

Communication Interference in Mobile Boxed Ambients

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joint with: M. Bugliesi, S. Crafa, M. Merro

Vladimiro Sassone

University of Sussex,



Mobile Ambients

Both administrative domains and computational environments (Cardelli-Gordon)

● Subjective movements

$$n[\text{in } m.P \mid Q] \mid m[R] \longrightarrow m[n[P \mid Q] \mid R]$$

$$m[n[\text{out } m.P \mid Q] \mid R] \longrightarrow n[P \mid Q] \mid m[R]$$

● Process interaction

$$n[\langle M \rangle.P \mid (x).Q] \longrightarrow n[P \mid Q\{x := M\}],$$

● Boundary dissolver

$$\text{open } n.P \mid n[Q] \longrightarrow P \mid Q.$$

Interferences in Mobile Ambients

● The inherent nondeterminism of movement may go wild: Grave Interferences.

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- Introducing Safe Ambients (Levi-Sangiorgi)

$$n[\text{in } m.P \mid Q] \mid m[\overline{\text{in}} m.R \mid S] \longrightarrow m[n[P \mid Q] \mid R \mid S]$$

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- Co-capabilities and single-threadedness rule out grave interferences

- Safe Ambients with passwords have a conveniently treatable semantics.
(Merro-Hennessy)

$$n[\text{in } (m, k).P \mid Q] \mid m[\overline{\text{in}} (m, k).R \mid S] \longrightarrow m[n[P \mid Q] \mid R \mid S]$$

Mobile Boxed Ambients

● **open**'s nature of ambient dissolver is a potential source of problems.

● Direct communication as alternative source of expressiveness: **Mobile Boxed Ambients** (Bugliesi et al.). Perform I/O on a subambient n 's local channel (viz. $(x)^n$) as well as from the parent's local channel (viz. $(x)^\uparrow$)

$$(x)^n.P \mid n[\langle M \rangle.Q \mid R] \longrightarrow P\{x := M\} \mid n[Q \mid R]$$

$$\langle M \rangle.P \mid n[(x)^\uparrow.Q \mid R] \longrightarrow P \mid n[Q\{x := M\} \mid R].$$

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● But it is a great source of **non-local nondeterminism** and **communication interference**.

$$m[(x)^n.P \mid n[\langle M \rangle \mid (x).Q \mid k[(x)^\uparrow.R]]]$$

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Introducing NBA: Communication

NBA: a fresh foundation based on: each ambient comes equipped with two mutually non-interfering channels, for *local* and *upward* communications.

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● Good algebraic laws; simple type system;

● Expressiveness??

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- Good algebraic laws; simple type system;
- Expressiveness??
- Hmm, rather poor: $n[P]$ cannot, for instance, communicate with children it doesn't know statically. It can never learn about incoming ambients, and will never be able to talk to them.

Introducing NBA: Mobility

- Essentially, our idea is to introduce co-actions of the form $\overline{\text{enter}}(x)$ which have the effect of binding the variable x .
- Such a purely binding mechanism does not provide a way control of access, but only to *registers* it. As a (realistic) access protocol where newly arrived agents must register themselves to be granted access to local resources.

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- Need a finer mechanism of access control:

$$a[\text{enter}\langle b, k \rangle.P_1 \mid P_2] \mid b[\overline{\text{enter}}(x, k).Q_1 \mid Q_2] \longrightarrow b[a[P_1 \mid P_2] \mid Q_1\{x := a\} \mid Q_2]$$

This represent an access protocol where the credentials of incoming processes (k in the rule above) are controlled, as a preliminary step to the registration protocol.

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NBA: Syntax

Names: $a, b, \dots, n, x, y, \dots \in \mathbf{N}$

Locations:

η	$::=$	a	nested names
		$\hat{}$	enclosing ambient
		\star	local

Processes:

P	$::=$	0	nil process
		$P_1 P_2$	composition
		$(\nu n)P$	restriction
		$!\pi.P$	replication
		$M[P]$	ambient
		$\pi.P$	prefixing

Messages:

M, N	$::=$	a	name
		$\text{enter}\langle M, N \rangle$	may enter
		$\text{exit}\langle M, N \rangle$	may exit
		$M.N$	path

Prefixes:

