VLASOV HYBRID SIMULATION- AN EFFICIENT AND STABLE ALGORITHM FOR THE NUMERICAL SIMULATION OF COLLISION-FREE PLASMA

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ABSTRACT

This paper presents a highly efficient and stable algorithm for the numerical simulation of collision free plasma. The algorithm has been successfully used to numerically model non linear electron cyclotron resonance in VLF band radio waves in space, and has produced good simulations of radio emissions such as ‘dawn chorus’ and ‘triggered VLF emissions’.

The algorithm fills the phase box with simulation particles which represent phase space trajectories. Particle trajectories are followed forwards continuously in time without restarts, and these particles are followed until either the simulation ends or the particles exit from the phase box. Liouville’s theorem states that distribution function $F$ is conserved along these trajectories, and so $F$ is known at the phase space points represented by the particle positions. No phase space volume is associated with these particles. At each time step $F$ is truly interpolated, and not distributed from particles to the regular phase space grid, solely for the purpose of calculating plasma charge and current fields. For this operation a cheap low order interpolation is useable. The algorithm is simple, robust and highly efficient. There is NO diffusion in phase space whatsoever, and fine structure is merely undersampled and does not cause any instability problems. Regions of phase space where $F=0$ are cost free as no particles need be provided. The algorithm readily accommodates situations where phase fluid flows into or out of the phase box.

The paper presents the latest simulations of VLF chorus and VLF triggered emissions from the VHS/VLF code, which are in excellent agreement with observations.
1. **Introduction**

The paper presents the VHS algorithm, a highly efficient and stable general purpose algorithm for the numerical simulation of collision free plasma. Numerical modelling for collision free space, fusion or laboratory plasmas is a highly effective investigative tool, particularly as problems in this domain tend to be highly complex and/or nonlinear.

The traditional approach to such plasma simulation problems is the classical particle-in-cell (PIC) method (Hockney and Eastwood, 1988). In the last two decades Vlasov methods have come to the fore, pioneered by Cheng and Knorr (1977) and by Denavit (1972). Vlasov methods seek to directly find numerical solutions to the Vlasov equation, as opposed to PIC codes which integrate forwards in time with an EM field and assembly of point particles. Given enough particles PIC codes should reproduce experimental reality. However, Vlasov codes are intrinsically more efficient and with a far lower noise level. In PIC codes the information as to what distribution function is, will be determined by the density of (weighted) simulation particles, whereas in Vlasov codes this information is carried as a floating point array of numbers defined on a phase space grid. Vlasov codes however do furnish distribution function directly, a diagnostic of considerable physical relevance when it comes to theoretical interpretation of the results. The superiority of Vlasov methods becomes particularly marked for situations, such as in wave particle interaction problems, where perturbation of distribution function dF about the ambient is small.

In recent years many authors have employed a variety of Vlasov codes to tackle a range of problems, invariably with impressive results, superior to that obtainable with PIC codes. The issues to be addressed concern numerical efficiency, accuracy, the level of numerical noise and algorithm robustness, particularly in the presence of distribution function filamentation. The paper presents the Vlasov Hybrid Simulation or VHS algorithm. The algorithm has been very successfully used for the numerical modelling of non linear cyclotron resonance between VLF band EM waves in the earth’s magnetosphere and energetic keV radiation belt
electrons. Such non linear wave particle interactions result in a variety of interesting VLF band radio emissions such as ‘triggered emissions’ and ‘dawn chorus’ (Nunn et al, 2002). The VHS algorithm was first reported in Nunn (1990,1993), but has since undergone many radical improvements.

The paper presents the essential basics of the VHS algorithm. So far its use has been confined to the domain of VLF radio science/space physics, VHS is completely general and is applicable to any collision free plasma simulation problem, except perhaps those of high dimensionality. The VHS algorithm has been found to be highly efficient, stable and low noise, and should be of interest to the Applied Mathematics and Fusion communities.

The algorithm will be demonstrated on an exemplar problems, which are the numerical modelling of VLF band chorus emissions, using data provided by the Cluster satellites, and simulation of VLF triggered emissions as observed at a terrestrial receiver in N Finland. This code will be designated VHS/VLF.

