

# Communication Interference in Mobile Boxed Ambients

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# Mobile Ambients

Both administrative domains and computational environments (Cardelli-Gordon)

- Subjective movements

$$n[\text{in } m.P \mid Q] \mid m[R] \longrightarrow m[n[P \mid Q] \mid R]$$

$$m[n[\text{out } m.P \mid Q] \mid R] \longrightarrow n[P \mid Q] \mid m[R]$$

- Process interaction

$$n[\langle M \rangle.P \mid (x).Q] \longrightarrow n[P \mid Q\{x := M\}],$$

- Boundary dissolver

$$\text{open } n.P \mid n[Q] \longrightarrow P \mid Q.$$

# Interferences in Mobile Ambients

- The inherent nondeterminism of movement may go wild: Grave Interferences.

$$k[n[\text{in } m.P \mid \text{out } k.R] \mid m[Q]]$$


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- Introducing **Safe Ambients** (Levi-Sangiorgi)

$$n[\text{in } m.P \mid Q] \mid m[\overline{\text{in}} \ m.R \mid S] \longrightarrow m[n[P \mid Q] \mid R \mid S]$$

- Co-capabilities and single-threadedness rule out grave interferences



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- Co-capabilities and single-threadedness rule out grave interferences
- Safe Ambients with **passwords** have a conveniently treatable semantics.  
(Merro-Hennessy)

$$n[\text{in } (m, k).P \mid Q] \mid m[\overline{\text{in}} \ (m, k).R \mid S] \longrightarrow m[n[P \mid Q] \mid R \mid S]$$



# Mobile Boxed Ambients

- open's nature of ambient dissolver is a potential source of problems.
- Direct communication as alternative source of expressiveness: **Mobile Boxed Ambients** (Bugliesi et al.). Perform I/O on a subambient  $n$ 's local channel (viz.  $(x)^n$ ) as well as from the parent's local channel (viz.  $(x)^\uparrow$ )

$$(x)^n.P \mid n[\langle M \rangle.Q \mid R] \longrightarrow P\{x := M\} \mid n[Q \mid R]$$

$$\langle M \rangle.P \mid n[(x)^\uparrow.Q \mid R] \longrightarrow P \mid n[Q\{x := M\} \mid R].$$



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- But it is a great source of non-local nondeterminism and communication interference.

$$m[(x)^n.P \mid n[\langle M \rangle \mid (x).Q \mid k[(x)^\uparrow.R]]]$$

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# Introducing NBA: Communication

**NBA**: a fresh foundation based on: each ambient comes equipped with two mutually non-interfering channels, for **local** and **upward** communications.

$$(x)^n.P \mid n[\langle M \rangle \hat{\cdot} Q \mid R] \longrightarrow P\{x := M\} \mid n[Q \mid R]$$
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- Good algebraic laws; simple type system;
- Expressiveness??



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$$\langle M \rangle \hat{\cdot} P \mid n[(x)^n \hat{\cdot} Q \mid R] \longrightarrow P \mid n[Q\{x := M\} \mid R]$$

- Good algebraic laws; simple type system;
- Expressiveness??
- Hmm, rather poor:  $n[P]$  cannot, for instance, communicate with children it doesn't know statically. It can never learn about incoming ambients, and will never be able to talk to them.

# Introducing NBA: Mobility

- Essentially, our idea is to introduce co-actions of the form  $\overline{\text{enter}}(x)$  which have the effect of binding the variable  $x$ .
- Such a purely binding mechanism does not provide a way control of access, but only to **registers** it. As a (realistic) access protocol where newly arrived agents must register themselves to be granted access to local resources.



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- Need a finer mechanism of **access control**:

$$a[\text{enter}\langle b, k \rangle.P_1 \mid P_2] \mid b[\overline{\text{enter}}(x, k).Q_1 \mid Q_2] \longrightarrow b[a[P_1 \mid P_2] \mid Q_1\{x := a\} \mid Q_2]$$

This represent an access protocol where the credentials of incoming processes ( $k$  in the rule above) are controlled, as a preliminary step to the registration protocol.



