Exact BER Performance of Asynchronous DS-CDMA Systems Using Quadriphase Spreading and QPSK Modulation over Rayleigh Channels

Xiang Liu and Lajos Hanzo

School of Electronics and Computer Science, University of Southampton, SO17 1BJ, UK, lh@ecs.soton.ac.uk

Abstract— **An accurate closed-form expression is derived for calculating the average BER in an asynchronous DS-CDMA system using random complex-valued spreading sequences for transmission over Rayleigh channels. This accurate solution is based on the Characteristic Function (CF) approach and only requires a single numerical integration. Our numerical simulation results verify its accuracy, and also demonstrate the relative inaccuracy of the Gaussian approximation.**

I. Introduction

CDMA systems have substantial benefits compared to FDMA and TDMA systems, and hence have found their way into the third generation wireless systems. The performance study of DS-CDMA systems using QPSK modulation has attracted substantial research interests [1]–[12].

In traditional QPSK modulated DS-CDMA systems the in-phase and quadrature-phase components are spreaded separately [1]–[12]. Complex-valued spreading sequences have been applied in W-CDMA systems [13], [14]. As a benefit of their relative simplicity in comparison to other polyphase sequences [15], a number of complex-valued spreading sequences, synonymously also referred to as quadriphase spreading sequences, have been designed in [16]–[21]. Some of them were claimed to outperform Gold and m-sequences in asynchronous DS-CDMA systems [20], [21]. Recently various receiver architectures using complex-valued spreading sequences and QPSK modulation were proposed in [22].

The average Bit Error Ratio (BER) performance of the family of DS-CDMA systems using complex-valued spreading and QPSK modulation has been investigated using a variety of techniques, including accurate analyses [21], [22], approximations [21], [23], [24], bounding techniques [24] or simulations [20], [23]. The achievable BER performance over AWGN [20], [22]–[24] and Rayleigh [21] channels was also documented. The BER performance attainable using both deterministic complex-valued spreading sequences [20]–[22], [24] and random complex-valued spreading sequences [23] was also studied.

In contrast to the performance analysis of [21], [22], we provide an accurate BER analysis of asynchronous DS-CDMA systems using random complex-valued spreading sequences and QPSK modulation for transmission over Rayleigh channels, rather than employing deterministic complex-valued spreading sequences [21], [22] for transmission over AWGN channels [22]. Furthermore, we do not assume the independence of the real and imaginary parts, which was implicitly assumed in [22]. In contrast to the performance analysis of [24], [25], we use quadriphase data and matching quadriphase spreading, rather than binary data.

This paper is organized as follows. In Section II a general asynchronous DS-CDMA system using complex-valued spreading sequences and QPSK modulation for transmission over Rayleigh channels is described. Then in Section III its exact BER performance using random complex-valued spreading sequences is investigated using the characteristic function based approach of [26] as well as the approximate Gaussian approach. Our numerical results are discussed in Section IV and finally, our conclusions are provided in Section V.

II. System Model

We consider a general asynchronous QPSK modulated DS-CDMA system using quadriphase spreading for transmission over a Rayleigh fading channel, and assume that there are K simultaneously transmitting users.

A. Notations

We distinguish symbols with a tilde, as in $\widetilde{x}$, for denoting complex-valued variables, while symbols without a tilde denote real-valued variables. Furthermore, $\widetilde{x}^*$ and $|\widetilde{x}|$ denote the complex conjugate and the modulo of the complex number $\widetilde{x}$, respectively, while $\Re\{\widetilde{x}\}$ and $\Im\{\widetilde{x}\}$ denote the real and imaginary parts of the complex number $\widetilde{x}$, respectively. Hence we have $\widetilde{x} = \Re\{\widetilde{x}\} + j\Im\{\widetilde{x}\}$, where $j = \sqrt{-1}$ denotes the imaginary unit.