2. Other Vlasov techniques

In 1976 the early pioneers Cheng and Knorr addressed the problem of the numerical integration of the Vlasov equation in configuration space. They used a well known splitting scheme to time advance distribution function F values on a phase space grid. This required three interpolations per time step and per grid point, from grid values of F to arbitrary points in phase space. Cubic interpolation was satisfactory, giving an acceptable level of distribution function diffusion. The Cheng and Knorr method is rather vulnerable to distribution function filamentation, but this may be remedied by using the non physical phase space diffusion resulting from the interpolation operations. Klimas and Farrell (1994) found that Fourier interpolation was superior, and no more expensive than cubic splines, and showed it was then a straightforward matter to filter out distribution function fine structure directly.

Many researchers have successfully used variants of Cheng and Knorr’s basic algorithm (Estabrook et al, 1971, Sakanaka et al, 1971)). Arguably the version of
Klimas and Farrell best addresses the dual problems of algorithm instability due to filamentation and non physical phase space diffusion. (Cheng, 1977). The filtering process is essentially non physical and the whole algorithm is complex and quite expensive. Recent work by Horne et al (2001) and Petkaki et al (2003) use MacCormack’s method, related to that of Cheng and Knorr, to simulate electrostatic beam instability and ion acoustic instability in non Maxwellian plasmas.

Another early approach to the problem was advanced by Denavit (1972,1985). Unlike Cheng and Knorr, Denavit used simulation trial particles in order to advance the distribution function in configuration space. The formalism uses Liouville’s theorem which states that F is constant along phase space trajectories. Particles are started off from grid points and followed forwards for N time steps. At each time step distribution function values F are distributed to nearest grid points, allowing distribution function to be reconstructed on the grid and charge/ current computed. The procedure in practice associates a phase volume with each particle. After ‘N’ time steps a new set of particles is defined starting again from phase space grid points. The process of reconstruction of F invokes unphysical phase space diffusion, and the algorithm is not immune to stability and noise problems. It has been very successfully applied to the electrostatic 1D two stream instability problem.

A rather different and interesting algorithm was proposed by Kotschenreuther et al (1990) and developed by Berk et al (1999). These may be described as ‘df particle algorithms’. The simulation uses a full set of simulation particles, and evaluates dF along each phase trajectory. At each time step dF values are assigned to the nearest grid points, effectively reconstructing the distribution function. As with Denavit’s method this is effectively associating a fixed phase volume with each simulation particle.

More recently Besse and Sonnendrucker,(2003) have proposed the the Semi-Lagrangian method, which exploits Liouville’s theorem, and at every new timestep numerically integrates particle trajectories backwards and interpolates distribution function from the values on the grid at the previous timestep onto the
end point of the backward integration. Third order spline interpolation is used. The interpolation procedure will invoke phase space diffusion, and the method has the disadvantage of being able to produce negative values for distribution function resulting from the higher order interpolation procedure.

3. **Description of the Vlasov Hybrid Simulation algorithm**

**The phase space box**

The code firstly defines the phase space simulation box and grid of dimensionality \( n \), which covers the region of configuration space known to be of physical interest. The VHS/VLF code uses a regular geometric grid in 4D space (1 spatial and three velocity coordinates), although the method could be readily modified to use an adaptive grid. The grid spacings must be sufficient to resolve relevant structures in configuration space. The phase box itself may be a function of time as the simulation proceeds.

**The simulation particle (SP’s) population**

It is assumed throughout that the particle population may be treated as a Vlasov fluid and that particle granularity is ignorable. What follows will assume that only one species of particle requires full VHS simulation. The generalisation to the multi species case is obvious. The phase box is filled with simulation or trial particles (SP’s) with a density of order 1/ phase space cell. These particles are followed forwards in time as the simulation proceeds, using a suitable integrator for the equations of motion, in this case a second order Runge-Kutta scheme (modified Euler). These particle trajectories are properly viewed as *phase space trajectories* although for ease of notation they will be referred to rather erroneously as particles. Now applying Liouville’s theorem is is known that distribution function \( F \) is conserved along these trajectories. Assuming \( F \) is known at the start of the simulation at all points in phase space occupied by particles, then \( F \) must be exactly known at all times at the locations in phase space of the particles. Unlike the methods of Kotschenreuther and Denavit no phase volume is
associated with the SP’s. The only purpose of the SP’s is to carry information on the value of distribution function at a point.