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# NBA: Syntax

Names:  $a, b, \dots, n, x, y, \dots \in \mathbf{N}$

Locations:

$\eta ::= a$	nested names
$\hat{\phantom{x}}$	enclosing ambient
$\star$	local

Messages:

$M, N ::= a$	name
$\text{enter}(M, N)$	may enter
$\text{exit}(M, N)$	may exit
$M.N$	path

Processes:

$P ::= 0$	nil process
$P_1   P_2$	composition
$(\nu n)P$	restriction
$!P$	replication
$M[P]$	ambient
$\pi.P$	prefixing

Prefixes:

$\pi ::= M$	messages
$(x_1, \dots, x_k)^\eta$	input
$\langle M_1, \dots, M_k \rangle^\eta$	output
$\overline{\text{enter}}(x, M)$	allow enter
$\overline{\text{exit}}(x, M)$	allow exit

# NBA: Reduction Semantics

## mobility

$$\begin{array}{lcl} n[\text{enter}\langle m, k \rangle.P_1 \mid P_2] \mid m[\overline{\text{enter}}(x, k).Q_1 \mid Q_2] & \longrightarrow & m[n[P_1 \mid P_2] \mid Q_1\{x := n\} \mid Q_2] \\ n[m[\text{exit}\langle n, k \rangle.P_1 \mid P_2] \mid Q] \mid \overline{\text{exit}}(x, k).R & \longrightarrow & m[P_1 \mid P_2] \mid n[Q] \mid R\{x := m\} \end{array}$$

## communication

$$\begin{array}{lcl} (\tilde{x}).P \mid \langle \tilde{M} \rangle.Q & \longrightarrow & P\{\tilde{x} := \tilde{M}\} \mid Q \\ (\tilde{x})^n.P \mid n[\langle \tilde{M} \rangle^{\hat{\wedge}}.Q \mid R] & \longrightarrow & P\{\tilde{x} := \tilde{M}\} \mid n[Q \mid R] \\ \langle \tilde{M} \rangle^n.P \mid n[(\tilde{x})^{\hat{\wedge}}.Q \mid R] & \longrightarrow & P \mid n[Q\{\tilde{x} := \tilde{M}\} \mid R] \end{array}$$

## structural congruence

$$P \equiv Q \quad Q \longrightarrow R \quad R \equiv S \text{ implies } P \longrightarrow S$$

# NBA: Behavioural Equivalence

## • Barbs

$P \downarrow_n$  iff  $P \equiv (\nu \vec{m})(n[\overline{\text{enter}}(x, k).Q \mid R] \mid S)$ , for  $\{n, k\} \cap \{\vec{m}\} = \emptyset$ .

$P \Downarrow_n$  iff  $P \implies P'$  and  $P' \downarrow_n$ .

# NBA: Behavioural Equivalence

## Barbs

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$P \Downarrow_n$  iff  $P \xrightarrow{} P'$  and  $P' \downarrow_n$ .

● A relation  $\mathcal{R}$  is **reduction closed** if

$P \mathcal{R} Q$  and  $P \rightarrow P'$  implies  $Q \Rightarrow Q'$  with  $P' \mathcal{R} Q'$ ;

it is **barb preserving** if  $P \mathcal{R} Q$  and  $P \downarrow_n$  implies  $Q \Downarrow_n$ .

● **Reduction barbed congruence**, written  $\cong$ , is the largest congruence relation over processes which is reduction closed and barb preserving.

● Note: We could equivalently observe  $\langle \cdot \rangle^{\hat{\wedge}}$ .

