Analytical BER Performance of DS-CDMA Ad Hoc Networks Using Large Area Synchronized Spreading Codes

Xiang Liu, Hua Wei and Lajos Hanzo

School of Electronics and Computer Science, University of Southampton, SO17 1BJ, UK, lh@ecs.soton.ac.uk

Abstract—The family of operational CDMA systems is interference-limited owing to the Inter Symbol Interference (ISI) and the Multiple Access Interference (MAI) encountered. They are interference-limited, because the orthogonality of the spreading codes is typically destroyed by the frequency-selective fading channel and hence complex multiuser detectors have to be used for mitigating these impairments. By contrast, the family of Large Area Synchronous (LAS) codes exhibits an Interference Free Window (IFW), which renders them attractive for employment in cost-efficient quasi-synchronous ad hoc networks dispensing with power control. In this contribution we investigate the performance of LAS DS-CDMA assisted ad hoc networks in the context of a simple infinite mesh of rectilinear node topology and benchmark it against classic DS-CDMA using both random spreading sequences as well as Walsh-Hadamard and Orthogonal Gold codes. It is demonstrated that LAS DS-CDMA exhibits a significantly better performance than the family of classic DS-CDMA systems operating in a quasi-synchronous scenario associated with a high node density, a low number of resolvable paths and a sufficiently high number of RAKE receiver branches.

I. INTRODUCTION

Code Division Multiple Access (CDMA) systems have substantial benefits and hence have found their way into the third-generation wireless systems. A set of ideal spreading sequences is expected to have zero off-peak auto-correlations and cross-correlations, which would eliminate both ISI and MAI imposed, regardless whether they operate in synchronous or asynchronous scenarios. Unfortunately, such ideal spreading sequences do not exist. But nonetheless, it is possible to design sequences, which exhibit an IFW over a limited duration [1]. A further design-trade-off is that maintaining low auto-correlations and cross-correlations outside the IFW contradict to each other [1] and these correlation side-lobes in fact are typically higher than those of classic spreading sequences. Despite these trade-offs, spreading codes having an IFW are attractive, since the ISI and MAI associated with the family of classic spreading sequences, such as orthogonal Walsh-Hadamard (WH) codes, Pseudo Noise (PN) sequences, Gold codes and Kasami codes [2], significantly impair the attainable capacity of traditional CDMA systems.

More explicitly, the achievable system capacity might be dramatically improved even with the aid of a limited-duration IFW, where the off-peak aperiodic auto-correlations $c_{a,a}$ and cross-correlations $c_{a,b}$ become zero [3]. These attractive correlation properties assist us in eliminating the effects of both ISI and MAI. The family of LAS [4] codes exhibits the above-mentioned favourable properties, which renders them a promising design alternative to traditional spreading codes. Initial comparative studies between LAS-CDMA and traditional CDMA were presented for example in [4].

LAS codes are generated by combining Large Area (LA) [5] codes and Loosely Synchronous (LS) [6] codes. Li [5] proposed various construction schemes for LA codes and analyzed the performance of LA-CDMA, which exhibited higher spectral efficiency than traditional CDMA. Stanczak et al. [6] focussed their attention on contriving systematic methods for the construction of LS codes. Choi and Hanzo [7] investigated the design of efficient and flexible LAS codes having an increased duty ratio, which resulted in a higher number of codes and hence a higher number of supported users than previous designs. An entire LAS-CDMA network was studied in [8].

Again, as a benefit of having an IFW, LAS codes have the ability to support asynchronous operation in ad hoc networks. In this paper we will focus our attention on a comparative study of LAS codes and traditional spreading sequences in DS-CDMA ad hoc networks. For the detailed construction methods of LA, LS and LAS codes please refer to [5], [6].

The outline of this paper is as follows. An infinite mesh of rectilinear node topology is introduced in Section II for the sake of evaluating the achievable BER performance. In Section II the system model of LAS DS-CDMA is also described. The BER performance of classic spreading sequences employed in such a system is investigated in Section III. Our comparative study between LAS codes and traditional spreading sequences is presented in Section IV. Finally, Section V provides our conclusions.