The rectangular pulse $p_T(t)$ of duration T is defined as:

$$p_T(t) = \begin{cases} 1, & t \in [0, T), \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

The complex data signal $\widetilde{b}_k(t)$ of the kth user is expressed as:

$$\widetilde{b}_k(t) = \sum_{m=-\infty}^{\infty} \widetilde{b}_{k,m} p_{T_s}(t - mT_s), \tag{2}$$

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where $\widetilde{b}_{k,m} \in \{\pm 1 \pm j\}$ is the quaternary data sequence and T_s is the symbol duration. The quadriphase spreading signal $\widetilde{a}_k(t)$ of the kth user is expressed as:

$$\widetilde{a}_k(t) = \sum_{m=-\infty}^{\infty} \widetilde{a}_{k,m} p_{T_c}(t - mT_c), \quad (3)$$

where $\widetilde{a}_{k,m} \in \{\pm 1 \pm j\}$ is the complex-valued spreading sequence having L chips and T_c is the chip duration satisfying $T_s = LT_c$. The aperiodic cross-correlation function between the pair of complex-valued spreading sequences $\{\widetilde{a}_{km} | m = 0, 1, ..., L-1\}$ and $\{\widetilde{a}_{im} | m = 0, 1, ..., L-1\}$ both having a length of L chips is defined as:

$$\widetilde{c}_{ki}(\xi) = \begin{cases} \sum_{m=0}^{L-1-\xi} \widetilde{a}_{k,m} \widetilde{a}^*_{i,m+\xi}, & 0 \leq \xi \leq L-1, \\ \sum_{m=0}^{L-1+\xi} \widetilde{a}_{k,m-\xi} \widetilde{a}^*_{i,m}, & -(L-1) \leq \xi < 0, \\ 0, & |\xi| \geq L. \end{cases} \quad (4)$$

The chip waveform $\Psi(t)$ is expressed as:

$$\Psi(t) = \sum_{m=-\infty}^{\infty} \psi_{T_c}(t - mT_c), \quad (5)$$

where ψ_{T_c} is an arbitrary time-limited function, which satisfies $\psi_{T_c}(t) = 0$ if $t \notin [0, T_c)$ and $\int_0^{T_c} \psi_{T_c}(t) = T_c$. The normalized partial auto-correlation functions, $R_\Psi(\nu)$ and $\hat{R}_\Psi(\nu)$, of the chip waveform $\Psi_{T_c}(t)$ are defined as:

$$R_\Psi(\nu) = \frac{1}{T_c} \int_0^{\nu} \Psi_{T_c}(t) \Psi_{T_c}(t + T_c - \nu) \mathrm{d}t, \quad (6)$$

$$\hat{R}_\Psi(\nu) = \frac{1}{T_c} \int_{\nu}^{T_c} \Psi_{T_c}(t) \Psi_{T_c}(t - \nu) \mathrm{d}t. \quad (7)$$

The complementary function $\mathrm{erfc}(x)$ and the Gaussian Q-function $Q(x)$ are defined as in [27].

B. Receiver Model

The received signal $r(t)$ at the input of the matched filter receiver is given by:

$$r(t) = \Re\left\{ \sum_{k=0}^{K-1} \frac{1}{\sqrt{2}} \Psi(t - \tau_k) \widetilde{a}_k(t - \tau_k) \widetilde{b}_k(t - \tau_k) \times h_k e^{j[\omega_c(t-\tau_k)+\theta_k]} \right\} + \eta(t), \quad (8)$$

where $\eta(t)$ is the zero-mean stationary white Gaussian noise with two-sided power density $\frac{N_0}{2}$, i.e. $\sigma^2_\eta = \frac{N_0}{2}$. The amplitudes $\{h_k\}$ are independent Rayleigh distributed random variables having a Probability Distribution Function (PDF) of $f_{h_k}(x)$ expressed as:

$$f_{h_k}(x) = \begin{cases} \frac{x}{\sigma_k^2} e^{-\frac{x^2}{2\sigma_k^2}}, & x \geq 0, \\ 0, & x < 0. \end{cases} \quad (9)$$

The phase shifts $\{\theta_k\}$ and the time delays $\{\tau_k\}$ are independently and uniformly distributed in $[0, T_s)$ and $[0, 2\pi)$, respectively, while ω_c is the common carrier frequency.