# The rest of the talk

- Two small examples
- A few equational laws
- LTS characterization of reduction barbed bisimulation congruence.
- A type system
- An encoding of BA into NBA:  $BA \lesssim_{\text{NBA}} \text{Guarded Choice}$



# A one-to-one communication server

- Let  $w(k)$  be a bidirectional forwarder for any pair of incoming ambients.

$$w(k) \triangleq w[ \overline{\text{enter}}(x, k). \overline{\text{enter}}(y, k). (!z)^x. \langle z \rangle^y \mid !z^y. \langle z \rangle^x ]$$

An agent can be defined as:  $A(a, k, P, Q) \triangleq a[\text{enter}\langle w, k \rangle.P \mid \text{exit}\langle w, k \rangle.Q]$  and a communication server as:

$$\text{o2o}(k) = (\nu r) ( r[\langle \rangle^{\hat{\wedge}}] \mid !().^r.(w(k) \mid \overline{\text{exit}}(\_, k). \overline{\text{exit}}(\_, k). r[\langle \rangle^{\hat{\wedge}}]) )$$

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- It can be proved that:

$$\begin{aligned} & (\nu k) (\text{o2o}(k) \mid A(k, a_1, \langle M \rangle^{\hat{\wedge}}.P_1, Q_1) \mid A(k, a_2, (x)^{\hat{\wedge}}.P_2\{x\}, Q_2) \mid \Pi_{i \in I} A(K, a_i, R_i, S_i) ) \\ & \implies \cong (\nu k) (\text{o2o}(k) \mid a_1[P_1 \mid Q_1] \mid a_2[P_2\{x := M\} \mid Q_2] \mid \Pi_{i \in I} A(K, a_i, R_i, S_i) ) \end{aligned}$$

that is, once two agents engage in communication no other agent knowing the key  $k$  can interfere with their completing the exchange.



# A print server

- The following process assigns a progressive number to incoming jobs.

$$\text{enqueue}_k \triangleq (\nu c) ( c[\langle 1 \rangle^{\hat{\wedge}}] \mid !(n)^c. \overline{\text{enter}}(x, k). \langle n \rangle^x. c[\langle n + 1 \rangle^{\hat{\wedge}}] )$$

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- We can turn it into a print server (which consumes such numbers).

$$\text{prtsrv}(k) \triangleq k[ \text{enqueue}_k \mid \text{print} ]$$

$$\text{print} \triangleq (\nu c) ( c[\langle 1 \rangle^\hat{\wedge}] \mid !(n)^c.\overline{\text{exit}}(x, n).(data)^x.(P\{data\} \mid c[\langle n + 1 \rangle^\hat{\wedge}]) )$$

- A client then acts as:

$$\text{job}(M, k) \triangleq (\nu p)p[ \text{enter}\langle k, k \rangle.(n)^\hat{\wedge}.(\nu q)q[\text{exit}\langle p, n \rangle.\langle M \rangle^\hat{\wedge}] ]$$

It enters the server  $\text{prtsrv}(k)$  (using  $\text{enqueue}$ ), it is assigned a number that it uses as a password to carry job  $M$  to  $\text{print}$  (which eventually will bind it to  $data$  in  $P$ . (Dynamic name discovery and passwords are fundamental here.)

# Some Equational Laws

## Garbage Collection laws

- $l[ (\tilde{x}_i)^n.P \mid (\tilde{x}).Q \mid \langle \tilde{M} \rangle^m.R ] \cong 0$
- $l[ (\tilde{x})^n.P \mid \langle \tilde{M} \rangle.P \mid \langle \tilde{M} \rangle^m.P ] \cong 0$

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## Communication laws

- $l[ \langle \tilde{M}_0 \rangle^{\hat{\wedge}} \mid \langle \tilde{M}_1 \rangle^{\hat{\wedge}} ] \cong l[ \langle \tilde{M}_0 \rangle^{\hat{\wedge}} ] \mid l[ \langle \tilde{M}_1 \rangle^{\hat{\wedge}} ]$
- $l[(\tilde{x}).P \mid \langle \tilde{M} \rangle.Q] \cong l[P\{\tilde{x} := \tilde{M}\} \mid Q]$
- $(\nu l)( (\tilde{x})^l.P \mid l[ \langle \tilde{M} \rangle^{\hat{\wedge}}.Q ] ) \cong (\nu l)( P\{\tilde{x} := \tilde{M}\} \mid l[Q] )$
- $m[(\tilde{x})^l.P \mid l[ \langle \tilde{M} \rangle^{\hat{\wedge}}.Q ]] \cong m[P\{\tilde{x} := \tilde{M}\} \mid l[Q]]$

