

Near-Optimum Nonlinear Soft Detection for Multiple-Antenna Assisted OFDM

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Abstract – In this contribution, a nonlinear hybrid detection scheme based on a novel soft-information assisted Genetic Algorithm (GA) is proposed for a Turbo Convolutional (TC) coded Space Division Multiplexing (SDM) aided Orthogonal Frequency Division Multiplexing (OFDM) system. Our numerical results show that the performance of the currently known GA-assisted system can be improved by about 2dB with the aid of the GA's population-based soft solution, approaching the optimum performance of the soft-information assisted Maximum Likelihood (ML) detection, while exhibiting a lower complexity, especially in high-throughput scenarios. Furthermore, the proposed technique is capable of achieving a good performance even in the so-called overloaded systems, where the number of transmit antennas is higher than the number of receiver antennas.

performing its counterpart based on hard-decision outputs, while achieving a similar performance to that attained by soft-decision assisted optimum Maximum Likelihood (ML) detection. Furthermore, the proposed scheme is capable of achieving a good performance even in the so-called *overloaded* scenarios, where the number of transmit antennas is higher than the number of receiver antennas. Finally, the computational complexity of the proposed scheme is significantly lower than that of the optimum ML system, especially in high-throughput scenarios.

The structure of this paper is as follows. The SDM-OFDM system model is introduced in Section 2, followed by a brief review of the conventional-GA assisted detection technique in Section 3. The proposed GA detection with population-based soft outputs is detailed in Section 4. Our numerical results are presented in Section 5, while Section 6 concludes our findings.

1. INTRODUCTION

Recently, Genetic Algorithm (GA) [1] assisted Multi-User Detection (MUD) techniques have been proposed for both multi-user Code Division Multiple Access (CDMA) [2–5] as well as Space Division Multiple Access (SDMA) type uplink Orthogonal Frequency Division Multiplexing (OFDM) [6] systems. However, to the best of our knowledge all GA-based detection schemes found in the open literature at the time of writing are only capable of providing a hard-decision output for the channel decoder, which inevitably limits the system's achievable performance. Against this background, we propose the novel concept of a GA-aided detection scheme benefitting from population-based soft solutions and quantify its performance gain in a Turbo Convolutional (TC) coded Multiple-Input-Multiple-Output (MIMO) OFDM architecture, which is referred to here as Space Division Multiplexing (SDM) assisted OFDM. We will demonstrate that the proposed Bell Labs Layered Space-Time (BLAST) [7] type SDM-OFDM system exploiting the GA's soft outputs is capable of out-

2. SDM-OFDM ARCHITECTURE

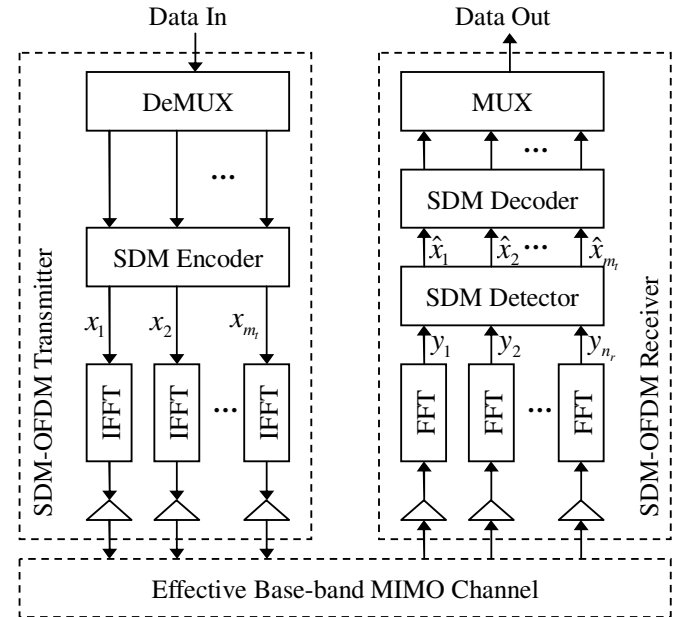


Figure 1: Schematic of a SDM-OFDM BLAST-type transceiver.

