A variety of MUD schemes, such as the Least-Squares (LS) [2, 3] and Minimum Mean-Square Error (MMSE) [2–4] detectors, or Successive Interference Cancellation (SIC) [2–4], Parallel Interference Cancellation (PIC) [2] and Maximum Likelihood Detection (MLD) [2, 4] schemes may be invoked for the sake of separating the different users at the BS on a per-subcarrier basis. Among these schemes, the ML detection arrangement was found to give the best performance, although this was achieved at the cost of a dramatically increased computational complexity, especially in the context of a high number of users and higher-order modulation schemes, such as 16QAM. By contrast, MMSE combining exhibits the lowest complexity in this set of detectors, while suffering from a performance loss [1].

Furthermore, the achievable performance can be significantly improved, if Forward Error Correction (FEC) schemes, such as for example Turbo Convolutional (TC) codes [5] are incorporated into the SDMA system. Various Coded Modulation (CM) schemes such as for example Trellis Coded Modulation (TCM) [5, 6], Turbo TCM (TTCM) [5, 7], Bit-Interleaved Coded Modulation (BICM) [5, 8] and Iteratively Decoded BICM (BICM-ID) [5, 9] have also attracted intensive research interests, since they are capable of achieving a substantial coding gain without bandwidth expansion. It was demonstrated in [10] that TTCM generally provides the best performance in the family of CM schemes in the specific context of the SDMA-OFDM system investigated. Hence in this paper we will adopt TTCM as the FEC scheme for our SDMA-OFDM system.

Genetic Algorithms (GAs) [11] have been applied to a number of problems, such as machine learning and modelling adaptive processes. Moreover, GA-aided transceiver research has been documented in the context of Code Division Multiple Access (CDMA) systems in [12, 13]. However, most of the GA-aided transceiver research mentioned above was conducted in the context of Code Division Multiple Access (CDMA) systems. In this contribution, we combine the GA with an MMSE-MUD assisted TTCM-aided SDMA-OFDM system. Our simulation results show that the proposed MMSE-GA assisted TTCM-SDMA-OFDM system is capable of achieving a similar performance to that attained by its optimum Maximum Likelihood (ML) MUD assisted counterpart at a significantly lower computational complexity, especially at high user loads.

The structure of this paper is as follows. The SDMA MIMO channel model is described in Section 2.1, while the overview of the GA-assisted TTCM-aided MMSE-SDMA-OFDM system is given in Section 2.2, followed by the introduction of the basic principles of the concatenated MMSE-GA MUD of Section 2.3. Our simulation results are provided in Section 3, while the associated complexity issues are discussed in Section 4. Our final conclusions are summarized in Section 5.

2. SYSTEM MODEL

2.1. SDMA MIMO Channel Model

Figure 1 shows an SDMA uplink MIMO channel model, where each of the L simultaneous mobile users employs a single transmit antenna, while the BS's receiver has P antennas. At the k^{th} subcarrier of the n^{th} OFDM symbol received by the P-element BS antenna array we have the received complex signal vector $\mathbf{x}[n,k]$, which is constituted by the superposition of the independently faded signals associated with the L mobile users and contaminated by the AWGN

Acknowledgements: The work reported in this paper has formed part of the (relevant work area) area of the (Core 2, or 1, or 3, or Elective Title, as appropriate) Research Programme of the Virtual Centre of Excellence in Mobile and Personal Communications, Mobile VCE, www.mobilevce.com, whose funding support, including that of EPSRC, is gratefully acknowledged. Fully detailed technical reports on this research are available to Industrial Members of Mobile VCE.