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- $l[(\tilde{x}).P \mid \langle \tilde{M} \rangle.Q] \cong l[P\{\tilde{x} := \tilde{M}\} \mid Q]$
- $(\nu l)( (\tilde{x})^l.P \mid l[ \langle \tilde{M} \rangle^{\hat{\wedge}}.Q ] ) \cong (\nu l)( P\{\tilde{x} := \tilde{M}\} \mid l[Q] )$
- $m[(\tilde{x})^l.P \mid l[ \langle \tilde{M} \rangle^{\hat{\wedge}}.Q ]] \cong m[P\{\tilde{x} := \tilde{M}\} \mid l[Q]]$

## Mobility laws

- $(\nu p)(m[\text{enter}\langle n, p \rangle.P] \mid n[\overline{\text{enter}}(x, p).Q]) \cong (\nu p)(n[Q\{x := m\} \mid m[P]])$
- $l[m[\text{enter}\langle n, p \rangle.P] \mid n[\overline{\text{enter}}(x, p).Q]] \cong l[n[Q\{x := m\} \mid m[P]]]$

# An LTS for NBA

Concretions:  $(\nu\tilde{p})\langle P \rangle Q$  and  $(\nu\tilde{p})\langle M \rangle Q$

(AMB CO-ENTER)

$$\frac{}{m[P] \xrightarrow{m \overline{\text{enter}}(n,k)} (\nu)\langle P' \rangle \mathbf{0}}$$

$$\frac{P \xrightarrow{\overline{\text{enter}}(n,k)} P'}{m[P] \xrightarrow{m \overline{\text{enter}}(n,k)} (\nu)\langle P' \rangle \mathbf{0}}$$

(CO-ENTER HO)

$$\frac{P \xrightarrow{m \overline{\text{enter}}(n,k)} (\nu\tilde{p})\langle P_1 \rangle P_2 \quad \tilde{p} \cap \text{fn}(Q) = \emptyset}{P \xrightarrow{m \overline{\text{enter}}(n,k)Q} (\nu\tilde{p})(m[n[Q] \mid P_1] \mid P_2)}$$

(EXIT)

$$\frac{P \xrightarrow{\text{exit}\langle n,k \rangle} (\nu\tilde{p})\langle m[P_1] \rangle P_2}{n[P] \xrightarrow{\text{exit}\langle k \rangle} (\nu\tilde{p})\langle m \rangle (m[P_1] \mid n[P_2])}$$

( $\tau$ -EXIT)

$$\frac{P \xrightarrow{\text{exit}\langle k \rangle} (\nu\tilde{p})\langle m \rangle P' \quad Q \xrightarrow{\overline{\text{exit}}(m,k)} Q'}{P \mid Q \xrightarrow{\tau} (\nu\tilde{p})(P' \mid Q')}$$

(EXIT HO)

$$\frac{P \xrightarrow{\text{exit}\langle n,k \rangle} (\nu\tilde{p})\langle m[P_1] \rangle P_2 \quad x \in \text{fn}(R) \quad \tilde{p} \cap \text{fn}(Q|R) = \emptyset}{P \xrightarrow{\text{exit}\langle n,k \rangle QR} (\nu\tilde{p})(m[P_1] \mid n[P_2 \mid Q] \mid R\{x := m\})}$$

# A Characterisation of Reduction Bisimulation

• Thm. If  $P \xrightarrow{\tau} P'$  then  $P \rightarrow P'$ . If  $P \rightarrow P'$  then  $P \xrightarrow{\tau} \equiv P'$ .

• Bisimilarity. A symmetric relation  $\mathcal{R}$  is a bisimulation if

$$P \mathcal{R} Q \quad \text{and} \quad P \xrightarrow{\alpha} P' \quad \text{implies} \quad \exists Q \xrightarrow{\hat{\alpha}} Q' \quad \text{with} \quad P' \mathcal{R} Q'.$$

•  $P \approx Q$  if  $P \mathcal{R} Q$  for some bisimulation  $\mathcal{R}$ .