Here the terminology of SDM is used for the sake of differ-

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entiating it from the conventional BLAST scheme [7], which usually refers to the point-to-point single-carrier MIMO architecture. The discrete-time model of the SDM-OFDM MIMO system is illustrated in Figure 1, which can be characterized by:

$$y_i[n, k] = \sum_{j=1}^{m_t} H_{ij}[n, k] x_j[n, k] + w_i[n, k], \quad (1)$$

where $n = 0, 1, \dots$, and $k = 0, \dots, K-1$ are the OFDM symbol and subcarrier indices, respectively, while $y_i[n, k]$, $x_j[n, k]$ and $w_i[n, k]$ denote the symbol received at the i^{th} ($i = 1, \dots, n_r$) receive antenna, the symbol transmitted from the j^{th} ($j = 1, \dots, m_t$) transmit antenna and the Gaussian noise sample encountered at the i^{th} receive antenna, respectively. Furthermore, $H_{ij}[n, k]$ represents the complex-valued Channel Transfer Function (CTHF) associated with the propagation link connecting the j^{th} transmit and the i^{th} receive antennas at the k^{th} OFDM subcarrier and time instance n . For the sake of mathematical convenience, Equation 1 can be expressed in the form of vectors and matrices, as:

$$\mathbf{y}[n, k] = \mathbf{H}[n, k] \mathbf{x}[n, k] + \mathbf{w}[n, k], \quad (2)$$

where we introduce the space-division related vectors \mathbf{y} , \mathbf{x} and \mathbf{w} , as well as the $(n_r \times m_t)$ -dimensional CHTF matrix \mathbf{H} , as follows:

$$\mathbf{y} = (y_1, y_2, \dots, y_{n_r})^T, \quad (3)$$

$$\mathbf{x} = (x_1, x_2, \dots, x_{m_t})^T, \quad (4)$$

$$\mathbf{w} = (w_1, w_2, \dots, w_{n_r})^T, \quad (5)$$

$$\mathbf{H} = \begin{pmatrix} H_{11} & H_{12} & \dots & H_{1m_t} \\ H_{21} & H_{22} & \dots & H_{2m_t} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n_r 1} & H_{n_r 2} & \dots & H_{n_r m_t} \end{pmatrix}. \quad (6)$$

Note that in Equations 3 to 6 the notation of $[n, k]$ has been omitted for brevity, but the above equations refer to any of the K subcarriers in the n^{th} OFDM symbol.

3. GA-BASED SDM DETECTION

It is well known that the optimum ML detection [8] uses an exhaustive search for finding the most likely transmitted signals. For the SDM-OFDM system employing m_t transmit antennas, the ML detection requires a total of $M = (2^{m_B})^{m_t}$ metric evaluations, where m_B denotes the number of bits per symbol (BPS), in order to detect the symbol vector $\hat{\mathbf{x}}_{ML}$ that consists of the most likely transmitted (2^{m_B}) -ary symbols of the m_t transmit antennas at a specific subcarrier. Explicitly, the ML complexity increases exponentially with both the number of BPS and the number of transmit antennas m_t . By contrast, the GA-based detection schemes [2–5] confine their search to a small fraction of the ML detector's search space by evaluating

a system-dependent *Objective Function (OF)* at a substantially reduced number of up to $X \cdot Y \ll M$ times, where X is the GA's population size and Y is the number of GA generations. In the context of the SDM-OFDM system, the OF can be formulated according to the ML decision metric as:

$$\Omega(\tilde{\mathbf{x}}) = \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}\|_2^2, \quad (7)$$

where $\tilde{\mathbf{x}}$ is a trial-vector in the set \mathcal{M}^{m_t} , which has a size of M and is defined by:

$$\mathcal{M}^{m_t} = \left\{ \tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{m_t})^T \mid \tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{m_t} \in \mathcal{M}_c \right\}, \quad (8)$$