Without loss of generality, we assume that the 0th user's signal is the desired one and that we have $\tau_0 = 0$ as well as $\theta_0 = 0$. The decision statistic of $\widetilde{Z}$ at the output of the correlation receiver matched to the 0th user's signal is given by:

$$\begin{aligned} \widetilde{Z} &= \frac{2}{T_c} \int_0^{T_s} r(t) \frac{1}{\sqrt{2}} \widetilde{a}^*_0(t) \Psi(t) e^{-j\omega_c t} \mathrm{d}t \\ &= \widetilde{D} + \sum_{k=1}^{K-1} \widetilde{I}_k + \widetilde{\eta}, \end{aligned} \quad (10)$$

where $\widetilde{D}$ is the desired signal component, $\widetilde{I}_k$ is the co-channel interference component incurred by the kth user and $\widetilde{\eta}$ is the noise component.

The desired signal $\widetilde{D}$ can be expressed as:

$$\widetilde{D} = h_0 L \widetilde{b}_{0,0}. \quad (11)$$

The noise component $\widetilde{\eta}$ can be shown to be a zero-mean complex-valued Gaussian distributed variable having a variance of $\sigma^2_{\widetilde{\eta}} = \frac{2N_0 L}{T_c}$. Hence its real and imaginary components, $\Re\{\widetilde{\eta}\}$ and $\Im\{\widetilde{\eta}\}$, are independent, zero-mean real-valued Gaussian variables having a variance of $\sigma^2_{\Re\{\widetilde{\eta}\}} = \sigma^2_{\Im\{\widetilde{\eta}\}} = \frac{N_0 L}{T_c}$.

The co-channel interference $\widetilde{I}_k$ incurred by the kth user can be expressed as:

$$\widetilde{I}_k = \frac{1}{2} h_k e^{j\Delta_k} \left\{ \widetilde{b}_{k,0} \left[\widetilde{c}_{k,0}(\xi_k) \hat{R}_\Psi(\nu_k) + \widetilde{c}_{k,0}(\xi_k + 1) R_\Psi(\nu_k) \right] + \widetilde{b}_{k,-1} \left[\widetilde{c}_{k,0}(\xi_k - L) \hat{R}_\Psi(\nu_k) + \widetilde{c}_{k,0}(\xi_k + 1 - L) R_\Psi(\nu_k) \right] \right\}, \quad (12)$$

where $\xi_k = \lfloor \frac{\tau_k - \tau_0}{T_c} \mod L \rfloor$, $\nu_k = (\frac{\tau_k - \tau_0}{T_c} \mod L) - \xi_k$, and $\Delta_k = -\omega_c(\tau_k - \tau_0) + (\theta_k - \theta_0)$ is the phase shift difference between the kth user and the 0th user.

III. BER Analysis

In this section, we analyze the BER performance of an asynchronous DS-CDMA system using a rectangular chip waveform and random complex spreading sequences conditioned on the 0th user's complex spreading sequence, $\{\widetilde{a}_{0,m}\}$. Hence we have $\psi_{T_c}(t) = p_{T_c}(t)$, $R_\Psi(\nu) = \nu$ and $\hat{R}_\Psi(\nu) = 1 - \nu$ for the rectangular chip waveform, $\{\Delta_k\}$ and $\{\nu_k\}$ are independently and uniformly distributed in $[0, 2\pi)$ and $[0, T_c)$, respectively, for the asynchronous system, $\{\widetilde{a}_{k,m}\}$ are mutual independent and uniformly distributed, i.e. we have $P\{\widetilde{a}_{k,m} = \pm 1 \pm j\} = \frac{1}{4}$ for random quadriphase spreading sequences. Furthermore, the QPSK data symbols $\{\widetilde{b}_{k,m}\}$ are also assumed to be mutually independent and uniformly distributed as $\{\widetilde{a}_{k,m}\}$.