• The closure under substitutions of  $\approx$  is denoted by  $\approx_c$ .

• Thm. If  $P \approx_c Q$  then  $P \cong Q$  and viceversa.



# A Type System for NBA

## Types

Message Types  $W ::= \mathbf{N}[E]$  ambient/password

|  $\mathbf{C}[E]$  capability

Exchange Types  $E, F ::= \mathbf{shh}$  no exchange

|  $W_1 \dots W_k$  tuples ( $k \geq 0$ )

Process Types  $T ::= [E, F]$  composite exchange

$\mathbf{N}[E]$  types both ambients and passwords;  $\mathbf{shh}$  is the silent type;  $\mathbf{N}[\mathbf{shh}]$  is an ambient with no upward exchanges or a password that reveal the visitor's name.



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## ● Types

Message Types	$W ::= \mathbf{N}[E] \quad   \quad \mathbf{C}[E]$	ambient/password capability
Exchange Types	$E, F ::= \mathbf{shh} \quad   \quad W_1 \dots W_k$	no exchange tuples ( $k \geq 0$ )
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## ● Type Environments

(ENV EMPTY)

$\emptyset \vdash \diamond$

(ENV NAME)

$\Gamma \vdash \diamond \quad a \notin \text{Dom}(\Gamma)$

$\Gamma, a : W \vdash \diamond$

# Typing Rules

## Messages

(PROJECTION)

$$\Gamma, a : W, \Gamma' \vdash \diamond$$

---

$$\Gamma, a : W, \Gamma' \vdash a : W$$

(PATH)

$$\Gamma \vdash M_1 : \mathbf{C}[E_1] \quad \Gamma \vdash M_2 : \mathbf{C}[E_2]$$

---

$$\Gamma \vdash M_1.M_2 : \mathbf{C}[E_1 \sqcup E_2]$$

(ENTER)

$$\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash N : \mathbf{N}[F] \quad (F \leqslant G)$$

---

$$\Gamma \vdash \mathbf{enter}\langle M, N \rangle : \mathbf{C}[G]$$

(EXIT)

$$\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash N : \mathbf{N}[F] \quad (F \leqslant G)$$

---

$$\Gamma \vdash \mathbf{exit}\langle M, N \rangle : \mathbf{C}[G]$$



# Typing Rules

## Messages

(PROJECTION)

$$\frac{}{\Gamma, a : W, \Gamma' \vdash \diamond}$$

$$\frac{}{\Gamma, a : W, \Gamma' \vdash a : W}$$

(PATH)

$$\frac{}{\Gamma \vdash M_1 : \mathbf{C}[E_1] \quad \Gamma \vdash M_2 : \mathbf{C}[E_2]}$$

$$\frac{}{\Gamma \vdash M_1.M_2 : \mathbf{C}[E_1 \sqcup E_2]}$$

(ENTER)

$$\frac{\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash N : \mathbf{N}[F] \quad (F \leqslant G)}{\Gamma \vdash \mathbf{enter}\langle M, N \rangle : \mathbf{C}[G]}$$

(EXIT)

$$\frac{\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash N : \mathbf{N}[F] \quad (F \leqslant G)}{\Gamma \vdash \mathbf{exit}\langle M, N \rangle : \mathbf{C}[G]}$$

$$\frac{}{\Gamma \vdash \mathbf{enter}\langle M, N \rangle : \mathbf{C}[G]}$$

$$\frac{}{\Gamma \vdash \mathbf{exit}\langle M, N \rangle : \mathbf{C}[G]}$$

## Processes

(PAR)

$$\frac{\Gamma \vdash P : [E, F] \quad \Gamma \vdash Q : [E, F]}{\Gamma \vdash P \mid Q : [E, F]}$$

(REPL)

$$\frac{\Gamma \vdash P : [E, F]}{\Gamma \vdash !P : [E, F]}$$

(DEAD)

$$\frac{}{\Gamma \vdash \diamond}$$

$$\frac{}{\Gamma \vdash P \mid Q : [E, F]}$$

$$\frac{}{\Gamma \vdash !P : [E, F]}$$

$$\frac{}{\Gamma \vdash \mathbf{0} : [E, F]}$$

(NEW)

$$\frac{\Gamma, n : \mathbf{N}[G] \vdash P : [E, F]}{\Gamma \vdash (\nu n : \mathbf{N}[G])P : [E, F]}$$