II. SYSTEM MODEL

For the sake of maintaining mathematical tractability and for gaining valuable insights into more realistic practical mobile
wireless networks, we consider an infinite mesh of rectilinear topology having infinite nodes. Its regular topology exactly enables us to examine the system performance [9]-[11]. Its infinite structure results in a scenario having the highest possible interference and conveniently eliminates any effect the size of the node-grid would impose on the validity of the results. We use \( (u, v) \) denoting the location of nodes on the two-dimensional plane, where \( u \) and \( v \) are integers and \( d \) is the minimum distance between any two adjacent nodes.

Every node in the network has four neighbours, connected by the edges. We assume that the maximum affordable transmit power only supports communications between neighbours. If a packet has to be sent from a source node to a destination node that is not an immediate neighbour, it has to be relayed by the intermediate nodes. Each node is assigned a spreading sequence for transmitting such that the shortest distance between any two nodes using the same spreading sequence is maximized. Each node is assumed to have the equal probability to transmit or receive signals to or from one of its neighbours. Under uniform traffic generation conditions, the network is homogeneous and hence it is possible for us to study a single receiving reference node, rather than considering all nodes.

We will focus our attention on the system’s single-hop performance under the above assumptions, rather than considering a particular routing strategy. Without loss of generality, the receiving reference node is assumed to be located at \((0, 0)\) in the two-dimensional plane and receives from the transmitting node located at \((0, d)\). Then all the other nodes are regarded as interferers with the probability 0.5.

Consider \( K \) simultaneously transmitting nodes in the context of wireless ad hoc networks, where each transmitting node transmits at the same constant power level \( P_t \), since cost-efficient ad hoc networks routinely dispense with power control. The multipath channel has the channel impulse response (CIR) of [12]:

\[
h_k(t) = \sum_{l=0}^{L_p-1} h_{kl} \delta(t - lT_c)e^{-j\theta_{kl}},
\]

where \( h_{kl} \) is the complex-valued low-pass equivalent CIR of the multipath channel experienced by the transmitting node \( k \). Furthermore, \( L_p \) is the total number of resolvable paths, which is assumed to be identical for all nodes, \( h_{kl} \) is the fading envelope of the \( l \)th path of the transmitting node \( k \), \( \delta(t) \) is the Dirac delta function, \( T_c \) is the chip duration and \( \theta_{kl} \) is the phase-shift of the \( l \)th path of the transmitting node \( k \). In general, the phase angles \( \{\theta_{kl}\} \) are deemed to be independently and uniformly distributed in \([0, 2\pi]\), while the fading amplitudes \( \{h_{kl}\} \) are independent Nakagami-\( m \) distributed random variables having a Probability Distribution Function (PDF) of \( p(h_{kl}) \) expressed as [12]:

\[
p(h_{kl}) = \frac{2m_{kl}h_{kl}^{2m_{kl}-1}}{\Gamma(m_{kl})\Omega_{kl}^{m_{kl}}} e^{-m_{kl}h_{kl}^2},
\]

In Equation 2 \( m_{kl} \) is the Nakagami-\( m \) fading parameter, which characterizes the severity of the fading on the \( l \)th path of the transmitting node \( k \). \( \Omega_{kl} = \frac{\Omega_{kl}}{\Omega_{kl}} \) is the second moment of \( h_{kl} \), which corresponds to the average path loss on the \( l \)th path of the transmitting node \( k \) [12], and \( \Gamma(\cdot) \) is the gamma function. We assume a negative exponentially decaying Multipath Intensity Profile (MIP) given by [13], [14]:

\[
\Omega_{kl} = \Omega_{kl}e^{-\eta l}, \quad k = 1, \ldots K \text{ and } l = 1, \ldots, L_p - 1,
\]

\[
\Omega_{kl} = \frac{\Omega_{kl}}{\Omega_{kl}} = e^{-\frac{\eta l}{\alpha}},
\]

where \( \eta > 0 \) is the rate of average power decay, which is assumed to be identical for all nodes, \( \tau_k \) is the distance between the reference receiving node and the transmitting node \( k \), and \( \alpha > 2 \) is the path loss exponent [12], which is also assumed to be identical for all nodes.