In contrast to the performance analysis of [22], we investigate the average BER rather than the average Symbol Error Ratio (SER). From Equation 12 we can observe that the real and imaginary interference components might be statistically dependent. This renders the SER calculation using the BERs of the real and imaginary components [22] somewhat inaccurate, since it implicitly assumes that the error probabilities of the real and imaginary components are independent. The exact SER calculation may lead to a double integration. However, the average BER can always be expressed as the average of the real and imaginary components' BERs without the assumption of independence, which enables us to evaluate the average BER performance with the aid of the marginal distributions [28] of the real and imaginary statistics.

A. Accurate Analysis

We will follow a procedure similar to that of [29] for simplifying the expression characterizing the co-channel interference. It has been shown in [29] that the interference imposed by the $(K-1)$ interfering users would be mutually independent, if and only if it was conditioned on the spreading sequence of the 0th user.

We will analyze the average BER P_e^r of the 0th user's real component. The average BER P_e^i of its imaginary component may be derived in the same way. For the sake of simplifying the expression of $\widetilde{I}_k$ in Equation 12, we define a set of $(L+1)$ complex-valued random variables $\widetilde{Y}_{k,m}, m=0,...,L$ by:

$$\widetilde{Y}_{k,m} = \begin{cases} \frac{1}{2}\widetilde{b}_{k,-1}\widetilde{a}_{k,m-\xi_k}\widetilde{a}_{0,m}^*, & m=0,...,\xi-1, \\ \frac{1}{2}\widetilde{b}_{k,0}\widetilde{a}_{k,m-\xi_k}\widetilde{a}_{0,m}^*, & m=\xi,...,L-2, \\ \frac{1}{2}\widetilde{b}_{k,-1}\widetilde{a}_{k,L-1-\xi_k}\widetilde{a}_{0,0}^*, & m=L-1, \\ \frac{1}{2}\widetilde{b}_{k,0}\widetilde{a}_{k,L-1-\xi_k}\widetilde{a}_{0,L-1}^*, & m=L. \end{cases} \tag{13}$$

It can be shown [29] that these random variables $\{\widetilde{Y}_{k,m}\}$ are mutually independent and uniformly distributed as $\{\widetilde{a}_{k,m}\}$ if conditioned on $\{\widetilde{a}_{0,m}\}$. Hence the interference component $\widetilde{I}_k$ in 12 can be rewritten as:

$$\widetilde{I}_k = h_k e^{j\Delta_k}\widetilde{X}_k, \tag{14}$$

where the random variable $\widetilde{X}_k$ is defined as:

$$\begin{aligned} \widetilde{X}_k &= \sum_{m=0}^{L-2}\widetilde{Y}_{k,m}\left[(1-\nu_k)+\frac{1}{2}\widetilde{a}_{0,m}\widetilde{a}_{0,m+1}^*\nu_k\right] \\ &\quad + \widetilde{Y}_{k,L-1}\nu_k + \widetilde{Y}_{k,L}(1-\nu_k), \end{aligned} \tag{15}$$

and where the $(L-1)$ possible chip combinations of $\frac{1}{2}\widetilde{a}_{0,m}\widetilde{a}_{0,m+1}^*, m=0,...,L-2$ can be categorized into four sets according to the relative change of the chip value. Let A, B, C and D denote the number of the relative phase changes of 0°, 180°, 270° and 90° between the adjacent two chips' values within the 0th user's spreading sequence, respectively. Then we have $A+B+C+D=L-1$ and $(A-B)+j(C-D)=\frac{1}{2}\sum_{m=0}^{L-2}\widetilde{a}_{0,m}\widetilde{a}_{0,m+1}^* = \frac{1}{2}\widetilde{c}_{0,0}(1)$.