The particle trajectories are followed forwards in time. The same particles are followed for the entire duration of the simulation or until they exit from the phase box. Unlike all other methods particles are not restarted from grid points at every time step or at regular intervals, and there is absolutely no non physical diffusion of the distribution function. Phase space diffusion always occurs when particles are restarted from grid points.

As no assumption may be made concerning the phase volume associated with any SP, it is necessary to perform a true interpolation of distribution function from the SP’s to the regular phase space grid. Unlike the algorithms of Denavit and Kotschenreuther an interpolation is performed and not a deposition as in PIC methods. The construction of distribution function on the regular grid is necessary in order to compute the plasma charge/current field, needed in order to push the EM fields. It may also be required, at relatively long intervals, to provide a diagnostic view of the distribution function at a given time.

**The interpolation**

At each timestep the distribution function must be constructed on the phase space grid, in order to compute the plasma charge current field correctly. In addition the simulationist will want to inspect distribution function at well spaced intervals for diagnostic purposes. Diagnostics apart, the distribution function construction is only performed in order to calculate $J/\rho$. The interpolation process will have no effect on the feedback dynamics of the simulation, as particle trajectories are never restarted, and hence no phase space diffusion will occur. As the constructed $F$ is only integrated over velocity space it is apparent that a simple, low cost, low order interpolation process is more than adequate. Interpolation errors are substantially averaged out in the integration process, and only give small stochastic errors in the charget current field. As to which low order interpolation method to use, there is plenty of latitude. In the VHS/VLF code the following method was used and found to be efficient and
entirely satisfactory. At each timestep and for each SP the value of \( F \) for that particle is *distributed* to the nearest phase space grid points using the well known Area Weighting coefficients, and additionally the weighting coefficients themselves are also distributed to the nearest grid points. The *interpolated* value of \( F \) on grid point \( ijk \ldots \) is then given by

\[
F_{ijk} = \frac{\sum_l F_l \beta_{ijkl}}{\sum_l \beta_{ijkl}}
\]

The sum \( l \) here is over all SP’s present in phase space cells that surround the grid point \( ijk \ldots \), by way of example assuming a 3D system. The coefficients \( \beta_{ijkl} \) are the familiar ‘Area Weighting Coefficients’, generalised to the dimensionality \( n \) of the problem in question. The procedure is readily implemented as simple and vectorisable code. Although only a first order interpolation, it is entirely adequate for the task. This interpolation process cannot generate negative distribution functions, unlike the third order interpolation used in Besse et al (2003). Charge and current are immediately secured by integrating grid values \( F_{ij} \) over velocity space. For production of distribution function plots for presentational purposes higher order interpolation may be used if required.

**Particle control**

Clearly if distribution function is going to be successfully interpolated from particles to the regular grid, then it is necessary that every grid point has at least one SP in the surrounding cells. The reader may be concerned that large numbers of grid points will be left without neighbouring SP’s, however Liouville’s theorem states that density of particles in phase space is conserved along phase trajectories. By the same token the density of simulation particles is also conserved, and hence if the initial density of SP’s is sufficient then it will remain
so. As was shown in Nunn (1993) the density of SP’s \( \rho \) (particles/cell) must exceed a floor or required minimum density of \( \rho_{\text{min}} \), where

\[
\rho \geq \rho_{\text{min}} \approx 4.8 / (2^n)
\]

Not surprisingly the required density per cell is less in systems of higher dimensionality \( n \).

The fact that \( F \) is interpolated to the grid rather than assigned to nearest grid points has profound implications which need emphasis.

- Particles only carry information as to what the value of \( F \) is, and are not associated with any specific volume of phase space, which would immediately give them the properties of quasi-particles.
- Particles may be inserted into or removed from the phase fluid at any time.
- No particular significance attaches to density of simulation particles, as long as it is above the minimum level. Excess density increases CPU loading in return for a modest increase in accuracy in the calculation of the \( J/\rho \) fields.
- No significance attaches to the exact positioning of SP’s in phase space.