The signal \( x_k(t) \) transmitted by node \( k \) is given by:

\[
x_k(t) = \sqrt{2P_t}a_k(t)h_k(t)\Psi(t)e^{j\phi_k},
\]

where \( \phi_k \) is the carrier phase of the transmitting node \( k \), which is assumed to be uniformly distributed in \([0, 2\pi]\). The spreading code \( a_k(t) \) is defined as \( a_k(t) = \sum_{j=-\infty}^{\infty} a_{kj}\varphi_{jk}(t-jT_c) \), where \( a_{kj} \in \{-1, 1\} \) is a binary spreading sequence having a period \( L \) and \( \varphi_{jk}(t) = 1 \) if \( t \in [0, T_c) \) or 0 otherwise. The data signal \( b_k(t) \) is defined as \( b_k(t) = \sum_{j=-\infty}^{\infty} b_{kj}\varphi_{jk}(t-jT_c) \), where \( b_{kj} \in \{-1, 1\} \) is the binary data sequence and \( T_c \) is the symbol duration, which satisfies \( T_c = LT_c \). The chip waveform \( \Psi(t) \) is defined as \( \Psi(t) = \sum_{j=-\infty}^{\infty} \varphi_{jk}(t-jT_c) \), where \( \Psi_{jk}(t) \) is an arbitrary time-limited function, which satisfies \( \Psi_{jk}(t) = 0 \) for \( t \notin [0, T_c) \) and is normalized to have an energy equal to \( T_c \), i.e., we have \( \int_{0}^{T_c} \Psi_{jk}(t)dt = T_c \). For a rectangular chip waveform we have \( \Psi_{jk}(t) = \varphi_{jk}(t) \).

The signal \( y(t) \) received by the reference node is given by:

\[
y(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_p-1} \sqrt{2P_t}a_k(t - lT_c - \tau_k)h_k(t - lT_c - \tau_k)\Psi(t - lT_c - \tau_k)h_k(t) e^{j\phi_k} + n(t),
\]

where \( \tau_k \) is the propagation delay of the transmitting node \( k \), \( \theta_{kl} = \theta_{kl} - \sum_{i=l}^{k} \theta_{il} \) is the phase-shift of the \( l \)th path of the transmitting node \( k \) at the receiving reference node and \( n(t) \) is the complex-valued low-pass equivalent AWGN having a double-sided spectral density of \( N_0 \). Generally \( \theta_{kl} \) is independently and uniformly distributed in \([0, 2\pi]\) and \( \tau_k \) is assumed to be uniformly distributed in the range of \([\tau_{k0}, \tau_{k0} + \tau_{max}) \), where \( \tau_{max} = \frac{\text{light speed}}{\text{light speed}} \) is the shortest delay or Light-Of-Sight (LOS) delay between the transmitting node \( k \) and the reference node. \( \tau_{max} \) is the maximum propagation delay relative to the LOS delay, which is assumed to be identical for all nodes.

### III. BER Analysis

Let us assume that the reference node is receiving signals from the transmitting node \( q \) using maximal ratio combining (MRC) and the RAKE receiver combines a total of \( L_r \) branches. Then the output \( Z_{qi} \) of its correlation receiver at the \( i \)th branch is given by:

\[
Z_{qi} = D_{qi} + I_q(s_{qi}) + I_q(m_{qi}) + n_{qi},
\]
where \( D_{qi} \) is the required signal, \( I_{(S)qi} \) is the self-interference incured by the other paths of node \( q \), \( I_{(M)qi} \) is the multiple access interference incured by other nodes and \( n_{qi} \) is the noise. Following the procedure in [14]–[17], we can derive \( D_{qi} \) and the associated variances of \( I_{(S)qi} \), \( I_{(M)qi} \) and \( n_{qi} \).