and \mathcal{M}_c denotes the set containing the 2^{m_B} number of legitimate complex constellation points associated with the specific modulation scheme employed. Based on Equation 7, the solution vector found by the GA is given by:

$$\hat{\mathbf{x}}_{GA} = \arg \left\{ \min_{\tilde{\mathbf{x}} \in \mathcal{M}^{m_t}} [\Omega(\tilde{\mathbf{x}})] \right\}, \quad (9)$$

which is the *individual* having the lowest *objective score* (i.e. the OF's output) in terms of GA terminology. With the aid of the GA's natural evolution-like search mechanism, the GA-based detection technique is capable of achieving near-optimum performance at a significantly lower complexity than that imposed by the full-search based ML detection. For more details on GA-based detection, the interested readers are referred to references [2, 3, 6]. Here we point out that the GA-aided detection schemes found in the open literature [2–6] can only provide a GA-individual based hard-decoded solution, which inevitably limits the GA-aided system's attainable performance. Furthermore, none of these previous schemes considered the so-called overloaded scenarios, where the number of transmit antennas is higher than the number of receiver antennas. In Section 4, the proposed GA with the ability to provide soft outputs will be introduced, which is capable of performing well even in overloaded systems.

4. GAS USING POPULATION-BASED SOFT OUTPUT

In this section we derive an algorithm that enables the GA to output soft information. The soft-bit value or Log-Likelihood Ratio (LLR) associated with the m^{th} bit position at the j^{th} ($j = 1, \dots, m_t$) transmit antenna can be formulated as [9]:

$$L_{j,m} = \ln \frac{P(b_{j,m} = 1 | \mathbf{y}, \mathbf{H})}{P(b_{j,m} = 0 | \mathbf{y}, \mathbf{H})}, \quad (10)$$

which is the natural logarithm of the quotient of probabilities that the bit considered has a value of $b_{j,m} = 1$ or $b_{j,m} = 0$. Here we have omitted again the indices of $[n, k]$. Note that the probability $P(b_{j,m} = b | \mathbf{y}, \mathbf{H})$ that the symbol transmitted by the j^{th} transmit antenna has the m^{th} bit value of $b_{j,m} =$

$b \in \{0, 1\}$, is given by the sum of all the probabilities of the symbol combinations which assume that $b_{j,m} = b$. Hence, Equation 10 can be equivalently rewritten as:

$$L_{j,m} = \ln \frac{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,1}^{m_t}} P(\tilde{\mathbf{x}}|\mathbf{y}, \mathbf{H})}{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,0}^{m_t}} P(\tilde{\mathbf{x}}|\mathbf{y}, \mathbf{H})}, \quad (11)$$

where $\mathcal{M}_{j,m,b}^{m_t}$ denotes the specific subset associated with the j^{th} transmit antenna, which is constituted by those specific trial vectors, whose j^{th} element's m^{th} bit has a value of b , which is expressed as:

$$\mathcal{M}_{j,m,b}^{m_t} = \left\{ \tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{m_t})^T \mid \{\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{m_t} \in \mathcal{M}_c\} \wedge \{b_{j,m} = b\} \right\}. \quad (12)$$

With the aid of Bayes' theorem [9], we have:

$$P(\tilde{\mathbf{x}}|\mathbf{y}, \mathbf{H}) = P(\mathbf{y}|\tilde{\mathbf{x}}, \mathbf{H}) \frac{P(\tilde{\mathbf{x}})}{P(\mathbf{y})}. \quad (13)$$

Upon substituting Equation 13 into Equation 11, we arrive at:

$$L_{j,m} = \ln \frac{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,1}^{m_t}} P(\mathbf{y}|\tilde{\mathbf{x}}, \mathbf{H})}{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,0}^{m_t}} P(\mathbf{y}|\tilde{\mathbf{x}}, \mathbf{H})}. \quad (14)$$