0-7803-8521-7/04/$20.00 © 2004 IEEE

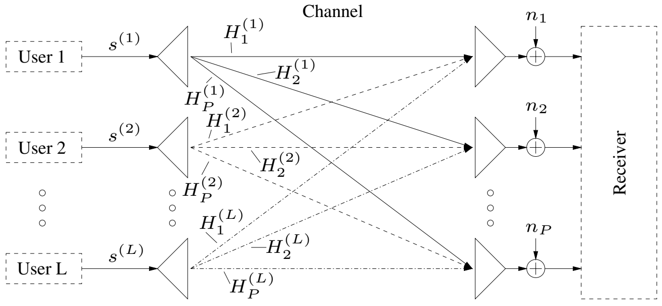

Figure 1: Schematic of the SDMA uplink MIMO channel model [1], where each of the L mobile users is equipped with a single transmit antenna and the BS's receiver is assisted by a P-element antenna front-end.

noise, expressed as:

$$\mathbf{x} \quad = \quad \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where the $(P \times 1)$-dimensional vector $\mathbf{x}$, the $(L \times 1)$-dimensional vector $\mathbf{s}$ and the $(P \times 1)$-dimensional vector $\mathbf{n}$ are the received, transmitted and noise signals, respectively. Here we have omitted the indices $[n, k]$ for each vector for the sake of notational convenience. Specifically, the vectors $\mathbf{x}$, $\mathbf{s}$ and $\mathbf{n}$ are given by:

$$\mathbf{x} \quad = \quad (x_1, x_2, \ldots, x_P)^T, \tag{2}$$

$$\mathbf{s} \quad = \quad (s^{(1)}, s^{(2)}, \ldots, s^{(L)})^T, \tag{3}$$

$$\mathbf{n} \quad = \quad (n_1, n_2, \ldots, n_P)^T. \tag{4}$$

The $(P \times L)$-dimensional matrix $\mathbf{H}$, which contains the frequency-domain channel transfer functions (FD-CHTF) of the L users, is given by:

$$\mathbf{H} \quad = \quad (\mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \ldots, \mathbf{H}^{(L)}), \tag{5}$$

where $\mathbf{H}^{(l)}$ $(l = 1, \ldots, L)$ is the vector of the FD-CHTFs associated with the transmission paths spanning the distance between the l^{th} user's transmit antenna and each element of the P-element receiver antenna array, which is expressed:

$$\mathbf{H}^{(l)} \quad = \quad (H_1^{(l)}, H_2^{(l)}, \ldots, H_P^{(l)})^T, \quad l = \{1, \ldots, L\}. \tag{6}$$

In Equations 1 to 6, we assume that the complex signal $s^{(l)}$ transmitted by the l^{th} user has zero-mean and a variance of σ_l^2. The AWGN noise signal n_p also exhibits a zero-mean and a variance of σ_n^2. The FD-CHTFs $H_p^{(l)}$ of the different receivers or users are independent, stationary, complex Gaussian distributed processes with zero-mean and unit variance.

2.2. System Overview

In Figure 2, we present the schematic of the proposed concatenated MMSE-GA MUD aided SDMA-OFDM uplink system. At the transmitter end, as seen at the top of Figure 2, the information bit sequences of the geographically separated L simultaneous mobile users are forwarded to the TTCM [5] encoders, where they are encoded into symbols. The encoded signals $s^{(l)}$ $(l = 1, \ldots, L)$ are then forwarded to the OFDM-related Inverse Fast Fourier Transform (IFFT) based modulator, which converts the frequency-domain signals to the time-domain modulated OFDM symbols. The OFDM symbols are then transmitted by the independent Mobile Stations (MSs) to the BS over the SDMA MIMO channel. Then each element of the receiver antenna array shown at the bottom of Figure 2 receives the superposition of the transmitted signals faded and contaminated by the channel and performs Fast Fourier Transform (FFT) based OFDM demodulation. The demodulated outputs x_p $(p = 1, \ldots, P)$ seen in Figure 2 are forwarded to the proposed concatenated MMSE-GA MUD for separating the different users' signals. The separated signals $\hat{s}^{(l)}$ $(l = 1, \ldots, L)$, namely the estimated versions of the transmitted signals, are then independently decoded by the TTCM decoders of Figure 2.

