The consideration of surrogate model accuracy in single-objective electromagnetic design optimization

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1 Introduction

Optimization problems in electromagnetic design are typified by features which present difficulties to most deterministic search algorithms, e.g. the existence of multiple local minima. Genetic Algorithms (GAs), on the other hand, with their ability to search more globally, are better suited for exploring complicated objective function landscapes. The high computational cost of evaluating the objective function in such problems, however, means that direct use of a GA is often not feasible or impractical, due to the general requirement for a large number of objective function evaluations. Additional cost-effective techniques must be used, with the aim to make the GA require fewer evaluations of the objective function. Techniques used include hybrid algorithms, GAs specially adapted for small population sizes, and simplifying the problem by removing irrelevant design variables. One technique, called *surrogate modelling*, is the focus of this paper.

A surrogate model is a functional relationship between the design variable space of an optimization problem, and the objective function space, which is constructed based on a set of design vectors which have their objective function values known. Having constructed a surrogate model, a GA can then use it to predict fitness values for unevaluated design vectors, rather than call the true expensive objective function, thus reducing computational costs. However, ideally the reliability of the model should be taken into account as well, when choosing points to evaluate; this is discussed further in Section 2. Different methods exist to construct surrogate models, including polynomial approximation, artificial neural networks (ANNs) and kriging; the use of these three types of surrogate model in electromagnetic design optimization is discussed in Section 3. Developments in this area outside the field of electromagnetic design optimization are discussed in Section 4.

2 Model Accuracy

Care should be taken when using a surrogate model to select design vectors to evaluate for optimization purposes. In particular, the existence of false optima (points which are optima of the surrogate model, but which are not optima of the true objective function space, see **Fig 1**) means that selecting points to evaluate based entirely on their predicted objective function value is not desirable. Instead, ideally some measure of the reliability of the predicted objective function value should also be considered, and so the choice of the next point to evaluate becomes a balance between attempting to locate the best points and aiming at improving the accuracy of the surrogate.

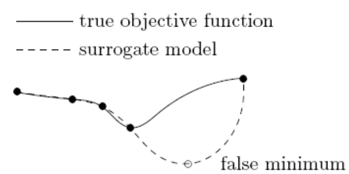

Fig. 1 False minimum in a surrogate model

3 Surrogate-assisted single-objective electromagnetic design optimization

3.1 Polynomial approximations

Polynomial approximation suffers in that inclusion of additional points into the model does not necessarily lead to increased model accuracy. In particular, if only the optimum of the surface is added, the model can converge very quickly to a false optimum [1].

In [2], model accuracy was considered in several ways. The initial set of examples was chosen so as to minimize the condition number of the matrix [M] which was to be inverted in order to determine the polynomial coefficients. A dynamic weighting factor was then used as the optimization process proceeded to place more emphasis on the region around the predicted optimum. Then, in order to ensure that [M] did not become ill-conditioned as the optimization process continued, additional *learning points* were evaluated, chosen specifically so as to minimize the condition number of [M]. The method was successfully used to optimize a brushless permanent magnet motor; an analysis of the errors on predicted optima and learning points indicated that the inclusion of learning points was effective in improving the accuracy of the polynomial surrogate model.

3.2 Artificial Neural Networks

A wide range of different types of ANNs exist which may be used to construct surrogate models. One popular type used is a radial basis function ANN. A method in [3] uses multiquadric radial basis functions to successfully optimize a C-core magnet and a magnetizer. In addition to evaluating the predicted optimum during on-line learning, design vectors in the most *unexplored* regions of design space were also evaluated, with the aim of avoiding local minima; however it is likely this has improved the model accuracy globally as well.

3.3 Kriging

Kriging has recently been recognized as a useful method for surrogate model construction for electromagnetic optimal design [4]. Due to its statistical nature, useful information may be extracted giving an indication of model accuracy and reliability.

The EGO algorithm [5] uses such information to build up an auxiliary function, known as the expected improvement, which automatically balances the objective function values predicted by the kriging model, with the uncertainty in this prediction. By optimizing this auxiliary function, model accuracy increases as the optimum is being searched for. A variation of EGO, known as superEGO, has been used to solve two electromagnetic design problems with expensive objective functions [6], and convergence was found to occur within tens of iterations.

4 Developments Elsewhere

Other algorithms have been developed outside the electromagnetic design community which also consider model reliability when searching for new points. One such approach, based on a radial basis function ANN surrogate model, known as *rbfsolve* [7], predicts the location of a potential new optimum (whose objective function value f^* is lower than the current minimum f_{min}) and evaluates a measure of the credibility of the response surface which would interpolate it and the existing data. A measure of the "bumpiness" of the resulting response surface serves as a measure of its credibility, with smoother surfaces being deemed more acceptable.

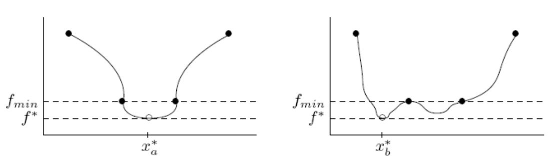

Fig. 2 Two response surfaces which pass through an existing set of examples and a predicted optimum

For example, in **Fig 2**, the proposed optimum (x_a*,f*) is preferred to the proposed optimum (x_b*,f*), as the surface which interpolates it and the existing set of points (shown as black dots) is less "bumpy" than the surface which interpolates (x_b*,f*). The algorithm has performed well on test functions, but has yet to be applied to electromagnetic optimal design problems.

5 Conclusion

Surrogate models have proven to be effective in reducing the cost of electromagnetic optimal design problems. Model reliability has been recognised as an important factor and attempts have been made to ensure model accuracy improves as the optimization search proceeds. However, suitable algorithms exist which are yet to be implemented in electromagnetic design optimization. The full paper will critically assess various surrogate modelling techniques.

6 Literature

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