Note that here we have assumed that the different (2^{m_b}) -ary symbol combination vectors $\tilde{\mathbf{x}}$ have the same probability, namely that $P(\tilde{\mathbf{x}})$, $\tilde{\mathbf{x}} \in \mathcal{M}_c$ is a constant. On the other hand, it can be observed from Equation 2 that \mathbf{y} is a random sample of the m_t -dimensional multi-variate complex Gaussian distribution, where the mean vector is $\mathbf{H}\mathbf{x}$, while the $(n_r \times n_r)$ -dimensional covariance matrix \mathbf{R}_n is given by [8]:

$$\mathbf{R}_n = E\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2 \mathbf{I}, \quad (15)$$

where σ_n^2 is the noise variance and \mathbf{I} is the identity matrix, while the noise encountered at the n_r number of receiver antennas is assumed to be uncorrelated. Hence, the above-mentioned multi-variate complex Gaussian distribution can be described by [10]:

$$f(\mathbf{y}|\mathbf{x}, \mathbf{H}) = \frac{1}{\pi^{n_r} |\mathbf{R}_n|} \exp\left\{ -(\mathbf{y} - \mathbf{H}\mathbf{x})^H \mathbf{R}_n^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x}) \right\}. \quad (16)$$

When substituting Equation 15 into Equation 16, we have [8]:

$$f(\mathbf{y}|\mathbf{x}, \mathbf{H}) = \frac{1}{(\pi \sigma_n^2)^{n_r}} \exp\left\{ -\frac{1}{\sigma_n^2} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 \right\}. \quad (17)$$

Note that $f(\mathbf{y}|\mathbf{x}, \mathbf{H}) = P(\mathbf{y}|\mathbf{x}, \mathbf{H})$ is the *a priori* probability that the vector \mathbf{y} has been received under the condition that the vector \mathbf{x} was transmitted over the MIMO channel characterized by the CHTF matrix \mathbf{H} . Thus, Equation 14 can be further

developed with the aid of Equation 17, yielding:

$$L_{j,m} = \ln \frac{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,1}^{m_t}} \frac{1}{(\pi \sigma_n^2)^{n_r}} \exp\left\{ -\frac{1}{\sigma_n^2} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}\|_2^2 \right\}}{\sum_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,0}^{m_t}} \frac{1}{(\pi \sigma_n^2)^{n_r}} \exp\left\{ -\frac{1}{\sigma_n^2} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}\|_2^2 \right\}}. \quad (18)$$

In order to avoid the exponential computation imposed by Equation 18, the maximum-approximation [8] can be applied, yielding:

$$L_{j,m} \approx -\frac{1}{\sigma_n^2} \left[\|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}_{j,m,1}\|_2^2 - \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}_{j,m,0}\|_2^2 \right], \quad (19)$$

where

$$\tilde{\mathbf{x}}_{j,m,b} = \arg \left\{ \min_{\tilde{\mathbf{x}} \in \mathcal{M}_{j,m,b}^{m_t}} \left[\|\mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}\|_2^2 \right] \right\}, \quad b = \{0, 1\}. \quad (20)$$

Furthermore, with the aid of Equation 7, Equation 19 can be represented as:

$$L_{j,m} \approx -\frac{1}{\sigma_n^2} \left[\bar{\Omega}_{j,m,1} - \bar{\Omega}_{j,m,0} \right], \quad (21)$$

where

$$\bar{\Omega}_{j,m,b} = \min [\Omega(\tilde{\mathbf{x}}_{j,m,b}), \omega], \quad b = \{0, 1\}, \quad (22)$$

and $\omega = m_t n_r$ is a normalization factor. Equation 21 suggests that the LLRs can be obtained by evaluating the GA's OF. More explicitly, in order to calculate the LLR of the m^{th} bit of the j^{th} ($j = 1, \dots, m_t$) transmit antenna at the specific subcarrier considered, the X number of individuals in the GA's final generation are divided into two groups, where the first (or second) group is constituted by those individuals that have a value of one (or zero) at the m^{th} bit of the j^{th} transmit antenna. The resultant lowest OF score calculated in each of the two groups is then compared to ω , and the smaller of the two will be used in Equation 21 for calculating the corresponding LLR, which can therefore assist the channel decoder in improving the SDM-OFDM system's performance.