# Typing Rules: II

## Processes: mobility

(AMB)

$$\Gamma \vdash M : \mathbf{N}[E] \quad \Gamma \vdash P : [F, E]$$

$$\Gamma \vdash M[P] : [G, H]$$

(PREFIX)

$$\Gamma \vdash M : \mathbf{C}[F] \quad \Gamma \vdash P : [E, G] \quad (F \leq G)$$

$$\Gamma \vdash M.P : [E, G]$$

(CO-ENTER)

$$\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, x : \mathbf{N}[\tilde{W}] \vdash P : [E, F]$$

$$\Gamma \vdash \overline{\text{enter}}(x, M).P : [E, F]$$

(CO-EXIT)

$$\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, x : \mathbf{N}[\tilde{W}] \vdash P : [E, F]$$

$$\Gamma \vdash \overline{\text{exit}}(x, M).P : [E, F]$$

(CO-ENTER-SILENT)

$$\Gamma \vdash M : \mathbf{N}[\mathbf{shh}] \quad \Gamma \vdash P : [E, F] \quad (x \notin \text{fv}(P))$$

$$\Gamma \vdash \overline{\text{enter}}(x, M).P : [E, F]$$

(CO-EXIT-SILENT)

$$\Gamma \vdash M : \mathbf{N}[\mathbf{shh}] \quad \Gamma \vdash P : [E, F] \quad (x \notin \text{fv}(P))$$

$$\Gamma \vdash \overline{\text{exit}}(x, M).P : [E, F]$$



# Typing Rules: II

## Processes: I/O

(INPUT)

$$\Gamma, \tilde{x}:\tilde{W} \vdash P : [\tilde{W}, E]$$

$$\frac{}{\Gamma \vdash (\tilde{x}:\tilde{W}).P : [\tilde{W}, E]}$$

(INPUT  $\hat{\cdot}$ )

$$\Gamma, \tilde{x}:\tilde{W} \vdash P : [E, \tilde{W}]$$

$$\frac{}{\Gamma \vdash (\tilde{x}:\tilde{W})^{\hat{\cdot}}.P : [E, \tilde{W}]}$$

(INPUT  $M$ )

$$\Gamma \vdash M : \mathbf{N}[\tilde{W}] \quad \Gamma, \tilde{x}:\tilde{W} \vdash P : [G, H]$$

$$\frac{}{\Gamma \vdash (\tilde{x}:\tilde{W})^M.P : [G, H]}$$

(OUTPUT)

$$\Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [\tilde{W}, E]$$

$$\frac{}{\Gamma \vdash \langle \tilde{M} \rangle.P : [\tilde{W}, E]}$$

(OUTPUT  $\hat{\cdot}$ )

$$\Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [E, \tilde{W}]$$

$$\frac{}{\Gamma \vdash \langle \tilde{M} \rangle^{\hat{\cdot}}.P : [E, \tilde{W}]}$$

(OUTPUT  $N$ )

$$\Gamma \vdash N : \mathbf{N}[\tilde{W}] \quad \Gamma \vdash \tilde{M} : \tilde{W} \quad \Gamma \vdash P : [G, H]$$

$$\frac{}{\Gamma \vdash \langle \tilde{M} \rangle^N.P : [G, H]}$$

Subject Reduction. If  $\Gamma \vdash P : T$  and  $P \rightarrow Q$ , then  $\Gamma \vdash Q : T$ .