The required signal \( D_{qi} \) can be expressed as:

\[
D_{qi} = \sqrt{2P_i T_s b_q} h_{qi}' \tag{8}
\]

The variance \( \sigma^2_{(S)qi} \) of the multipath interference \( I_{(S)qi} \) is given by:

\[
\sigma^2_{(S)qi} = P_i T_s^2 h_{qi}^2 \sum_{i=0}^{L_p-1} \Omega_{qi} e^{-\eta I} \left[ c_{qq}^2 (\xi - L) + c_{qq}^2 (\xi) \right] \tag{9}
\]

where we have \( \xi = t - i \), and \( c_{hq}(\xi) \) is the discrete aperiodic cross-correlation function of the spreading sequences \( \{a_{hj}\} \) and \( \{a_{qj}\} \), mod \( L \) and the partial integral \( S_j^{(1)}(t) \) is given by:

\[
S_j^{(1)}(t) = \frac{1}{4} \omega_1(\xi) t^4 + \frac{1}{3} [\omega_1(\xi) (j - t) + 2 \omega_2(\xi)] t^3 + \frac{1}{2} [2 \omega_2(\xi) (j - t) + \omega_3(\xi)] t^2 + \omega_3(\xi) (j - t), \tag{10}
\]

where we have \( \xi = (t - i + j) \) mod \( L \) and the partial integral \( S_j^{(2)}(t) \) is given by:

\[
S_j^{(2)}(t) = S_j^{(1)}(t) - S_j^{(1)}(t_1), \tag{11}
\]

The coefficients \( \omega_1(\xi) \), \( \omega_2(\xi) \) and \( \omega_3(\xi) \) in Equation 11 are given by:

\[
\omega_1(\xi) = [c_{hq}(\xi + 1 - L) - c_{hq}(\xi - L)]^2 + [c_{hq}(\xi + 1) - c_{hq}(\xi)]^2, \\
\omega_2(\xi) = c_{hq}(\xi - L) [c_{hq}(\xi + 1) - c_{hq}(\xi - L)] + c_{hq}(\xi) [c_{hq}(\xi + 1) - c_{hq}(\xi)], \\
\omega_3(\xi) = c_{hq}^2 (\xi - L) + c_{hq}^2 (\xi). \tag{12}
\]

The parameters \( \tau_-, \tau_0, \tau_+ \) defining the integral area and their integer parts \( \lambda_-, \lambda_0, \lambda_+ \) are given by:

\[
\tau_- = \frac{\tau_0 - \tau_+ - \tau_{\max}}{T_s}, \quad \lambda_- = \lfloor \tau_- \rfloor, \\
\tau_0 = \frac{\tau_0 - \tau_- T_s}{T_s}, \quad \lambda_0 = \lfloor \tau_0 \rfloor, \\
\tau_+ = \frac{\tau_0 - \tau_- T_s + \tau_{\max}}{T_s}, \quad \lambda_+ = \lfloor \tau_+ \rfloor. \tag{13}
\]

The variance \( \sigma^2_{(N)qi} \) of the Gaussian noise \( n_{qi} \) is given by:

\[
\sigma^2_{(N)qi} = h_{qi}^2 N_0 T_s. \tag{14}
\]

Consequently, the SINR encountered on the \( i \)th path of the transmitting node \( q \) at the receiving reference node is denoted by \( 2\gamma_i \), where \( \gamma_i \) is given by:

\[
\gamma_i = \frac{D_{qi}}{\alpha_1^2 (\sigma^2_{(S)qi} + \sigma^2_{(M)qi} + \sigma^2_{(N)qi})}. \tag{15}
\]

Therefore the Bit Error Probability (BEP), a term which we will use interchangeably with the BER \( P_{b;h}(\gamma) \), is conditioned on the fading envelope vector \( h \) and when using BPSK for transmitting from node \( n \) to the reference receiving node, the BEP is given by [17]:

\[
P_{b;h}(\gamma) = Q \left( \sqrt{\frac{\sum_{i=0}^{L_p-1} 2\gamma_i}{\gamma_1}} \right), \tag{16}
\]

where \( h \) and \( \gamma \) are the vectors constituted by \( \{h_{qi}\} \) and \( \{\gamma_i\} \), respectively, \( L_p \) is the total number of RAKE receiver branches and \( Q(x) \) is the Gaussian Q-function. For the sake of conveniently evaluating the average BER, we will use an alternative definite integral form of \( Q(x) \), which was formulated in Equation 2 of [17].