Since h_k is a Rayleigh distributed random variable defined in Equation 9 and Δ_k is uniformly distributed in $[0,2\pi)$, the complex-valued random variable, $h_k e^{j\Delta_k}$, is complex Gaussian distributed with zero-mean and variance of $2\sigma_k^2$ [26], [28]. Hence $\Re\left\{\widetilde{I}_k\right\}$ is Gaussian distributed conditioned on $\widetilde{X}_k$ as follows:

$$f_{\Re\{\widetilde{I}_k\}|\widetilde{X}_k}(x) = \frac{1}{\sqrt{2\pi}\sigma_k\left|\widetilde{X}_k\right|}\exp\left(-\frac{x^2}{2\sigma_k^2\left|\widetilde{X}_k\right|^2}\right). \tag{16}$$

Following a similar derivation to that in [26], the characteristic function of $\Re\left\{\widetilde{I}_k\right\}$ conditioned on A, B and C can be expressed as:

$$\begin{aligned} \Phi_{\Re\{\widetilde{I}_k\}|A,B,C}(y) &= 2^{-2(L+1)}\sum_{\widetilde{d}_1\in\mathcal{A}}\sum_{\widetilde{d}_2\in\mathcal{B}}\sum_{\widetilde{d}_3\in\mathcal{C}}\sum_{\widetilde{d}_4\in\mathcal{D}} \\ &\binom{A}{\frac{\Re\{\widetilde{d}_1\}+A}{2}}\binom{A}{\frac{\Im\{\widetilde{d}_1\}+A}{2}}\binom{B}{\frac{\Re\{\widetilde{d}_2\}+B}{2}}\binom{B}{\frac{\Im\{\widetilde{d}_2\}+B}{2}} \\ &\binom{C}{\frac{\Re\{\widetilde{d}_3\}+C}{2}}\binom{C}{\frac{\Im\{\widetilde{d}_3\}+C}{2}}\binom{D}{\frac{\Re\{\widetilde{d}_4\}+D}{2}}\binom{D}{\frac{\Im\{\widetilde{d}_4\}+D}{2}} \\ &\sum_{\widetilde{Y}_{k,L-1},\widetilde{Y}_{k,L}} W(\sigma_k y), \end{aligned} \tag{17}$$

where the sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are defined as:

$$\begin{aligned} \mathcal{A} &= \left\{\widetilde{d}_1\middle|\Re\left\{\widetilde{d}_1\right\},\Im\left\{\widetilde{d}_1\right\}\in\{-A,-(A-2),...,A\}\right\}, \\ \mathcal{B} &= \left\{\widetilde{d}_2\middle|\Re\left\{\widetilde{d}_2\right\},\Im\left\{\widetilde{d}_2\right\}\in\{-B,-(B-2),...,B\}\right\}, \\ \mathcal{C} &= \left\{\widetilde{d}_3\middle|\Re\left\{\widetilde{d}_3\right\},\Im\left\{\widetilde{d}_3\right\}\in\{-C,-(C-2),...,C\}\right\}, \\ \mathcal{D} &= \left\{\widetilde{d}_4\middle|\Re\left\{\widetilde{d}_4\right\},\Im\left\{\widetilde{d}_4\right\}\in\{-D,-(D-2),...,D\}\right\}. \end{aligned} \tag{18}$$

The function $W(y)$ is defined as:

$$W_l(y) = \begin{cases} \exp\left(-\frac{1}{2}y^2\lambda_0\right), & \text{if } \lambda_1=\lambda_2=0, \\ \frac{1}{y^2\lambda_1}\left[1-\exp\left(-y^2\lambda_1\right)\right]\exp\left(-\frac{1}{2}y^2\lambda_0\right), & \\ & \text{if } \lambda_1\neq 0, \lambda_2=0, \\ \frac{\sqrt{\pi}}{y\sqrt{2\lambda_2}}\exp\left[\frac{1}{2}y^2\left(\frac{\lambda_1^2}{\lambda_2}-\lambda_0\right)\right] & \\ \times\left\{\operatorname{erfc}\left(\frac{y\lambda_1}{\sqrt{2\lambda_2}}\right)-\operatorname{erfc}\left[y\sqrt{\frac{\lambda_2}{2}}\left(1+\frac{\lambda_1}{\lambda_2}\right)\right]\right\}, & \\ & \text{if } \lambda_2\neq 0, \end{cases} \tag{19}$$