At the commencement of the simulation the phase box is filled with SP’s, with a density at or above the floor level. Each SP is assigned the value of \( F \) appropriate to the start of the simulation \( F_0(\mathbf{x},\mathbf{v},t=0) \). Since SP’s are not associated with any phase volume, there is never any need for a *quiet start*. Where very exceptionally a grid point is found not to have any SP’s in the neighbouring cells, a grid value of \( F \) may be derived by interpolating from the next grid points. The unperturbed distribution function may have step discontinuities but delta functions in \( F_0 \) are not permissible. Any delta functions need to be given a finite width and a correspondingly fine grid employed.
Phase fluid flow across the phase box boundary

The VHS algorithm easily handles the situation where phase fluid flows out of or into the phase box across the boundary. Where SP’s exit the current phase box and are providing information no longer required they are immediately discarded from the simulation. Where phase fluid flows into the phase box a careful approach is required. New SP’s must be continuously inserted into the incoming phase fluid in such a way that their density is above the minimum, but not excessive. In the VHS/VLF code the technique for ensuring this is as follows. All the boundary cells are interrogated at every timestep to see if they possess an SP. If they do not one is inserted at a carefully chosen point. Assigning a value of F to these new particles may be a problem, since there is no knowledge of the past history of the particle outside the box. The normal procedure is to assign the unperturbed value of F. If this is inadequate then the phase box needs to be larger. In principle it is possible to remove SP’s from regions where there is SP over density, and to insert new particles in the interior of the box where density is too low. In operational practice with VHS/VLF this was never found to be desirable.

It is seen that the VHS algorithm can have a dynamic particle population, only particles of interest being followed. The reader needs to note that PIC codes will find it impossible to accommodate particle flux into the phase box owing to the difficulty of introducing new particles with exactly the right density. With VHS SP density in the incoming Vlasov fluid need only be above the floor value.

**Distribution function filamentation**

It is well known that other Vlasov algorithms have difficulty remaining stable in the presence of distribution fine structure, and have to actually remove fine structure either through the low pass character of imbedded interpolation procedures or by low pass filtering in the case of Fourier interpolation. How VHS responds to filamentation is a matter of some interest. All that happens is that the particle assembly actually undersamples the fine structure, which will give very small stochastic errors in the plasma charge/current fields. This will produce a
small ‘white’ broadband component in the wavefield which will probably be removed by simulation wavefield low pass filters or by the low pass character of the plasma processes in question. In the unlikely event that non resolution of distribution function fine structure gives unacceptable errors, then it would be necessary to use a higher density of SP’s and arguably also a finer grid. There is nothing in the VHS algorithm to cause it to be destabilised by filamentation, the fine structure merely gets ‘lost’ due to undersampling. There is certainly no need to counteract filamentation by phase space diffusion or by low pass filtering of the actual distribution function.

**Zero distribution function**

What happens when sizeable regions of the phase box contain Vlasov fluid for which F=0? Classic Vlasov techniques like that of Cheng and Knorr need to include these regions in the simulation, at some cost. In the case of VHS however it is permissible to not provide SP’s where phase fluid has zero F. The best approach is to use an SP density somewhat greater than the minimum, for example 20% higher. Then whenever a grid point has no SP’s in the vicinity, the value of F at that grid point may be put equal to zero, as it is almost certainly in a zero F region of phase space. There is however one scenario when errors are made. If there is unresolved fine structure in F involving phase fluid with F=0, the resulting ‘smeared’ distribution function will be in error. A solution to this may readily be devised. If every region of phase space with non zero F is surrounded by a layer with zero F that IS provided with SP’s, then systematic undersampling errors will not occur. This will be the subject of future research.