π	$::=$	M	messages
		$(x_1, \dots, x_k)^\eta$	input
		$\langle M_1, \dots, M_k \rangle^\eta$	output
		$\overline{\text{enter}}(x, M)$	allow enter
		$\overline{\text{exit}}(x, M)$	allow exit

NBA: Reduction Semantics

mobility

$$\begin{aligned} n[\text{enter}\langle m, k \rangle.P_1 \mid P_2] \mid m[\overline{\text{enter}}(x, k).Q_1 \mid Q_2] &\longrightarrow m[n[P_1 \mid P_2] \mid Q_1\{x := n\} \mid Q_2] \\ n[m[\text{exit}\langle n, k \rangle.P_1 \mid P_2] \mid Q] \mid \overline{\text{exit}}(x, k).R &\longrightarrow m[P_1 \mid P_2] \mid n[Q] \mid R\{x := m\} \end{aligned}$$

communication

$$\begin{aligned} (\tilde{x}).P \mid \langle \tilde{M} \rangle.Q &\longrightarrow P\{\tilde{x} := \tilde{M}\} \mid Q \\ (\tilde{x})^n.P \mid n[\langle \tilde{M} \rangle^\wedge.Q \mid R] &\longrightarrow P\{\tilde{x} := \tilde{M}\} \mid n[Q \mid R] \\ \langle \tilde{M} \rangle^n.P \mid n[(\tilde{x})^\wedge.Q \mid R] &\longrightarrow P \mid n[Q\{\tilde{x} := \tilde{M}\} \mid R] \end{aligned}$$

structural congruence

$$P \equiv Q \quad Q \longrightarrow R \quad R \equiv S \text{ implies } P \longrightarrow S$$

NBA: Behavioural Equivalence

Barbs

$P \downarrow_n$ iff $P \equiv (\nu \vec{m})(n[\overline{\text{enter}}(x, k).Q \mid R] \mid S)$, for $\{n, k\} \cap \{\vec{m}\} = \emptyset$.

$P \Downarrow_n$ iff $P \Longrightarrow P'$ and $P' \downarrow_n$.

NBA: Behavioural Equivalence

● Barbs

$P \downarrow_n$ iff $P \equiv (\nu \vec{m})(n[\overline{\text{enter}}(x, k).Q \mid R] \mid S)$, for $\{n, k\} \cap \{\vec{m}\} = \emptyset$.

$P \Downarrow_n$ iff $P \Longrightarrow P'$ and $P' \downarrow_n$.

● A relation \mathcal{R} is **reduction closed** if

$P \mathcal{R} Q$ and $P \rightarrow P'$ implies $Q \Rightarrow Q'$ with $P' \mathcal{R} Q'$;

it is **barb preserving** if $P \mathcal{R} Q$ and $P \downarrow_n$ implies $Q \Downarrow_n$.

● **Reduction barbed congruence**, written \cong , is the largest congruence relation over processes which is reduction closed and barb preserving.

● **Note:** We could equivalently observe $\langle \cdot \rangle^\wedge$.

The rest of the talk

- Two small examples
- A few equational laws
- LTS characterization of reduction barbed bisimulation congruence.
- A type system
- An encoding of BA into NBA: $BA \lesssim NBA + \text{Guarded Choice}$

A one-to-one communication server

Let $w(k)$ be a bidirectional forwarder for any pair of incoming ambients.

$$w(k) \triangleq w[\overline{\text{enter}}(x, k). \overline{\text{enter}}(y, k). (! (z)^x. \langle z \rangle^y \mid ! (z)^y. \langle z \rangle^x)]$$

An agent can be defined as: $A(a, k, P, Q) \triangleq a[\text{enter}\langle w, k \rangle. P \mid \text{exit}\langle w, k \rangle. Q]$ and a communication server as:

$$\text{o2o}(k) = (\nu r) (r[\langle \rangle^{\hat{}}] \mid ! ()^r. (w(k) \mid \overline{\text{exit}}(-, k). \overline{\text{exit}}(-, k). r[\langle \rangle^{\hat{}}]))$$

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It can be proved that:

$$\begin{aligned} & (\nu k) (\text{o2o}(k) \mid A(k, a_1, \langle M \rangle^{\hat{}}.P_1, Q_1) \mid A(k, a_2, (x)^{\hat{}}.P_2\{x\}, Q_2) \mid \Pi_{i \in I} A(K, a_i, R_i, S_i)) \\ & \implies \cong (\nu k) (\text{o2o}(k) \mid a_1[P_1 \mid Q_1] \mid a_2[P_1\{x := M\} \mid Q_2] \mid \Pi_{i \in I} A(K, a_i, R_i, S_i)) \end{aligned}$$

that is, once two agents engage in communication no other agent knowing the key k can interfere with their completing the exchange.