It is worth pointing out that the proposed population-based GA only imposes a modest complexity increase in comparison to the conventional hard-decision aided individual-based GAs [2–6]. This is because the only additional operation required by the proposed scheme is to compare ω to the objective scores, which are already available, since the results of the OF evaluation carried out by the conventional GAs can be readily used.

5. SIMULATION RESULTS

The OFDM modem used in our simulations employed 128 subcarriers. The half-rate TC [11] code employed two Recursive Systematic Convolutional (RSC) component codes having a constraint-length of $K = 3$ and the standard 124-bit Wideband

CDMA (WDM) Universal Mobile Telecommunications System's (UMTS) turbo code interleaver of [12]. The octally represented RSC generator polynomial of (7 5) was used. The 8-path dispersive fading channel model of [13] was employed and the channel estimation was assumed to be perfect. The Minimum Mean Square Error (MMSE) algorithm was used for creating the GA's initial population and the Biased Q -function based Mutation (BQM) scheme of [14] was employed. For the reader's convenience, the simulation parameters are summarized in Table 1. For more details on the GA's configuration, the interested reader is referred to [2, 6, 14].

TC code parameters	Modem	QPSK
	Code rate	1/2
	Component codes	RSC
	Constraint length	3
	Octal generator polynomial	(7 5)
	Turbo interleaver length	124 bits
GA parameters	Population initialization	MMSE
	Mating pool strategy	Pareto-Optimality [15]
	Selection method	Fitness-Proportionate
	Cross-over	Uniform cross-over
	Mutation	BQM [14]
	Elitism	Enabled
	Incest prevention	Enabled
	Population size	500
	Generations	5
	Mutation probability	0.1
Channel parameters	CIRs	BUG [13]
	Paths	8
	Maximum path delay	40 μ s
	Symbol duration	160 μ s
	Subcarriers	128
	Channel interleaver length	248 bits

Table 1: Basic simulation parameters used in Section 5.

Figure 2 provides the Bit Error Rate (BER) performance of the TC-aided GA-SDM-OFDM system employing both the conventional individual-based hard-decoded GA solution [2, 4–6, 14] and the proposed GA using population-based soft information, respectively. For the sake of benchmarking, the BER performances of the systems using hard-decoded and/or soft-information aided MMSE and/or ML detection are also shown in Figure 2, where we had $m_t = n_r = 8$ for all schemes, implying an effective throughput of $(2 \cdot 8 \cdot \frac{1}{2}) = 8$ BPS. It can be seen in Figure 2 that the GA-aided system was capable of achieving a similar BER performance to that attained by the optimum ML-aided arrangement, while outperforming the linear MMSE detected system. Furthermore, an E_b/N_0 gain of

