BiLog

A Framework for Structural Logics

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with G. Conforti and D. Macedonio

Spatial v Separation Logics

Spatial and Separation Logics are relatives but not twins.

- Spatial logics: Separation in space

$$\ell_1[\ a@\ell.P\ ] \mid \ell_2[\ \bar{a}@\ell.Q\ ]$$

- Separation logics: Separation of resources

$$\ell[\ a.nil \mid b.nil\ ]$$

- **Why not together?**

  A logic to support:

  - Multiple kinds of separation;
  - Multiple distribution models.

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A Unifying Logical Framework?

We need:

- An expressive framework –

E.G.

$$PC_a(in_c \otimes \mathbf{T}) \overset{c}{\otimes} PC_b(out_c \otimes \mathbf{T})$$

- A metalogical framework –

Language Independent

One general framework to capture multiple calculi and models.

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Bigraphs: A Universal Model

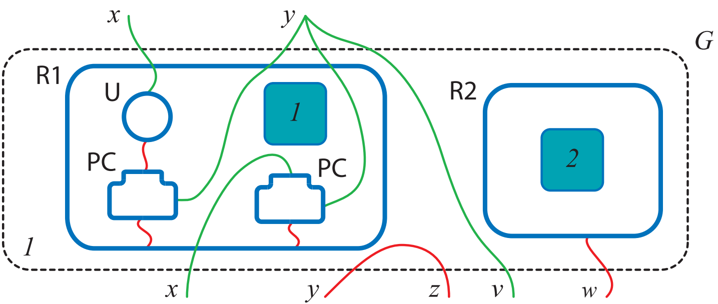

Figure: A bigraph $G : \langle 2, \{x, y, z, v, w\}\rangle \to \langle 1, \{x, y\}\rangle$.

The Bigraph $(\otimes, \circ)$-Algebra

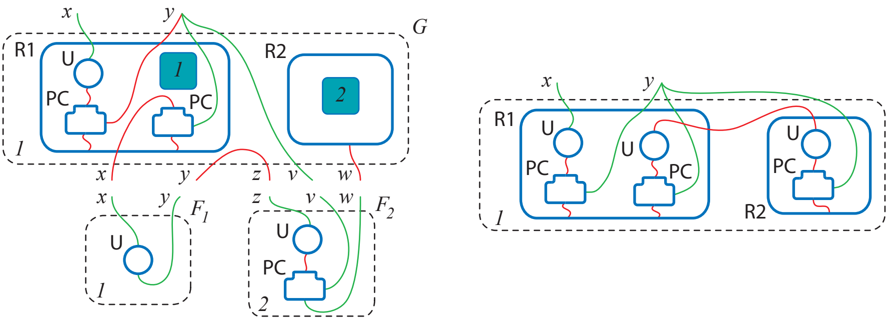

Figure: Bigraphical composition, $H \equiv G \circ (F_1 \otimes F_2)$.

A tensor-and-composition logic

- Two structural connectives: Horizontal $\otimes$ and Vertical $\circ$.
- Intuition: $\otimes$ separated parallel composition, no sharing. $\circ$ 'contextual' composition, with sharing.
- $(M, \otimes, \epsilon)$ a partial monoid of 'resources.'
  Tensor defined only if resources are 'disjoint.'
  Elements of discourse $G : I \to J$, with $\otimes$ lifted and $\circ$ as obvious.

BiLog$(M, \otimes, \epsilon, \Theta, \equiv, \tau)$

Ω ::= transparent constructors in Θ (ignore the opaque ones)

$A, B ::=$	$\mathbf{F}$	false	$A \Rightarrow B$	implication
	$\mathbf{id}$	identity	$\mathbf{\Omega}$	constants (transparent)
	$A \otimes B$	tensor product	$A \circ B$	composition
	$A \multimapinv B$	left comp. adjunct	$A \multimap B$	right comp. adjunct
	$A \otimes\!- B$	left prod. adjunct	$A -\!\otimes B$	right prod. adjunct

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The Forcing Relation

$G \models \mathbf{F}$ never
$G \models \mathbf{\Omega} \stackrel{def}{=} G \equiv \Omega$
$G \models \mathbf{id} \stackrel{def}{=} G \equiv id_I$