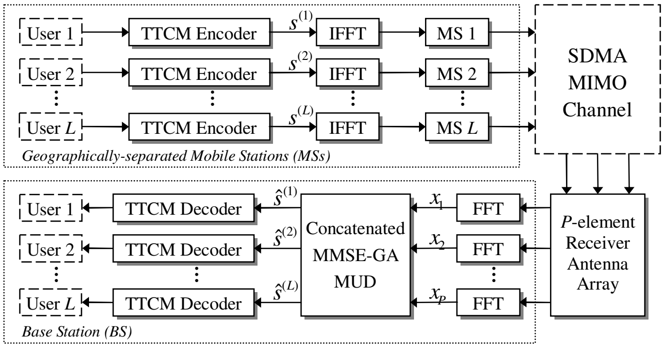

Figure 2: Schematic of the MMSE-GA-concatenated multi-user detected SDMA-OFDM uplink system.

2.3. MMSE-GA Multi-User Detector

2.3.1. MMSE MUD

In the MMSE-based MUD the estimates of the different users' transmitted signals are generated with the aid of the linear MMSE combiner. More specifically, the estimated signal vector $\hat{\mathbf{s}} \in \mathbb{C}^{L \times 1}$ generated from the transmitted signal $\mathbf{s}$ of the L simultaneous users, as shown in Figure 2, is obtained by linearly combining the signals received by the P different receiver antenna elements with the aid of the array weight matrix, as follows [1]:

$$\hat{\mathbf{s}}_{\text{MMSE}} \quad = \quad \mathbf{W}_{\text{MMSE}}^H \mathbf{x}, \tag{7}$$

where the superscript H denotes the Hermitian transpose, and $\mathbf{W}_{\text{MMSE}} \in \mathbb{C}^{P \times L}$ is the MMSE-based weight matrix given by [1]:

$$\mathbf{W}_{\text{MMSE}} \quad = \quad (\mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I})^{-1} \mathbf{H}, \tag{8}$$

while $\mathbf{I}$ is the identity matrix and σ_n^2 is the AWGN noise variance.

2.3.2. Optimization Metric for the GA MUD

The optimum ML MUD [1] uses an exhaustive search, which requires 2^{mL} evaluations of the decision metric of Equation 9 for finding the most likely transmitted L-user symbol vector $\hat{\mathbf{s}}_{\text{ML}}$:

$$\hat{\mathbf{s}}_{\text{ML}} = \arg\left\{ \min_{\check{\mathbf{s}} \in \mathcal{M}^L} ||\mathbf{x} - \mathbf{H}\check{\mathbf{s}}||_2^2 \right\}, \tag{9}$$

where the set of $\mathcal{M}^L$ number of trial-vectors is given by [1]:

$$\mathcal{M}^L = \left\{ \check{\mathbf{s}} = \begin{pmatrix} \check{s}^{(1)} \\ \vdots \\ \check{s}^{(L)} \end{pmatrix} \middle| \check{s}^{(1)}, \ldots, \check{s}^{(L)} \in \mathcal{M}_c \right\}, \tag{10}$$

and where $\mathcal{M}_c$ denotes the set containing the 2^m number of legitimate complex constellation points associated with the specific modulation scheme employed, while m denotes the number of bits per symbol.