Many real plasma physical problems, for example the 2 beam instability problem, have significant regions where F=0. Not providing particles in these regions will give significant savings. In the case of the Semi-Lagrangian algorithm Sonnendrucker and co-workers (this volume) have devised an ingenious adaptive grid algorithm to address this problem.
Relationship between VHS and PIC methods

Since the VHS algorithm numerically follows particle trajectories for the
duration of the simulation, the question might be asked, how does this algorithm
differ from PIC codes? The answer lies in the fact that VHS performs a true
interpolation of distribution function to the phase space grid, while PIC performs
a weighted distribution or assignment of particle charge to a spatial grid only. If
VHS particles had some phase volume associated with them (as in the algorithm
of Denavit) then the SP’s would begin to have the properties of real particles. In
that sense VHS is truly a Vlasov method.

4. The exemplar simulation

As an exemplar application the latest results from the VHS/VLF code are
presented, in which triggered VLF emissions and VLF chorus are simulated.
Triggered VLF emissions are radio emissions in the 2-30kHz band originating in
the earth’s magnetosphere. They consists of band limited signal of bandwidth
~50Hz that have a sweeping frequency with sweep rates ~kHz/s. Emissions may
be risers, fallers or upward or downward hooks. They are often observed to be
‘triggered’ by another signal, such as a lightning whistler, pulses from a terrestrial
VLF transmitter, or power line harmonic radiation. The emissions are commonly
observed on the ground, but are also seen on board satellites.

These emissions are generated in the equatorial zone in a region extending ~3000
km either side of the equator and at distances ~1-6 earth radii from the earth’s
surface. They arise from a plasma instability due to the non linear interaction
between keV radiation belt cyclotron resonant electrons and the band limited VLF
wave field. The dominant non linear feature of this wave particle interaction is so
called ‘phase trapping’ (Nunn,1993) in which electrons remain in resonance with
the wave for long periods. A vitally important aspect of the particle dynamics
derives from the inhomogeneity of the medium, mainly due not only to the
parabolic variation of magnetic field strength along the field line, but also to the
sweeping frequency of the emission. Nunn (1993) has shown that trapped
resonant electrons in inhomogeneous media undergo relatively large changes in energy and magnetic moment, resulting in regions of velocity space with enhanced or reduced distribution function. These large scale features in configuration space dominate the dynamics of the non linear wave particle interaction process and result in growth rates many times that of the linear growth rate. Also, where particle trapping predominates, the phase of the resonant particle current is determined by the phase locking angle which is controlled by the inhomogeneity. It is the space and time dependence of the phase of the resonant particle current that results in the observed frequency sweeping of triggered emissions.(Nunn,1990,1993,2002). Two examples of VLF emissions are shown in figs 1 and 2, which are spectrograms of VLF received at an experimental station in N. Finland, obtained during a field campaign of Sodankyla Geophysical Observatory. The first example is of an ‘N’ shaped emission, in which frequency sweep rate twice changes sign. The two subsequent signals have undergone a two hop reflection, together with frequency dispersion. Figure 2 shows a very long rising frequency emission which spans a huge frequency range ~3kHz with an almost constant sweep rate of 1.5kHz/s.

A phenomenon closely related to triggered VLF emissions is VLF chorus. This is a naturally occurring phenomenon consisting of sequences of short VLF rising frequency emissions (Santolik et al,2003). It is generally believed that in the interaction regions of both VLF emissions and chorus the wave vector k is closely parallel to the ambient magnetic field. The free energy for the instability derives from the pitch angle anisotropy of the energetic electron distribution function which derives from the influence of the loss cone.

**The VHS/VLF code**

The code is 4D, having one spatial dimension z (along the magnetic field line) and three velocity dimensions, namely perpendicular velocity |V⊥|, parallel velocity Vz and gyrophase ψ. The ‘mean’ or base frequency is divided out of the formalism, leaving slowly varying field variables with timescales ~ the trapping period (~20ms) and trapping length (~ 200km). The base frequency is sweeping, and so the reduction to baseband process needs to be repeated at frequent
intervals. The wavefield is band pass filtered at every timestep, by FFT and IFFT, in order to keep the bandwidth below the value that is resolvable by the spatial grid. The coordinate $|V_{\perp}|$ or pitch angle is relatively ‘weak’, with only a pitch angle range of 40-65 degrees being significant. Hence the number of grid points in $|V_{\perp}|$ can be quite small ~3-10, and in fact the code can be written to run separate VHS processes for a limited number of pitch angles. The spatial domain of the simulation about the equator need only include the non linear trapping region and is usually divided into 2048 grid points, which is sufficient to resolve the typical event bandwidth ~50Hz and the characteristic length of the trapping process.