A print server

● The following process assigns a progressive number to incoming jobs.

$$\text{enqueue}_k \triangleq (\nu c) (c[\langle 1 \rangle^{\wedge}] \mid !(n)^c.\overline{\text{enter}}(x, k).\langle n \rangle^x.c[\langle n + 1 \rangle^{\wedge}])$$

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$$\text{enqueue}_k \triangleq (\nu c) (c[\langle 1 \rangle^\wedge] \mid !(n)^c. \overline{\text{enter}}(x, k). \langle n \rangle^x. c[\langle n + 1 \rangle^\wedge])$$

- We can turn it into a print server (which consumes such numbers).

$$\text{prtsrv}(k) \triangleq k[\text{enqueue}_k \mid \text{print}]$$

$$\text{print} \triangleq (\nu c) (c[\langle 1 \rangle^\wedge] \mid !(n)^c. \overline{\text{exit}}(x, n). (data)^x. (P\{data\} \mid c[\langle n + 1 \rangle^\wedge]))$$

- A client then acts as:

$$\text{job}(M, k) \triangleq (\nu p) p[\text{enter} \langle k, k \rangle. (n)^\wedge. (\nu q) q[\text{exit} \langle p, n \rangle. \langle M \rangle^\wedge]]$$

It enters the server $\text{prtsrv}(k)$ (using enqueue), it is assigned a number that it uses as a password to carry job M to print (which eventually will bind it to $data$ in P). (Dynamic name discovery and passwords are fundamental here.)

Some Equational Laws

Garbage Collection laws

$$\bullet l[(\tilde{x}_i)^n . P \mid (\tilde{x}) . Q \mid \langle \tilde{M} \rangle^m . R] \cong 0$$

$$\bullet l[(\tilde{x})^n . P \mid \langle \tilde{M} \rangle . P \mid \langle \tilde{M} \rangle^m . P] \cong 0$$

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Communication laws

$$\bullet l[\langle \tilde{M}_0 \rangle^{\hat{}} \mid \langle \tilde{M}_1 \rangle^{\hat{}}] \cong l[\langle \tilde{M}_0 \rangle^{\hat{}}] \mid l[\langle \tilde{M}_1 \rangle^{\hat{}}]$$

$$\bullet l[(\tilde{x}) . P \mid \langle \tilde{M} \rangle . Q] \cong l[P\{\tilde{x} := \tilde{M}\} \mid Q]$$

$$\bullet (\nu l)((\tilde{x})^l . P \mid l[\langle \tilde{M} \rangle^{\hat{}} . Q]) \cong (\nu l)(P\{\tilde{x} := \tilde{M}\} \mid l[Q])$$

$$\bullet m[(\tilde{x})^l . P \mid l[\langle \tilde{M} \rangle^{\hat{}} . Q]] \cong m[P\{\tilde{x} := \tilde{M}\} \mid l[Q]]$$

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Communication laws

$$\bullet l[\langle \tilde{M}_0 \rangle^{\hat{}} \mid \langle \tilde{M}_1 \rangle^{\hat{}}] \cong l[\langle \tilde{M}_0 \rangle^{\hat{}}] \mid l[\langle \tilde{M}_1 \rangle^{\hat{}}]$$

$$\bullet l[(\tilde{x}) . P \mid \langle \tilde{M} \rangle . Q] \cong l[P\{\tilde{x} := \tilde{M}\} \mid Q]$$

$$\bullet (\nu l)((\tilde{x})^l . P \mid l[\langle \tilde{M} \rangle^{\hat{}} . Q]) \cong (\nu l)(P\{\tilde{x} := \tilde{M}\} \mid l[Q])$$

$$\bullet m[(\tilde{x})^l . P \mid l[\langle \tilde{M} \rangle^{\hat{}} . Q]] \cong m[P\{\tilde{x} := \tilde{M}\} \mid l[Q]]$$