$G \models A \Rightarrow B \stackrel{def}{=} G \models A$ implies $G \models B$

$G \models A \otimes B \stackrel{def}{=} G \equiv G_1 \otimes G_2$ and $G_1 \models A$ and $G_2 \models B$
$G \models A \circ B \stackrel{def}{=} G \equiv G_1 \circ G_2$ and $\tau(G_1)$ and $G_1 \models A$ and $G_2 \models B$

$G \models A \circ\!\!- B \stackrel{def}{=} \forall G'.\ G' \models A, \tau(G')$ and $(G' \circ G)\downarrow$ implies $G' \circ G \models B$
$G \models A \multimap B \stackrel{def}{=} \tau(G), \forall G'.\ G' \models A$ and $(G \circ G')\downarrow$ implies $G \circ G' \models B$

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Some Useful Derived Operators

$\mathbf{T}, \wedge, \vee, \Leftrightarrow, \Leftarrow, \neg$ as usual for classical logics

A_I	$\stackrel{def}{=} A \circ \mathbf{id}_I$	Source to be I
$A_{\to J}$	$\stackrel{def}{=} \mathbf{id}_J \circ A$	Target to be J
$A_{I\to J}$	$\stackrel{def}{=} (A_I)_{\to J}$	Interface to be $I \to J$
$A \circ_I B$	$\stackrel{def}{=} A \circ \mathbf{id}_I \circ B$	Composition along interface I
$A \multimapinv_J B$	$\stackrel{def}{=} A_{\to J} \multimapinv B$	Contexts with J as target
$A \multimap_I B$	$\stackrel{def}{=} A_I \multimap B$	Composing with I as source
$A \ominus B$	$\stackrel{def}{=} \neg(\neg A \otimes \neg B)$	Dual of tensor product
$A \bullet B$	$\stackrel{def}{=} \neg(\neg A \circ \neg B)$	Dual of composition
$A^{\exists\otimes}$	$\stackrel{def}{=} \mathbf{T} \otimes A \otimes \mathbf{T}$	Some horizontal component satisfies A
$A^{\exists\circ}$	$\stackrel{def}{=} \mathbf{T} \circ A \circ \mathbf{T}$	Some vertical component satisfies A
$\square A$	$\stackrel{def}{=} A^{\exists\otimes\exists\circ}$	Somewhere modality
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$A^{\exists\circ}$	$\stackrel{def}{=} \mathbf{T} \circ A \circ \mathbf{T}$	Some vertical component satisfies A
$\Box A$	$\stackrel{def}{=} A^{\exists\otimes\exists\circ}$	Somewhere modality
$\blacksquare A$	$\stackrel{def}{=} \neg\Box\neg A$	Anywhere modality

Some Useful Derived Operators

$\mathbf{T}, \wedge, \vee, \Leftrightarrow, \Leftarrow, \neg$ as usual for classical logics

A_I	$\stackrel{def}{=}$	$A \circ \mathbf{id}_I$	Source to be I
$A_{\to J}$	$\stackrel{def}{=}$	$\mathbf{id}_J \circ A$	Target to be J
$A_{I \to J}$	$\stackrel{def}{=}$	$(A_I)_{\to J}$	Interface to be $I \to J$
$A \circ_I B$	$\stackrel{def}{=}$	$A \circ \mathbf{id}_I \circ B$	Composition along interface I
$A \circ\!\!-_J B$	$\stackrel{def}{=}$	$A_{\to J} \circ\!\!- B$	Contexts with J as target
$A \multimap_I B$	$\stackrel{def}{=}$	$A_I \multimap B$	Composing with I as source
$A \ominus B$	$\stackrel{def}{=}$	$\neg(\neg A \otimes \neg B)$	Dual of tensor product
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How new is all this?

Not very much, I'm afraid...

The news is that this is simply the (internal language of) a free (partial) monoidal category.

- Let **C** be the free partial monoidal category generated by $(M, \otimes, \epsilon)$ and $\Omega : I \to J$ in Θ.
- Let G range over the arrows of **C** and let $\equiv$ be equality in **C**.

Then:

Theorem

If all constants are transparent, then

$$G =_L G' \quad \textit{if and only if} \quad G \equiv G',$$

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BiLog as a Logic for Bigraphs

This amounts to instantiate the parameters

$$M, \quad \otimes, \quad \epsilon, \quad \Theta, \quad \equiv, \quad \tau$$

appropriately.