Furthermore, the optimum ML-based decision metric of Equation 9 may also be used in the GA MUD for the sake of detecting the estimated transmitted symbol vector $\hat{\mathbf{s}}_{\text{GA}}$, where the decision metric required for the p^{th} receiver antenna, namely the antenna-specific *objective function* is defined by:

$$\Omega_p(\mathbf{s}) \quad = \quad |x_p - \mathbf{H}_p \mathbf{s}|^2, \tag{11}$$

and x_p is the received symbol at the input of the p^{th} receiver at a specific OFDM subcarrier, while $\mathbf{H}_p$ is the p^{th} row of the $(P \times L)$-dimensional channel transfer function matrix $\mathbf{H}$. Therefore the decision rule for the optimum multiuser detector associated with the p^{th} antenna is to choose the specific L-symbol vector $\mathbf{s}$, which minimizes the metric given in Equation 11. Thus, the estimated transmitted symbol vector of the L users based on the knowledge of the received signal at the p^{th} receiver antenna and a specific subcarrier is given by:

$$\hat{\mathbf{s}}_{\text{GA}_p} \quad = \quad \arg\left\{\min_{\mathbf{s}}\left[\Omega_p(\mathbf{s})\right]\right\}. \tag{12}$$

It transpires from the above derivation that we will have P metrics in total for the P receiver antennas. Since the CIRs of each of the P antennas are statistically independent, the L-symbol vector that is considered optimum at antenna 1 may not be considered optimum at antenna 2, etc. In other words, this implies that a decision conflict is encountered, which may be expressed as:

$$\arg\left\{\min_{\mathbf{s}}\left[\Omega_i(\mathbf{s})\right]\right\} = \hat{\mathbf{s}}_{\text{GA}_i} \neq \hat{\mathbf{s}}_{\text{GA}_j} = \arg\left\{\min_{\mathbf{s}}\left[\Omega_j(\mathbf{s})\right]\right\}, \tag{13}$$

where $\forall\ i, j \in \{1, \ldots, P\},\ i \neq j$. This decision conflict therefore leads to a so-called multi-objective optimization problem, since the optimization of the P metrics may result in more than one possible L-symbol solutions. In order to resolve this problem, we may amalgamate the P number of antenna-specific L-symbol metrics into a 'joint' metric as follows:

$$\Omega(\mathbf{s}) \quad = \quad \sum_{p=1}^{P} \Omega_p(\mathbf{s}). \tag{14}$$

Hence, the decision rule is to find the specific estimated transmitted L-symbol vector $\hat{\mathbf{s}}_{\text{GA}}$ that minimizes $\Omega(\mathbf{s})$ in Equation 14 for the OFDM subcarrier considered.

2.3.3. Concatenated MMSE-GA Multi-User Detection

In order to avoid an inefficient, entirely random search, it is beneficial to supply the GA MUD with a good initial guess of the estimated transmitted L-symbol vectors to be detected. A low-cost design option may include the simple MMSE-based OFDM MUD's L-user output vector in the GA's initial population, which imposes a fairly low complexity compared to the interference-cancellation based schemes of [1]. More specifically, the GA invoked in the SDMA-OFDM system commences its search for the optimum L-symbol solution at the initial *generation* with the aid of the MMSE combiner. In other words, using GA parlance, the so-called *individuals* of the $y = 0^{th}$ generation having a *population* size of X are created from the estimated length-L transmitted symbol vector provided by the MMSE combiner, where the i^{th} individual is expressed as $\tilde{\mathbf{s}}_i^{(y)} = [\tilde{s}_{i,1}^{(y)}, \tilde{s}_{i,2}^{(y)}, \ldots, \tilde{s}_{i,L}^{(y)}]$, and we have $\tilde{s}_{i,l}^{(y)} \in \mathcal{M}_c, l \in \{1, \ldots, L\}$. Hence, a complex symbol representation based on the *binary encoding* of the individuals is employed. Then the GA-based optimization selects some of the L-symbol candidates from a total of X legitimate individuals in order to create a so-called *mating pool* of T number of L-symbol *parent* vectors [12]. Two L-symbol parent vectors are then combined using specific GA operations for the sake of creating two L-symbol *offspring* [12] and this 'genetic evolution-like' process of generating new L-symbol offspring continues over Y number of consecutive generations, so that the optimum L-symbol solution may be found.