where the coefficients λ_0, λ_1 and λ_2 are given by:

$$\begin{aligned} \lambda_0 &= \left|\widetilde{d}_1+\widetilde{d}_2+\widetilde{d}_3+\widetilde{d}_4+\widetilde{Y}_{k,L}\right|^2, \\ \lambda_1 &= \Re\left\{\left(\widetilde{d}_1+\widetilde{d}_2+\widetilde{d}_3+\widetilde{d}_4+\widetilde{Y}_{k,L}\right)\right. \\ &\quad\left.\times\left[-2\widetilde{d}_2-(1-j)\widetilde{d}_3-(1+j)\widetilde{d}_4+\widetilde{Y}_{k,L-1}-\widetilde{Y}_{k,L}\right]^*\right\}, \\ \lambda_2 &= \left|-2\widetilde{d}_2-(1-j)\widetilde{d}_3-(1+j)\widetilde{d}_4+\widetilde{Y}_{k,L-1}-\widetilde{Y}_{k,L}\right|^2. \end{aligned} \tag{20}$$

Since the co-channel interference contribution $\left\{\widetilde{I}_k\right\}$ of the users $k=1,...,K-1$ conditioned on A, B and C are independent [26], [29], the characteristic function of the real component of the total interference $\Re\left\{\widetilde{I}\right\} = \sum_{k=1}^{K-1}\Re\left\{\widetilde{I}_k\right\}$

conditioned on A, B and C is given by [26]:

$$\Phi_{\Re\{\widetilde{I}\}|A,B,C}(y) = \prod_{k=1}^{K-1} \Phi_{\Re\{\widetilde{I}_k\}|A,B,C}(y). \quad (21)$$

Hence the BER of the 0th user's real component conditioned on A, B and C can be shown to be [26]:

$$P^r_{e|A,B,C} = \frac{1}{2} - \frac{\sigma_0 L}{\sqrt{2\pi}} \times \int_0^\infty \Phi_{\Re\{\widetilde{I}\}|A,B,C}(y)\Phi_{\Re\{\widetilde{\eta}\}}(y) \exp\left(-\frac{1}{2}y^2\sigma_0^2 L^2\right) \mathrm{d}y, \quad (22)$$

where $\Phi_{\Re\{\widetilde{\eta}\}}(y)$ is the characteristic function of the noise's real component $\Re\{\widetilde{\eta}\}$:

$$\Phi_{\Re\{\widetilde{\eta}\}}(y) = \exp\left(-\frac{1}{2}\sigma^2_{\Re\{\widetilde{\eta}\}}y^2\right). \quad (23)$$

Then the average BER of the 0th user's real component is obtained by averaging $P^r_{e|A,B,C}$ over all spreading sequences [26]:

$$P^r_e = 4^{-(L-1)} \sum_{A=0}^{L-1} \sum_{B=0}^{L-1-A} \sum_{C=0}^{L-1-A-B} \binom{L-1}{A} \times \binom{L-1-A}{B}\binom{L-1-A-B}{C} P^r_{e|A,B,C}. \quad (24)$$

Following the same approach, we may conclude that the average BER of the 0th user's imaginary component, P^i_e, has the same value as the average BER of the real component, P^r_e. Finally, we arrive at the overall BER, P_e, averaged over both the real and imaginary components of the 0th user, yielding:

$$P_e = \frac{1}{2}(P^r_e + P^i_e) = P^r_e. \quad (25)$$

B. Standard Gaussian Approximation

Similar to the derivations found in [26], the average BER approximated by the SGA can be shown to be:

$$P^r_e \approx \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{\sigma^2_{\Re\{\widetilde{\eta}\}}}{\sigma_0^2 L^2} + \frac{4}{3L}\sum_{k=1}^{K-1}\frac{\sigma_k^2}{\sigma_0^2}}}\right). \quad (26)$$

IV. Numerical Results

We will compare the BER results obtained by our accurate analysis provided in Section III-A to that by the SGA of Section III-B, to that of the BPSK system of [26] and to those of our simulations described in this section.