In coordinate $V_z$ the simulation box covers a narrow range centred on the local resonance velocity $V_{\text{res}} = (\omega - \Omega)/k$ which is a function of both $z$ and $t$. Hence the phase box itself must be slowly varying with time. The width must not only be enough to encompass all the significant trapping dynamics (~6 trapping widths’) but also include the range of resonant velocities appertaining to the selected bandwidth. To resolve resonant particle trap structures in velocity space ~40 grid points were found to be sufficient. Similarly 16 points in gyrophase provided adequate resolution.

The phase box only covers a narrow band of $V_z$ values centred on $V_{\text{res}}$. An important feature of this problem is that phase fluid is constantly entering and leaving the phase box, since as a result of the inhomogeneity many electrons are swept right through resonance. This is one reason PIC codes are so ineffectual for this problem, the other being the high noise level due to $dF$ being small. New SP’s must be constantly created at the phase box boundary. This is very much a feature of wave particle interaction problems in the case of narrow band wavefields, particularly in the presence of inhomogeneity.

**Simulation aspects.**

The code runs on Convex Exemplar and SG Origin 2000 platforms. All computationally intensive parts vectorise and the code has been parallelised with MPI by dividing the spatial domain into $n$ equal segments, where $n$ is the number
of processors. An event simulation requires about 20000 timesteps with ~5 million particles, and the run time on one Origin2000 processor is ~ 3 days.

5. Simulation of triggered emission observed in N. Finland

The first simulation using VHS/VLF is of a rising frequency triggered emission, similar to the one shown in fig 2. The following data was used in this simulation:
(a) L shell = 5.5
(b) Electron distribution function anisotropy A=2
(c) Linear growth rate at the equator = 100dB/s
(d) Cold electron density = 100/cc
(e) Saturation amplitude 7pT
(f) Triggering pulse; amplitude 0.1pT, length 186 ms.

Figure 3 shows a spectrogram of the wavefield exiting from the simulation box, which corresponds to the field received on the ground. Initially a falling tone is produced but then a strong riser is spontaneously triggered from noise with a sweep rate ~+400Hz/s. Figure 4 gives a history of the entire simulation and plots wave amplitude in pT as a 2D shaded contour plot as a function of time and position z. The generation region of the initial faller extends quite far in negative z (‘upstream’) and that of the strong riser is confined mainly downstream of the equator. Figures 5 and 6 present the plasma currents, as functions of z and t, due to the nonlinear electron cyclotron resonance. Figure 5 presents -Jr, the negative component parallel to the instantaneous E field of the EM whistler wave. This is the component that provides power input and wave amplification, and sustains the generation region as a quasi static feature. Figure 6 presents –Ji, the negative of the component parallel to the EM wave magnetic field, as a function of z and t. It is this component that radiates a wave field 90 degrees out of phase with the ambient one, and hence directly changes wavefield phase and is instrumental in causing the observed frequency sweep. Figure 7 is an example of a diagnostic view of distribution function. It is a 2D contour plot in the gyrophase,Vz plane, of dW (in dimensionless units)), the integrated energy change undergone by the particle at that point in phase space. This is at t=5 secs, at position z=489.7 (dimensionless units) and at a pitch angle of 63 degrees. Here dF is directly
proportional to dW. The large region of high values of dW will be noted, centred on Vz=Vres and with a centre phase of about 180 degrees relative to the wave electric field. This is called the resonant particle ‘trap’, the region in velocity space occupied by electrons whose gyrophase is locked to that of the wave and which stay in resonance with the wavefield for relatively long periods (Nunn, 1990).