The selection of the L-symbol individuals for creating the mating pool containing T number of L-symbol parents is vital in determining the GA's achievable performance. In our research the individual-selection strategy based on the concept of the so-called *Pareto Optimality* [11] was employed. This strategy favours the so-called *non-dominated* individuals and ignores the so-called *dominated* individuals [12]. More specifically, the u^{th} L-symbol individual is considered to be dominated by the v^{th} individual, if we have:

$$\begin{aligned} &\forall i \in \{1, \ldots, L\}:\ \Omega_i\left(\tilde{\mathbf{s}}_v^{(y)}\right) \leq \Omega_i\left(\tilde{\mathbf{s}}_u^{(y)}\right) \\ \wedge\ &\exists j \in \{1, \ldots, L\}:\ \Omega_j\left(\tilde{\mathbf{s}}_v^{(y)}\right) < \Omega_j\left(\tilde{\mathbf{s}}_u^{(y)}\right). \end{aligned} \tag{15}$$

If an individual is not dominated in the sense of Equation 15 by any other individuals in the population, then it is considered to be non-dominated. All the non-dominated individuals are then selected and placed in the mating pool, which will have a size of $2 < T \leq X$ [12]. Two of the T number of L-symbol individuals in the mating pool are then selected as parents based on their corresponding diversity-based *fitness* values calculated with the aid of Equation 14 according to the so-called *fitness-proportionate* selection scheme [12]. The so-called *windowing-mapping* [14] technique is invoked in order to get the fitness value $f_i^{(y)}$ associated with the i^{th} individual, which is given by:

$$f_i^{(y)} \quad = \quad \Omega_T^{(y)} - \Omega\left(\tilde{\mathbf{s}}_i^{(y)}\right) + c, \tag{16}$$

where

$$\Omega_T^{(y)} = \max_{t \in \{1, \ldots, T\}} \{\Omega\left(\tilde{\mathbf{s}}_t^{(y)}\right)\} \tag{17}$$

is the maximum objective score[1] achieved by evaluating the T number of individuals in the mating pool at the y^{th} generation, and c is a small positive constant, which is used for the sake of ensuring the positiveness of $f_i^{(y)}$. Then the fitness-proportionate probability of selection p_i of the i^{th} individual can be formulated as:

$$p_i = \frac{f_i^{(y)}}{\sum_{j=1}^{T} f_j^{(y)}}. \tag{18}$$

The selection of L-symbol parents from the mating pool is repeated, which is followed by the *uniform cross-over*, *mutation* and *elitism* operations [12], until a new population of offspring is created. Furthermore, the so-called *incest prevention* [12] technique was invoked during the selection process, which only allows different individuals to be selected for the cross-over operation. Finally, the GA terminates after $(Y - 1)$ number of generations and thus the L-symbol individual having the highest diversity-based fitness value will be considered as the detected L-user transmitted symbol vector corresponding to the specific OFDM subcarrier considered.

---

[1] Note that the individual having the maximum objective score out of the pool of the T candidates is considered as the worst solution in the context of the current mating pool, since the GA searches for the optimum solution which minimizes Equation 14.

3. SIMULATION RESULTS

Population initialization method	Output of the MMSE MUD
Mating Pool Creation Strategy	Pareto-Optimality
Selection method	Fitness-Proportionate
Cross-over operation	Uniform cross-over
Mutation operation	M-ary mutation
Elitism	Enabled
Incest prevention	Enabled
Population size X	Varied
Number of generations Y	Varied
Mutation probability p_m	0.1

Table 1: The various techniques and parameters used in the GA MUD.