Mobility laws

$$\bullet (\nu p)(m[\text{enter} \langle n, p \rangle . P] \mid n[\overline{\text{enter}}(x, p) . Q]) \cong (\nu p)(n[Q\{x := m\} \mid m[P]])$$

$$\bullet l[m[\text{enter} \langle n, p \rangle . P] \mid n[\overline{\text{enter}}(x, p) . Q]] \cong l[n[Q\{x := m\} \mid m[P]]]$$



An LTS for NBA

Concretions: $(\nu\tilde{p})\langle P \rangle Q$ and $(\nu\tilde{p})\langle M \rangle Q$

(AMB CO-ENTER)

$$\frac{P \xrightarrow{\overline{\text{enter}}(n,k)} P'}{m[P] \xrightarrow{m \overline{\text{enter}}(n,k)} (\nu)\langle P' \rangle \mathbf{0}}$$

(CO-ENTER HO)

$$\frac{P \xrightarrow{m \overline{\text{enter}}(n,k)} (\nu\tilde{p})\langle P_1 \rangle P_2 \quad \tilde{p} \cap \text{fn}(Q) = \emptyset}{P \xrightarrow{m \overline{\text{enter}}(n,k)Q} (\nu\tilde{p})(m[n[Q] \mid P_1] \mid P_2)}$$

(EXIT)

$$\frac{P \xrightarrow{\text{exit}\langle n,k \rangle} (\nu\tilde{p})\langle m[P_1] \rangle P_2}{n[P] \xrightarrow{\text{exit}\langle k \rangle} (\nu\tilde{p})\langle m \rangle (m[P_1] \mid n[P_2])}$$

(τ -EXIT)

$$\frac{P \xrightarrow{\text{exit}\langle k \rangle} (\nu\tilde{p})\langle m \rangle P' \quad Q \xrightarrow{\overline{\text{exit}}(m,k)} Q'}{P \mid Q \xrightarrow{\tau} (\nu\tilde{p})(P' \mid Q')}$$

(EXIT HO)

$$\frac{P \xrightarrow{\text{exit}\langle n,k \rangle} (\nu\tilde{p})\langle m[P_1] \rangle P_2 \quad x \in \text{fn}(R) \quad \tilde{p} \cap \text{fn}(Q \mid R) = \emptyset}{P \xrightarrow{\text{exit}\langle n,k \rangle QR} (\nu\tilde{p})(m[P_1] \mid n[P_2 \mid Q] \mid R\{x := m\})}$$

A Characterisation of Reduction Bisimulation

● Thm. If $P \xrightarrow{\tau} P'$ then $P \longrightarrow P'$. If $P \longrightarrow P'$ then $P \xrightarrow{\tau} \equiv P'$.

● Bisimilarity. A symmetric relation \mathcal{R} is a bisimulation if

$$P \mathcal{R} Q \text{ and } P \xrightarrow{\alpha} P' \text{ implies } \exists Q' \xRightarrow{\hat{\alpha}} Q' \text{ with } P' \mathcal{R} Q'.$$

● $P \approx Q$ if $P \mathcal{R} Q$ for some bisimulation \mathcal{R} .

● The closure under substitutions of \approx is denoted by \approx_c .

● Thm. If $P \approx_c Q$ then $P \cong Q$ and viceversa.

A Type System for NBA

Types

Message Types	W	$::=$	$N[E]$	ambient/password
			$ $ $C[E]$	capability
Exchange Types	E, F	$::=$	shh	no exchange
			$ $ $W_1 \dots W_k$	tuples ($k \geq 0$)
Process Types	T	$::=$	$[E, F]$	composite exchange

$N[E]$ types both ambients and passwords; shh is the *silent type*; $N[\text{shh}]$ is an ambient with no upward exchanges or a password that reveal the visitor's name.

A Type System for NBA

Types

Message Types $W ::= N[E]$ ambient/password
 | $C[E]$ capability

Exchange Types $E, F ::= \text{shh}$ no exchange
 | $W_1 \dots W_k$ tuples ($k \geq 0$)

Process Types $T ::= [E, F]$ composite exchange

$N[E]$ types both ambients and passwords; shh is the *silent type*; $N[\text{shh}]$ is an ambient with no upward exchanges or a password that reveal the visitor's name.