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# Encoding: BA in NBA

We can encode BA into NBA enriched with a focused form of nondeterminism.

$$\begin{aligned}
 \{P\}_n &= \text{cross} \mid \langle\langle P \rangle\rangle_n \\
 \langle\langle m[P] \rangle\rangle_n &= m[\{P\}_m] \\
 \langle\langle (x)^a P \rangle\rangle_n &= (x)^a \langle\langle P \rangle\rangle_n \\
 \langle\langle (x)P \rangle\rangle_n &= (x) \langle\langle P \rangle\rangle_n + (x)^\hat{\wedge} \langle\langle P \rangle\rangle_n + \overline{\text{exit}}(y, \text{pw})(x)^y \langle\langle P \rangle\rangle_n & y \notin \text{fn}(P) \\
 \langle\langle (x)^\uparrow P \rangle\rangle_n &= (\nu p)p[\text{exit}\langle n, \text{pr} \rangle.(x)^\hat{\wedge}.\text{enter}\langle n, p \rangle.\langle x \rangle^\hat{\wedge}] \mid \overline{\text{enter}}(y, p)(x)^y \langle\langle P \rangle\rangle_n & p, y \notin \text{fn}(P) \\
 \langle\langle \langle M \rangle^a P \rangle\rangle_n &= \langle M \rangle^a \langle\langle P \rangle\rangle_n \\
 \langle\langle \langle M \rangle P \rangle\rangle_n &= \langle M \rangle \langle\langle P \rangle\rangle_n + \langle M \rangle^\hat{\wedge} \langle\langle P \rangle\rangle_n + \overline{\text{exit}}(y, \text{pr})\langle M \rangle^y \langle\langle P \rangle\rangle_n & y \notin \text{fn}(P) \\
 \langle\langle \langle M \rangle^\uparrow P \rangle\rangle_n &= (\nu p)p[\text{exit}\langle n, \text{pw} \rangle.\langle M \rangle^\hat{\wedge}.\text{enter}\langle n, p \rangle.\langle \cdot \rangle^\hat{\wedge}] \mid \overline{\text{enter}}(y, p)(\_)^y \langle\langle P \rangle\rangle_n & p, y \notin \text{fn}(P)
 \end{aligned}$$

where  $\text{cross} = !\overline{\text{enter}}(x, \text{mv}) \mid !\overline{\text{exit}}(x, \text{mv})$ ,  $\text{in } n = \text{enter}\langle n, \text{mv} \rangle$ , and  $\text{out } n = \text{exit}\langle n, \text{mv} \rangle$ .

# Encoding: BA in NBA

We can encode BA into NBA enriched with a focused form of nondeterminism.

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 \langle\langle \langle M \rangle P \rangle\rangle_n &= \langle M \rangle \langle\langle P \rangle\rangle_n + \langle M \rangle^\hat{\wedge} \langle\langle P \rangle\rangle_n + \overline{\text{exit}}(y, \text{pr})\langle M \rangle^y \langle\langle P \rangle\rangle_n & y \notin \text{fn}(P) \\
 \langle\langle \langle M \rangle^\uparrow P \rangle\rangle_n &= (\nu p)p[\text{exit}\langle n, \text{pw} \rangle.\langle M \rangle^\hat{\wedge}.\text{enter}\langle n, p \rangle.\langle \cdot \rangle^\hat{\wedge}] \mid \overline{\text{enter}}(y, p)(\_)^y \langle\langle P \rangle\rangle_n & p, y \notin \text{fn}(P)
 \end{aligned}$$

where  $\text{cross} = !\overline{\text{enter}}(x, \text{mv}) \mid !\overline{\text{exit}}(x, \text{mv})$ ,  $\text{in } n = \text{enter}\langle n, \text{mv} \rangle$ , and  $\text{out } n = \text{exit}\langle n, \text{mv} \rangle$ .

Thm. If  $P \xrightarrow{\tau} P'$  then  $\{P\} \xrightarrow{\tau} \gtrsim \{P'\}$ .

If  $\{P\} \xrightarrow{\tau} Q$ , then  $\exists P \xrightarrow{\tau} P'$  with  $Q \gtrsim \{P'\}$ .

If  $P$  and  $Q$  are **single-threaded**, then  $\{P\}_n \cong \{Q\}_n$  implies  $P \cong Q$ .

# Conclusion and Future Work

- Type inference.
- Information flow analysis.
- Comparison with Seal calculus.
- Implementation.
- Logics.