Finally, the average BER at the reference node receiving from the transmitting node \( q \) is given by [17]:

\[
P_b(\gamma) = \frac{1}{\pi} \int_0^\infty \prod_{i=0}^{L_p-1} \frac{m_i \sin^2 \theta}{(\gamma_i + m_i \sin^2 \theta)} d\theta, \tag{17}
\]

where \( 2\gamma_i \) is the average SINR of the transmitting node \( q \) at the receiving reference node, and the SINR-related term \( \gamma_i \) in Equation 17 is given by:

\[
\gamma_i = \frac{\Upsilon_S(i)}{L^2} + \frac{\Upsilon_M(i)}{L^2} + \frac{1}{\gamma_{\text{SNR}}} e^{-\eta_i}, \tag{18}
\]

where \( \Upsilon_S(i) \) and \( \Upsilon_M(i) \) are the multipath interference (MPI) and the MAI related terms, respectively, and \( \gamma_{\text{SNR}} \) is the received per-bit signal-to-interference ratio (SNR), which does not take into account the interference power. They are formulated as:

\[
\Upsilon_S(i) = \sum_{l=0}^{L_p-1} e^{-\eta I} [c_{qq}^2 (\xi - L) + c_{qq}^2 (\xi)], \tag{19}
\]

\[
\Upsilon_M(i) = \sum_{k=1}^{L_p-1} \sum_{l=0}^{L_p-1} \Omega_{kl} e^{-\eta I} \frac{T_s^2}{\gamma_{\text{SNR}}} \left[ \sum_{j=\lambda_-}^{\lambda_+} S_j^{(1)}(t) \right]_{\max (\tau_0 - j, 0)} - \sum_{j=\lambda_0}^{\lambda_+} S_j^{(2)}(t) \right]_{\max (\tau_0 - j, 0)}, \tag{20}
\]

\[
\gamma_{\text{SNR}} = \frac{P_i L T_s \Omega q_0}{N_0}. \tag{21}
\]
IV. PERFORMANCE OF LAS DS-CDMA

A. Benchmark Systems

In this section we will compare the achievable performance of a number of systems, namely that of quasi-synchronous LAS DS-CDMA and quasi-synchronous DS-CDMA using WH codes and Orthogonal Gold (OG) codes, as well as asynchronous LAS DS-CDMA and asynchronous DS-CDMA using random signature sequences. BPSK modulation is used in all of the above systems.

We investigate the attainable performance of the LAS-CDMA 2000 system’s physical layer [18], which was originally conceived for base-station-aided communications, rather than for ad hoc networks. In the LAS-CDMA 2000 system modified versions of the LA(LA, MA, KA) and LS(N, P, WO) codes are combined for the sake of generating the LAS codes [7], [18], where the length of the LA code becomes \( L_A = 2552 \) chips. The minimum spacing between non-zero spreading code pulses of the LA code becomes \( M_A = 136 \) chip durations, which is equal to the length of the constituent LS code based on Equation 10 in [7]. The number of non-zero pulses is \( K_A = 17 \) based on Figure 1 in [5], the length of the complementary code used in the generation of the LS code is \( N = 4 \) [7], the dimension of the WH matrix used for generating the LS code is \( P = 32 \) [7] and the number of zeros at the beginning and in the center of the complementary code pair is \( W_0 = 4 \) [7]. Hence such a LAS code exhibits an IFW length of \([-\epsilon, \epsilon]\), where we have \( \epsilon = \min W_0, N - 1 \) = 3. The corresponding spreading gain becomes \( G = \frac{K_A}{P, A} = 151 \).

For the sake of a fair comparison we would need both WH and OG codes having a length of \( L = 151 \) chips. However, no such codes exist, and the most similar ones are those having a length of \( L = 128 \) and \( L = 256 \) chips. For the sake of fair comparability, when considering asynchronous DS-CDMA using random sequences, we assume that the spreading gain of the random signature sequences is the same as that of the LAS-CDMA 2000 codes, namely \( G = 151 \), which is also equal to the length \( L \) of the random signature sequences.