Type Environments

(ENV EMPTY)

$\emptyset \vdash \diamond$

(ENV NAME)

$\Gamma \vdash \diamond \quad a \notin \text{Dom}(\Gamma)$

$\Gamma, a : W \vdash \diamond$

Typing Rules

Messages

(PROJECTION)

$$\Gamma, a : W, \Gamma' \vdash \diamond$$

$$\Gamma, a : W, \Gamma' \vdash a : W$$

(PATH)

$$\Gamma \vdash M_1 : \mathsf{C}[E_1] \quad \Gamma \vdash M_2 : \mathsf{C}[E_2]$$

$$\Gamma \vdash M_1.M_2 : \mathsf{C}[E_1 \sqcup E_2]$$

(ENTER)

$$\Gamma \vdash M : \mathsf{N}[E] \quad \Gamma \vdash N : \mathsf{N}[F] \quad (F \leqslant G)$$

$$\Gamma \vdash \text{enter}\langle M, N \rangle : \mathsf{C}[G]$$

(EXIT)

$$\Gamma \vdash M : \mathsf{N}[E] \quad \Gamma \vdash N : \mathsf{N}[F] \quad (F \leqslant G)$$

$$\Gamma \vdash \text{exit}\langle M, N \rangle : \mathsf{C}[G]$$

Typing Rules

Messages

(PROJECTION)

$$\Gamma, a : W, \Gamma' \vdash \diamond$$

$$\Gamma, a : W, \Gamma' \vdash a : W$$

(PATH)

$$\Gamma \vdash M_1 : C[E_1] \quad \Gamma \vdash M_2 : C[E_2]$$

$$\Gamma \vdash M_1.M_2 : C[E_1 \sqcup E_2]$$

(ENTER)

$$\Gamma \vdash M : N[E] \quad \Gamma \vdash N : N[F] \quad (F \leq G)$$

$$\Gamma \vdash \text{enter}\langle M, N \rangle : C[G]$$

(EXIT)

$$\Gamma \vdash M : N[E] \quad \Gamma \vdash N : N[F] \quad (F \leq G)$$

$$\Gamma \vdash \text{exit}\langle M, N \rangle : C[G]$$

Processes

(PAR)

$$\Gamma \vdash P : [E, F] \quad \Gamma \vdash Q : [E, F]$$

$$\Gamma \vdash P \mid Q : [E, F]$$

(REPL)

$$\Gamma \vdash P : [E, F]$$

$$\Gamma \vdash !P : [E, F]$$

(DEAD)

$$\Gamma \vdash \diamond$$

$$\Gamma \vdash \mathbf{0} : [E, F]$$

(NEW)

$$\Gamma, n : N[G] \vdash P : [E, F]$$

$$\Gamma \vdash (\nu n : N[G])P : [E, F]$$

Typing Rules: II

Processes: mobility

(AMB)

$$\frac{\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash P : [F, E]}{\Gamma \vdash M[P] : [G, H]}$$

(PREFIX)

$$\frac{\Gamma \vdash M : \mathbf{C}[F] \quad \Gamma \vdash P : [E, G] \quad (F \leq G)}{\Gamma \vdash M.P : [E, G]}$$

(CO-ENTER)

$$\frac{\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, x : \mathbf{N}[\tilde{W}] \vdash P : [E, F]}{\Gamma \vdash \overline{\text{enter}}(x, M).P : [E, F]}$$

(CO-EXIT)

$$\frac{\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, x : \mathbf{N}[\tilde{W}] \vdash P : [E, F]}{\Gamma \vdash \overline{\text{exit}}(x, M).P : [E, F]}$$

(CO-ENTER-SILENT)

$$\frac{\Gamma \vdash M : \mathbf{N}[\text{shh}] \quad \Gamma \vdash P : [E, F] \quad (x \notin \text{fv}(P))}{\Gamma \vdash \overline{\text{enter}}(x, M).P : [E, F]}$$

(CO-EXIT-SILENT)

$$\frac{\Gamma \vdash M : \mathbf{N}[\text{shh}] \quad \Gamma \vdash P : [E, F] \quad (x \notin \text{fv}(P))}{\Gamma \vdash \overline{\text{exit}}(x, M).P : [E, F]}$$