In this section, we characterize the achievable performance of the proposed TTCM-assisted concatenated MMSE-GA multi-user detected SDMA-OFDM system. The various techniques and parameters used in the GA MUD are summarized in Table 1. The channel is assumed to be 'OFDM symbol-invariant', implying that the taps of the impulse response are assumed to be constant for the duration of one OFDM symbol, but they are faded at the beginning of each OFDM symbol [1]. The simulation results were obtained using a 4QAM scheme communicating over a channel characterized by the 3-path Short Wireless Asynchronous Transfer Mode (SWATM) CIR given on page 78 of [1], assuming that the channels' transfer functions are perfectly known. Each of the paths experiences independent Rayleigh fading having the same normalized Doppler frequencies of $f'_d = 1.235 \times 10^{-5}$. The specific channel parameters are given in Table 2.

Channel	τ_{max}	f'_d	n	K	cp
SWATM	48.9 ns	1.235×10^{-5}	3	512	64

Table 2: Maximum path delay τ_{max}, normalized Doppler frequency f'_d, number of paths n, FFT length K and cyclic prefix length cp of the SWATM [1] channel.

For the iterative TTCM scheme [5] employed, the code memory ν is fixed to 3, while the number of iterations is set to 4. Hence the total number of trellis states is $2^3 \cdot 4 \cdot 2 = 64$, since there are two 8-state trellis decoders which are invoked in four iterations. The generator polynomial expressed in octal format for the TTCM scheme considered is (13 6), while the TTCM codeword length N and channel interleaver depth are fixed to 1024.

The Bit Error Rate (BER) performance of the TTCM-assisted MMSE-GA-SDMA-OFDM system employing a 4QAM scheme for transmission over the SWATM channel is portrayed in Figure 3, where six users are supported with the aid of six receiver antenna elements. The performance of the TTCM-aided MMSE-detected SDMA-OFDM system, the TTCM-assisted optimum ML-detected system, and the uncoded single-user scheme employing either a single receiver or invoking Maximum Ratio Combining (MRC) when communicating over an AWGN channel are also provided for reference, respectively. It is observed from Figure 3 that the BER performance of the TTCM-assisted MMSE-SDMA-OFDM system was significantly improved with the aid of the GA having a sufficiently large population size X and/or a larger number of generations Y. This improvement was achieved, since a larger population may contain a higher variety of L-symbol individuals. Similarly, a larger number of generations implies that again, a more diverse set of individuals may be evaluated, thus extending the GA's search space, which may be expected to increase the chance of finding a lower-BER solution. On the other hand, it may be observed that when we have the same total number of $(X \times Y)$ correlation metric evaluations according to Equation 11, the performance improvement achieved by increasing the population size X was more substantial than that upon increasing the number of generations Y. For example, when we have $(X \times Y) = 100$, the GA-assisted scheme employing a population size of $X = 20$ and $Y = 5$ number of generations achieved about 1.5dB E_b/N_0 gain over its corresponding counterpart that has $X = 10$ and $Y = 10$, as evidenced by Figure 3. However, when the affordable complexity increases, the improvement achieved by a larger-population GA at a certain value of $(X \times Y)$ becomes modest, since most of the achievable performance gain of the system is likely to have been attained. This may suggest that in the TTCM-assisted MMSE-SDMA-OFDM system investigated, the GA's convergence speed tends to be faster, when we have a larger population size X instead of a higher number of generations Y, since we tend to invoke a low-complexity GA and rely on the TTCM scheme to remove the residual errors.

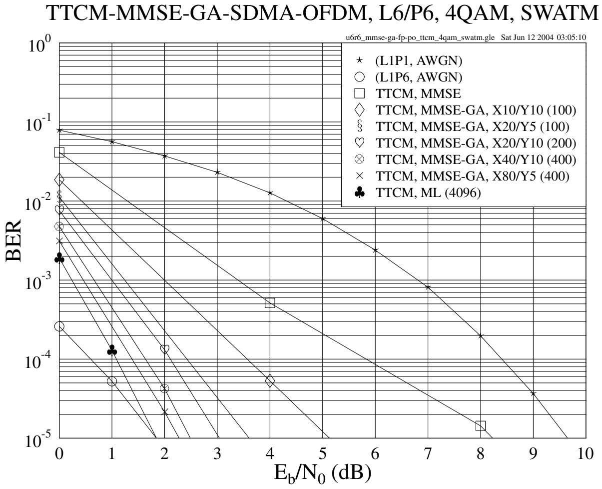

Figure 3: **BER** versus E_b/N_0 performance of the **TTCM-assisted MMSE-GA-SDMA-OFDM** system employing a 4QAM scheme for transmission over the **SWATM** channel, where **six** users ($L = 6$) are supported with the aid of **six** receiver antenna elements ($P = 6$). The GA-related system configuration is given in Table 1.