6. Simulation of VLF chorus observed on Cluster

Data has been provided by Santolik on observations of VLF chorus on board the Cluster satellites (Santolik et al, 2003). Using Cluster data VHS/VLF was used to simulate two consecutive elements of chorus. The data used in the simulation is as follows:
(a) L shell = 4.4
(b) Electron distribution function anisotropy A=2. The unperturbed distribution has a smoothed out discontinuity in Vz, centred on the resonance velocity at 3600Hz. This gives a linear growth rate which peaks at 3600Hz but falls off quickly either side.
(c) Linear equatorial growth rate = 1200 dB/s
(d) Cold plasma density 2 eV/nc
(e) Saturation amplitude 200pT
(f) Trigger signal is a random noise burst, length 17ms, rms amplitude 8pT

Figure 8 shows the received wave amplitude in pT. Figure 9 is the spectrogram of the received field amplitude. Each chorus element has a positive sweep rate of about 10kHz/s, in exact agreement with observations aboard Cluster (Santolik, pc). The first element ‘dies’ at f~4700Hz due to the decreasing power input resulting from falling linear growth rate. The second element is then regrown from random noise in the code. The time for this to occur is ~0.2 seconds, again in good agreement with experimental observations. Finally fig 10 plots wave amplitude in pT in the simulation box as a function of z and t.
7. Conclusions

A general purpose algorithm, Vlasov Hybrid Simulation (VHS) has been presented that is rather unique and clearly different from other Vlasov algorithms on offer. It is distinguished by simulation particles or phase trajectories that are followed continuously throughout the simulation and not restarted at grid points, which prevents non physical smoothing of distribution function in configuration space. From Liouville’s theorem distribution function is precisely known at the locations of the simulation particles. The key element of VHS is the use of a simple low order interpolation of distribution function from particles to the grid. This is solely to compute plasma charge/current and does not significantly effect the ongoing simulation dynamics. The use of interpolation implies that there is no phase volume associated with any particle, and that fine structure is not a problem, it is merely undersampled. The algorithm is efficient because only low order interpolation is needed, and that with one proviso particles need not be provided in regions of configuration space where \( F=0 \). Another powerful feature is the ability to easily accommodate flux of phase fluid into and out of the phase box, which is particularly relevant to wave particle interaction problems.

The demonstrator application gave a convincing example of the power of the method in solving what is quite a complex problem. VHS should have plentiful applications in fusion plasma and space plasma research, where collisions may be neglected. Vlasov methods are well known for their rigour but for systems of high dimensionality can be impossibly demanding of CPU time. The future role of VHS should be in providing very accurate numerical solutions of plasma physical problems of modest dimensionality, say involving up to 4D configuration space.

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Fig 1. Triggered VLF emissions as observed at a ground station in N. Finland. Note the rapidly sweeping frequency, and the unusual N shaped form. The two subsequent emissions arise from 2 hop reflection along the magnetospheric field line, and clearly show the dispersion expected for whistler waves.
Fig 2. Long triggered rising frequency VLF emission as observed on the ground in N. Finland.
Fig 3. Spectrogram of triggered VLF emissions simulated by the VHS/VLF code
Figure 4. Plot of simulation wave amplitude as a function of position $z$ and time. This gives a history of the entire simulation as far as wavefield amplitude is concerned.
Fig 5. Plot of the negative of plasma resonant particle current $-J_r$, where $J_r$ is the component of current parallel to the VLF wave electric field.
Fig 6. Plot of $-J_i$ in the $z,t$ plane, where $J_i$ is the component of resonant particle plasma current parallel to the VLF wave magnetic field.
Fig 7. Plot of integrated energy change of a test particle, $dW$, in the 2 dimensional plane whose axes are parallel velocity and electron gyrophase, at $z=489.7$ and $t=5$ secs. Pitch angle is 62 degrees. The quantity $dW$ is directly proportional to perturbation in distribution function $dF$. 
Fig 8. VHS/VLF code simulation of two chorus elements as observed on the Cluster satellite. Wave amplitude received in pT at the end of the simulation box.
Fig 9. Frequency time spectrogram of received wave field. Note the two consecutive chorus elements with sweep rates ~ 10kHz/s, exactly as observed by Cluster.
Fig 10. Wave amplitude as a two dimensional contour plot in the z,t plane. This presents a complete history of the simulation box for wave amplitude.