Typing Rules: II

Processes: I/O

(INPUT)

$$\frac{\Gamma, \tilde{x}:\tilde{W} \vdash P : [\tilde{W}, E]}{\Gamma \vdash (\tilde{x}:\tilde{W}).P : [\tilde{W}, E]}$$

(INPUT M)

$$\frac{\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, \tilde{x}:\tilde{W} \vdash P : [G, H]}{\Gamma \vdash (\tilde{x}:\tilde{W})^M.P : [G, H]}$$

(OUTPUT $\hat{}$)

$$\frac{\Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [E, \tilde{W}]}{\Gamma \vdash \langle \tilde{M} \rangle^{\hat{}}.P : [E, \tilde{W}]}$$

(INPUT $\hat{}$)

$$\frac{\Gamma, \tilde{x}:\tilde{W} \vdash P : [E, \tilde{W}]}{\Gamma \vdash (\tilde{x}:\tilde{W})^{\hat{}}.P : [E, \tilde{W}]}$$

(OUTPUT)

$$\frac{\Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [\tilde{W}, E]}{\Gamma \vdash \langle \tilde{M} \rangle.P : [\tilde{W}, E]}$$

(OUTPUT N)

$$\frac{\Gamma \vdash N : \mathbf{N}[\tilde{W}] \quad \Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [G, H]}{\Gamma \vdash \langle \tilde{M} \rangle^N.P : [G, H]}$$

Subject Reduction. If $\Gamma \vdash P : T$ and $P \longrightarrow Q$, then $\Gamma \vdash Q : T$.

Encoding: BA in NBA

We can encode BA into NBA enriched with a focused form of nondeterminism.

$$\{ P \}_n = \overline{\text{cross}} \mid \langle P \rangle_n$$

$$\langle m[P] \rangle_n = m[\{ P \}_m]$$

$$\langle (x)^a P \rangle_n = (x)^a \langle P \rangle_n$$

$$\langle (x)P \rangle_n = (x) \langle P \rangle_n + (x)^\wedge \langle P \rangle_n + \overline{\text{exit}}(y, \text{pw})(x)^y \langle P \rangle_n \quad y \notin \text{fn}(P)$$

$$\langle (x)^\uparrow P \rangle_n = (\nu p)p[\text{exit}\langle n, \text{pr} \rangle.(x)^\wedge.\text{enter}\langle n, p \rangle.\langle x \rangle^\wedge] \mid \overline{\text{enter}}(y, p)(x)^y \langle P \rangle_n \quad p, y \notin \text{fn}(P)$$

$$\langle \langle M \rangle^a P \rangle_n = \langle M \rangle^a \langle P \rangle_n$$

$$\langle \langle M \rangle P \rangle_n = \langle M \rangle \langle P \rangle_n + \langle M \rangle^\wedge \langle P \rangle_n + \overline{\text{exit}}(y, \text{pr})\langle M \rangle^y \langle P \rangle_n \quad y \notin \text{fn}(P)$$

$$\langle \langle M \rangle^\uparrow P \rangle_n = (\nu p)p[\text{exit}\langle n, \text{pw} \rangle.\langle M \rangle^\wedge.\text{enter}\langle n, p \rangle.\langle \cdot \rangle^\wedge] \mid \overline{\text{enter}}(y, p)(-) ^y \langle P \rangle_n \quad p, y \notin \text{fn}(P)$$

where $\overline{\text{cross}} = !\overline{\text{enter}}(x, \text{mv}) \mid !\overline{\text{exit}}(x, \text{mv})$, in $n = \text{enter}\langle n, \text{mv} \rangle$, and out $n = \text{exit}\langle n, \text{mv} \rangle$.

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 **Thm.** If $P \xrightarrow{\tau} P'$ then $\{ P \} \xrightarrow{\tau} \gtrsim \{ P' \}$.

If $\{ P \} \xrightarrow{\tau} Q$, then $\exists P \xrightarrow{\tau} P'$ with $Q \gtrsim \{ P' \}$.

If P and Q are **single-threaded**, then $\{ P \}_n \cong \{ Q \}_n$ implies $P \cong Q$.



Conclusion and Future Work

- Type inference.
- Information flow analysis.
- Comparison with Seal calculus.
- Implementation.
- Logics.