Furthermore, it can be seen in Figure 3 that the MMSE-GA-aided TTCM-SDMA-OFDM system was slightly outperformed by its optimum ML-detected counterpart, since the GAs are unable to *guarantee* that the optimum ML solution would be found [12]. However, the MMSE-aided TTCM-SDMA-OFDM system was capable of achieving a performance close to that of the ML-aided system with the beneficial assistance of the GA at a significantly lower computational complexity than that imposed by the ML-aided system, as it will be demonstrated in Section 4.

4. COMPLEXITY ANALYSIS

For the sake of simplicity, we only compare the GA MUD's complexity to that of the optimum ML MUD, since the simple MMSE MUD is used for providing a single initial solution and imposes a significantly lower complexity than that of its concatenated GA-aided counterpart. More specifically, since the proposed GA-aided MUD optimizes the metric of Equation 11 [2], we will quantify the complexity imposed

---

[2] Similarly, the ML-aided MUD optimizes the metric of Equation 9, from which Equation 11 is derived.

in terms of the number of GA metric computations required by the optimization process.

As mentioned in Section 2.3.2, for the ML MUD, 2^{mL} number of metric computations have to be carried out for finding the optimum solution [1], namely the most likely transmitted L-user vector, where m denotes the number of bits per symbol. By contrast, our proposed GA MUD requires a maximum of $(X \times Y)$ metric evaluations, since X number of L-symbol vectors are evaluated during each of the Y number of generations, as shown in round brackets in the legends of Figure 3. Furthermore, the number of such metric evaluations may readily be reduced by avoiding repeated evaluations of identical individuals, either within the same generation or across the entire iterative process, provided that the receiver has the necessary memory for storing the corresponding evaluation history.

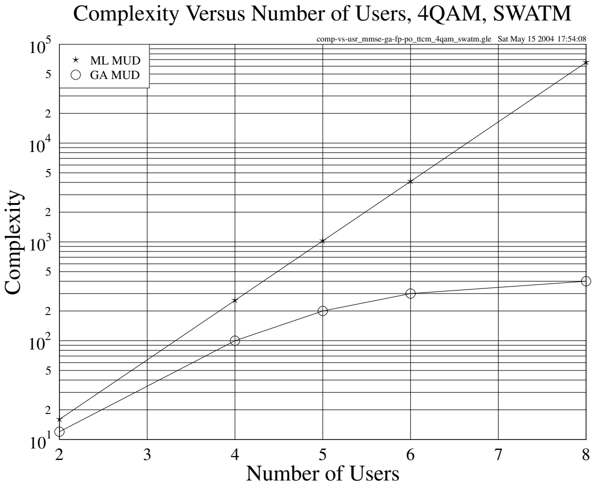

Figure 4: Comparison of the MUD complexity in terms of the number of metric evaluations, versus the number of users performance of the 4QAM TTCM-MMSE-GA-SDMA-OFDM and TTCM-ML-SDMA-OFDM systems. The number of receiver antenna elements employed is equivalent to the number of users supported, i.e. $L = P$.

In Figure 4, we compare both the ML- and the GA-aided schemes in terms of their complexity, i.e. the number of metric computations. At a specific user load, we always select an appropriate GA-aided scheme for comparison, which suffers from less than 1dB E_b/N_0 loss at the BER of 10^{-5} compared to the ML-aided system. As shown in Figure 4, the ML-aided system imposes an exponentially increasing complexity on the order of $O(2^{mL})$, when the number of users increases, while the complexity of the GA-aided system required for maintaining a near-optimum performance increases only slowly.