B. Numerical Results

The system parameters used in our investigations are listed in Table I, unless otherwise stated. The effects of the received per-bit SNR \( \gamma_{SNR} \) and the maximum propagation delay \( \tau_{max} \) are characterized in Figures 1 and 2, respectively.

<table>
<thead>
<tr>
<th>Chip duration (µs)</th>
<th>( T_c = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum separation (light speed ( \times T_c ))</td>
<td>( d = 0.1 )</td>
</tr>
<tr>
<td>Maximum propagation delay (( T_p ))</td>
<td>( \tau_{max} = 2 )</td>
</tr>
<tr>
<td>Total number of resolvable paths</td>
<td>( L_p = 4 )</td>
</tr>
<tr>
<td>Total number of RAKE branches</td>
<td>( L_r = 3 )</td>
</tr>
<tr>
<td>IFW width</td>
<td>( \mu = 3 )</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>( \alpha = 4.0 )</td>
</tr>
<tr>
<td>Rate of average power decay</td>
<td>( \eta = 0.2 )</td>
</tr>
<tr>
<td>Nakagami-m fading parameter</td>
<td>( m = 1.0 )</td>
</tr>
</tbody>
</table>

**TABLE I**

**THE SYSTEM PARAMETERS USED.**

Figure 1 shows that the BERs of all the systems considered decrease, as the SNR increases. This can be readily seen from Equation 18, where \( P_b(\gamma) \) is a monotonically decreasing function of \( \gamma_{SNR} \). However, it may also be concluded from Equation 18 that increasing the transmitted power is only capable of mitigating the effects of the background noise, but not of the MPI or MAI. This is the reason that increasing the transmitted power substantially improves the achievable BER performance for the quasi-synchronous LAS DS-CDMA system. The quasi-synchronous DS-CDMA systems using both 256-chip and 128-chip WH codes exhibit a poor performance owing to their high auto-correlation function side-lobes. The quasi-synchronous DS-CDMA systems using 256-chip and 128-chip Orthogonal Gold (OG) codes exhibit a moderate performance owing to their good correlation properties. Since the performance of quasi-synchronous DS-CDMA systems is mainly determined by the correlation values near \( \xi = 0 \) between the spreading sequences used by the desired node and interferers, and there are several cross-correlation values at \( \xi = 0 \) in OG codes, which are in proportion to their spreading gain. This is the reason that the quasi-synchronous DS-CDMA systems using 256-chip and 128-chip OG codes exhibit almost the same performance. The asynchronous LAS DS-CDMA system performs slightly better than the asynchronous DS-CDMA system using random sequences, as a consequence of its lower MPI resulting from its IFW, given the parameters of \( L_p = 4 \) and \( \mu = 3 \). The quasi-synchronous LAS DS-CDMA system performs best, since its IFW cancels most of the MPI, given the parameters of \( L_p = 4 \) and \( \mu = 3 \). Hence the quasi-synchronous LAS DS-CDMA system becomes essentially noise-limited, rather than interference-limited, which is also the reason that it benefits most from increasing the transmitted power.

In order to demonstrate the capability of LAS codes for mitigating MPI and MAI, we let \( \gamma_{SNR} \rightarrow \infty \) in Figure 2.
therefore requires a low number of resolvable components and a sufficiently high number of RAKE receiver branches. All these parameters are related to the chip rate. As the chip rate increases, the node density expressed in terms of the normalized distance travelled by the radio waves within a chip duration increases, while the total number of resolvable paths decreases, which therefore requires a low number of RAKE receiver branches. Hence, LAS-CDMA benefits from having a low chip rate. We may adjust the chip rate to correspond to the channel’s coherence bandwidth, which avoids encountering frequency non-selective fading that would result in having no multipath diversity. If a high bit rate is required, multicarrier LAS-CDMA might be invoked for the sake of providing low-chip-rate parallel transmissions mapped to several parallel subcarriers, which has however the following disadvantage. The number of resolvable multipath components is reduced proportionately to the number of subcarriers used, which reduces the achievable diversity gain. Investigation of these design trade-offs constitutes our future research.

**References**


