Authors' response to the referees' second reports

Paper title: Algorithms for deterministic balanced subspace identification
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We thank the referees for their relevant and useful comments.

In this document, we quote in **bold face** statements from the reports. Our replies follow in ordinary print.

Reviewer #1 (ID 10223)

. . . once the impulse responses are computed, then we can easily compute the (balanced) realization, and the solution is given before returning to the original data. In fact, defining $\mathcal{O}_{\mathbf{bal}}$ and $\mathcal{C}_{\mathbf{bal}}$ as in p. 12, we can easily extract $A_{\mathbf{bal}}, B_{\mathbf{bal}}, C_{\mathbf{bal}}$ by using the procedure of Zeiger-McEwen (1974). Also, D is given by H^0, obtained in Algorithm 2 of p. 9.

Thus I do not still understand why you stick to the algorithm of (2). I think the above procedure is very simple and natural. To my experience, (2) will be better for noisy case.

For the noise free case the procedure of computing the impulse response and then applying a realization algorithm is indeed simpler and more logical than the basic algorithm. We included in the introduction an outline of this procedure (see Algorithm 1 in the second revised version) as an alternative to the basic algorithm.

In the noisy case, however, our experience is that "going back to the data" leads to improved accuracy. We included in the introduction also a MOESP-type algorithm for balanced identification (see Algorithm 2 in the second revised version). With respect to the extent to which the data is used after the impulse response is computed, Algorithm 2 is in between Algorithm 1 and the basic algorithm, because A_{bal} and C_{bal} are obtained directly from $\mathcal{O}_{\mathrm{bal}}$ but B_{bal} and D_{bal} are obtained using back the data.

In Algorithm 3, the index k is not well defined. In fact, in Step 3.2, Y_0^k is defined, but in the output Y_0 contains only Y_0^0 to Y_0^{k-1} (up to $k-1$). This is misleading.

The index k is initialized with 0 and incremented *after* the computation of the kth piece of the response and *before* the check for the termination of the loop. Therefore at the check for the termination condition, $Y_0^{(k)}$ is not yet defined. On exit $\mathrm{col}(Y_0^{(0)}, \ldots, Y_0^{(k-1)})$ is the the computed sequence and it has length kL (L samples per piece). In Algorithm 3, however, we did correct a few typos.

Reviewer #3 (ID 10225)

. . . it would be nice if the authors would cleary answer these [identifiability] questions for the three algorithms considered

The fundamental lemma gives the following sufficient condition for identifiability:

> $\tilde{u}$ is persistently exciting of order $\mathtt{l}_{\max} + 1 + \mathtt{n}_{\max}$.

We proved that in the SISO case it is also necessary and conjecture (we have no proof yet) that the same holds in the MIMO case. The minimal length T of the available data $\tilde{w} = (\tilde{u}, \tilde{y})$ that ensures identifiability follows easily from the required persistency of excitation: T should be large enough to ensure that the matrix $\mathcal{H}_{\mathtt{l}_{\max}+1+\mathtt{n}_{\max}}(\tilde{u})$ has at least as many columns as rows. So, as long as sufficient conditions are concerned, they are given for the proposed algorithm in Theorem 2. The corresponding conditions for the alternative algorithms are stated in Proposition 3 of the second revised version.

In the present paper the answer is somewhat hidden in the text.

We agree. The reason is that we understood better this issue only after the first draft was submitted. See also our recent paper [WRMM04, Sec. 4].

Once again I would like to comment on the "answer to the long standing question...". In the deterministic case in a certain sense this question is not important, since "any" choice recovers the true system. Different choices "only" effect the minimal requirements for identifiability, the numerical efficiency and precision. The answer given by the authors only refers to the first point. I.e. they choose this splitting only on the basis of minimal requirements for identifiability.

This is correct. We adapted the last sentence of the abstract accordingly and separated the discussion on the past/future splitting question in a new section, see Section 7 of the second revised version.

I think there is a typo in the formula for row $\dim(\tilde{u})$ **on top of page 15:** $\{1, \ldots, i - \mathtt{l}_{\max} - \mathtt{n}_{\max}\}$

It is correct: $L \in \{1, \ldots, i - \mathtt{l}_{\max} - \mathtt{n}_{\max}\}$ is exactly the persistency of excitation condition of Theorem 2.

Even in the deterministic case (due to numerical problems) in the SVD step a choice on the rank of the Hankel matrix H has to be made (at least if the true (minimal) system order is not known.) This of course is of even more relevance in the case of noisy data, as considered in the simulation example. In other words model reduction is a part of the balancing algorithm presented.

The effect of the numerical errors can indeed be viewed as a (small) noise on the data, so that the data is no longer exact and a model reduction step is needed. Note, however, that model reduction is not done on the level the SVD (step 3 of the basic algorithm) but on the level of the solution of the linear system of equations, as explained in Note 2.

A potential reason for confusion is that by SVD we mean *restricted* SVD. (This has been done explicit in the second revised version.) Of course, numerically the restriction of the SVD is made according to a certain tolerance, so that approximation due to (small) numerical errors is indeed build in the exact identification algoorithms. More numerically oriented reader will think of the restricted SVD as a model reduction step

(in practice it is). Theoretically (*i.e.*, in exact arithmetic), however, the restricted SVD is an *exact* operation that cuts the singular values (and corresponding singular vectors) that are *exactly* zero.

We would like to reserve the term model reduction for the (typically) significant approximation via the restriction of the state sequence $\tilde{X}_{\mathrm{bal}}$ to $\tilde{X}_{\mathrm{red}}$ and the approximation on the level of the SVD for the (typically) small numerical errors.

$\mathcal{O}/\mathcal{C}$ are used for the observability/controllability matrix on page 12, whereas on page 17 they refer to the grammians?

Corrected.

References

[WRMM04] J. C. Willems, P. Rapisarda, I. Markovsky, and B. De Moor. A note on persistency of excitation. *Systems & Control Letters*, 2004. Available at: `ftp://ftp.esat.kuleuven.ac.be/pub/SISTA/markovsky/abstracts/04-101.html`