5. CONCLUSIONS

From the investigations conducted, we conclude that the GA-assisted TTCM-aided MMSE-SDMA-OFDM system is capable of achieving a similar performance to that of the optimum ML-assisted TTCM-SDMA-OFDM system. Furthermore, this is attained at a significantly lower computational complexity than that imposed by the ML-assisted system, especially when the number of users is high. For example, a complexity reduction in excess of a factor of 100 can be achieved by the proposed system for $L = P = 8$, as evidenced by Figure 4.

6. REFERENCES

[1] L. Hanzo, M. Münster, B. Choi, and T.Keller, *OFDM and MC-CDMA for Broadband Multi-user Communications, WLANs and Broadcasting*. IEEE Press - John Wiley & Sons Ltd., 2003.

[2] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.

[3] C. Sweatman, J. Thompson, B. Mulgrew, and P. M. Grant, “A Comparison of Detection Algorithms including BLAST for Wireless Communication using Multiple Antennas,” in *Proceedings of International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, (Hilton London Metropole Hotel, London, UK), pp. 698–703, IEEE, September 18-21 2000.

[4] P. Vandenameele, L. Van der Perre, M. Engels, B. Gyselinckx, and H. Man, “A novel class of uplink OFDM/SDMA algorithms: A statistical performance analysis,” in *Proceedings of Vehicular Technology Conference*, vol. 1, (Amsterdam, Netherlands), pp. 324–328, IEEE, 19-22 September 1999.

[5] L. Hanzo, T. Liew, and B. Yeap, *Turbo Coding, Turbo Equalisation and Space-Time Coding for Transmission Over Fading Channels*. New York, USA: IEEE Press - John Wiley & Sons Ltd., 2002.

[6] G. Ungerböck, “Channel coding with multilevel/phase signals,” *IEEE Transactions on Information Theory*, vol. IT-28, pp. 55–67, January 1982.

[7] P. Robertson and T. Wörz, “Bandwidth efficient turbo trellis-coded modulation using punctured component codes,” *IEEE Journal on Selected Area on Communications*, vol. 16, pp. 206–218, February 1998.

[8] E. Zehavi, “8-PSK trellis codes for a Rayleigh fading channel,” *IEEE Transactions on Communications*, vol. 40, pp. 873–883, May 1992.

[9] X. Li and J. Ritcey, “Bit-interleaved coded modulation with iterative decoding using soft feedback,” *IEE Electronics Letters*, vol. 34, pp. 942–943, May 1998.

[10] M. Jiang, S. Ng, and L. Hanzo, “TCM, TTCM, BICM and BICM-ID Assisted MMSE Multi-User Detected SDMA-OFDM Using Walsh-Hadamard Spreading,” (Milan, Italy), May 17-19 2004. IEEE Vehicular Technology Conference ’04 Spring.

[11] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. Reading, Massachusetts: Addison-Wesley, 1989.

[12] L. Hanzo, L.-L. Yang, E.-L. Kuan, and K. Yen, *Single- and Multi-Carrier DS-CDMA: Multi-User Detection, Space-Time Spreading, Synchronisation and Standards*. IEEE Press - John Wiley & Sons Ltd., 2003.

[13] S. Abedi and R. Tafazolli, “Genetically modified multiuser detection for code division multiple access systems,” *Journal on Selected Areas in Communications*, vol. 20, pp. 463–473, Feb. 2002.

[14] X. Wang, W.-S. Lu, and A. Antoniou, “A genetic-algorithm-based multiuser detector for multiple-access communications,” in *Proceedings of the 1998 IEEE International Symposium on Circuits and Systems*, vol. 4, pp. 534–537, 31 May-3 June 1998.