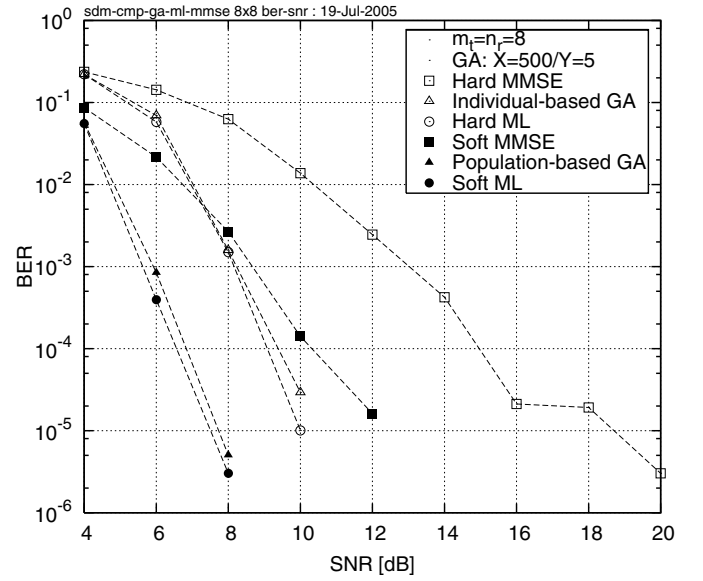


Figure 2: BER performance comparison of the 8-receiver TC-SDM-OFDM system using the individual-based hard-decoded and population-based soft-output GA detection, while employing a QPSK scheme having an effective throughput of 8 BPS.

about 2dB was achieved by the GA using population-based soft outputs over its counterpart using the individual-based hard-decoded outputs. In Figure 3 we provide the BER performance recorded in the overloaded scenario, where $m_t = 8$ transmit antennas and $n_r = 6$ receiver antennas were employed. In overloaded scenarios, the weight matrix calculated by the MMSE algorithm becomes a singular matrix, which will lead to a theoretically unresolvable detection problem and thus resulting in a dramatic performance degradation, as shown in Figure 3. By contrast, the system aided by the GA providing soft-outputs was capable of attaining an undistinguishable performance from that of the optimum soft-ML detected arrangement in the overloaded scenario of Figure 3. Again, a 2dB E_b/N_0 gain was observed, when comparing the proposed GA to the GAs found in the literature [2, 4–6]. Furthermore, the associated complexity of the proposed GA-aided detection is significantly lower than that of the ML detection. The complexity is quantified here in terms of the maximal number of OF evaluations required by the ML or GA detection, where the OF is given by Equation 7. In the overloaded scenario associated with Figure 3, for example, the corresponding complexity of the ML detection is $M = 2^{2 \cdot 8} = 65536$ OF evaluations, while that of the GA-aided detection is only $(X \cdot Y) = 500 \cdot 5 = 2500$ OF evaluations.

6. CONCLUSIONS

In conclusion, the performance of the TC-SDM-OFDM system using the individual-based hard-decoded GA's output can be

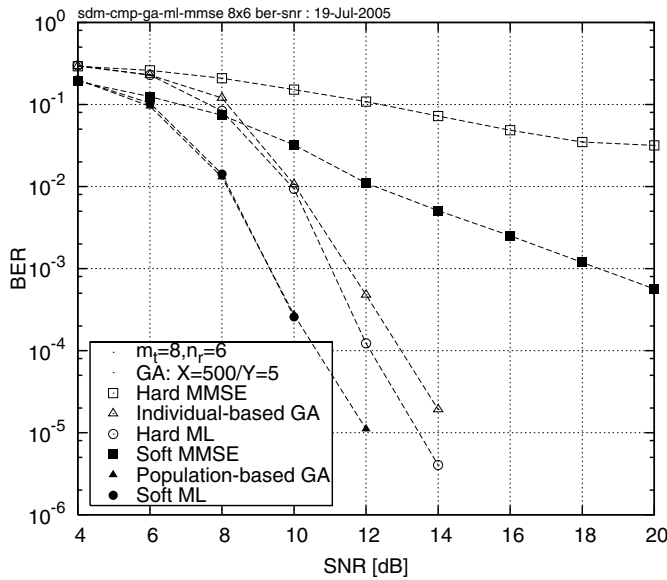


Figure 3: BER performance comparison of the 6-receiver TC-SDM-OFDM system using the individual-based hard-decoded and population-based soft-output GA detection, while employing a QPSK scheme having an effective throughput of 8 BPS.

improved by about 2dB, when the proposed population-based soft-output GA is employed. Furthermore, the GA-detected TC-SDM-OFDM system is capable of achieving an undistinguishable performance to that of the optimum soft-ML detected arrangement, while reducing the ML complexity by a factor of $65536/2500 \approx 26$ for $m_t = 8$. When a higher number of transmit antennas is used, the achievable complexity reduction becomes even more significant.

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