Figure 1 shows that the results obtained by our accurate analysis exactly match those obtained by simulations for two different-length random spreading sequences, when using $L = 7$ and 31. However, the SGA over-estimates the BER, especially in the scenario, where there is a low number of interfering users and when short spreading sequences are used. The BER of the QPSK system is higher than that of BPSK systems due to the cross-talk between the real and imaginary components.

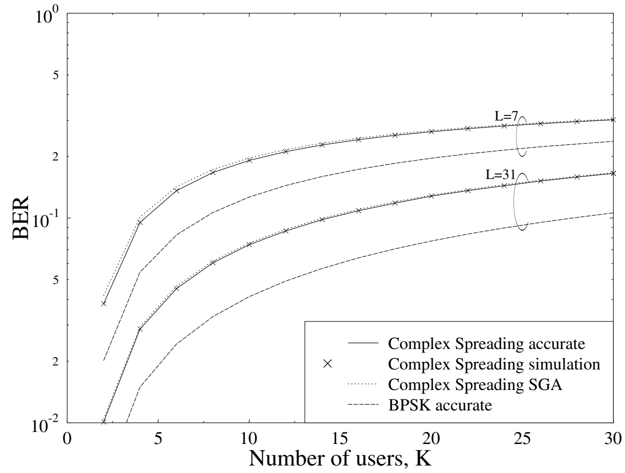

Fig. 1. The BER versus the number of users K in an asynchronous DS-CDMA system using random complex spreading sequences and QPSK modulation. The length of the random complex spreading sequences is $L = 7$ and 31, respectively, the average power of all users at the receiver is equal and the background noise is ignored, i.e. when $\gamma_{\mathrm{SNR}} = \infty$.

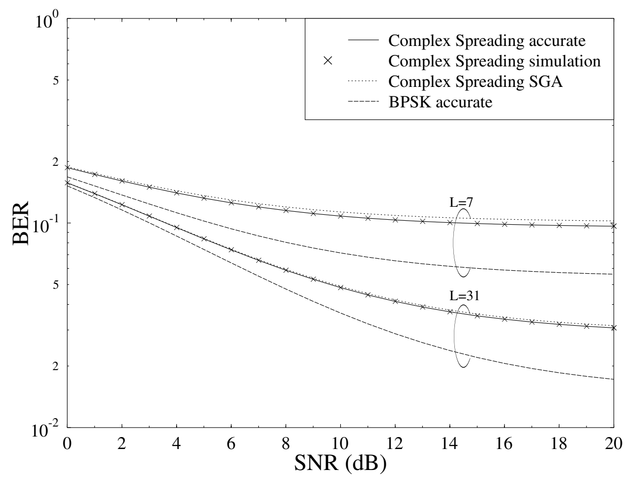

Fig. 2. The BER versus per-bit SNR in an asynchronous DS-CDMA system using random complex spreading sequences and QPSK modulation. The length of the random complex spreading sequences is $L = 7$ and 31, respectively, the average power of all users at the receiver is equal. The number of users is $K = 4$.

Similar to Figure 1, Figure 2 also confirms that the BER results obtained by our accurate analysis match those obtained by simulation for both different-length random spreading sequences, i.e. for $L = 7$ and 31. By contrast, the SGA slightly over-estimates the BER, particularly, when the SNR is high and where short spreading sequences are used. The BER of QPSK systems is higher than that of BPSK systems due to the cross-talk between the real and imaginary components.

V. Conclusion

In this paper we derived an exact closed-form expression for calculating the average BER of an asynchronous DS-CDMA system using QPSK modulation and random quadriphase spreading sequences for transmission over Rayleigh channels. Our analysis was based on the CF technique. Our accurate solution required only a single numerical integration. Furthermore, our simulation results verified the accuracy of our derivation and also demonstrated the limited accuracy of the Gaussian approximation.

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