As bigraphs consist of two superimposed graph structures, the place graphs and the link graphs, it is convenient to describe BiLog by superimposing two logics:

The Place Graph Logic (PGL) and the Link Graph Logic (LGL)

Place Graph Logic

- Place graphs are ordered lists of labelled trees with holes. The labels of the trees correspond to controls K belonging to the fixed signature $\mathcal{K}$.
- Monoid $(\mathbb{N}, +, 0)$: interfaces here represent the number of holes and regions of place graphs.
- Θ contains:

$$1 : 0 \to 1, \quad id_n : n \to n, \quad join : 2 \to 1,$$
$$\gamma_{m,n} : m + n \to n + m, \quad K : 1 \to 1, \text{for } K \in \mathcal{K}.$$

- $\equiv$ is augmented with the usual axioms for symmetry of $\gamma_{m,n}$ and commutativity of $join \circ (\_ \otimes \_)$.

PGL

Fixed the transparency predicate τ, $PGL(\mathcal{K}, \tau)$ is

$$BiLog(\mathbb{N}, +, 0, \equiv, \mathcal{K} \cup \{1, join, \gamma_{m,n}\}, \tau).$$

Encoding STL into PGL

BiLog restricted to prime ground place graphs (and the always-true transparency predicate) is equivalent to the propositional spatial tree logic (STL).

$$T ::= 0 \quad | \quad a[T] \quad | \quad T_1 \mid T_2.$$

$$A ::= 0 \quad | \quad a[A] \quad | \quad A \mid B \quad | \quad A@a \quad | \quad A \triangleright B.$$

Trees into Prime Ground Place Graphs

$$[\![0]\!] \stackrel{def}{=} 1 \qquad [\![a[T]]\!] \stackrel{def}{=} K(a) \circ [\![T]\!] \qquad [\![T_1 \mid T_2]\!] \stackrel{def}{=} join \circ ([\![T_1]\!] \otimes [\![T_2]\!])$$

STL formulae into PGL formulae

$$[\![\mathbf{0}]\!] \stackrel{def}{=} \mathbf{1} \qquad [\![a[A]]\!] \stackrel{def}{=} \mathbf{K}(a) \circ_1 [\![A]\!]$$

$$[\![\mathbf{F}]\!] \stackrel{def}{=} \mathbf{F} \qquad [\![A@a]\!] \stackrel{def}{=} \mathbf{K}(a) \circ\!\!-_1 [\![A]\!]$$

$$[\![A \Rightarrow B]\!] \stackrel{def}{=} [\![A]\!] \Rightarrow [\![B]\!] \qquad [\![A \mid B]\!] \stackrel{def}{=} [\![A]\!] \mid [\![B]\!]$$

$$[\![A \triangleright B]\!] \stackrel{def}{=} ([\![A]\!] \mid \mathbf{id}_1) \circ\!\!-_1 [\![B]\!]$$

where $A \mid B \stackrel{def}{=} \mathbf{join} \circ (A_{\to 1} \otimes B_{\to 1})$ is the *parallel composition*

Encoding STL into PGL (ctd)

Theorem

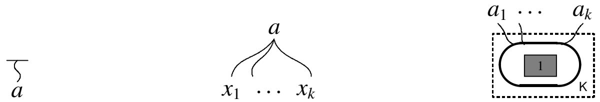
For all trees T and formulae A of STL we have that

$$T \models_{STL} A \quad \text{if and only if} \quad [\![T]\!] \models [\![A]\!].$$

Link Graph Logic

- Link graphs are simply directed graphs where nodes (aka ports) are connected by named links.
- Monoid $(\mathcal{P}(\Lambda), \uplus, \emptyset)$, for Λ a set of names and $\uplus$ is the union on disjoint pairs of sets and undefined otherwise.
- Θ contains ions and wirings, to map inner name to outer names.

$$/a : \{a\} \to \emptyset \qquad {}^{a}/_{X} : \{a\} \to X \qquad K_{\vec{a}} : \emptyset \to \vec{a}$$



closure $/a$ substitution ${}^{a}/_{X}$ discrete ion $\mathsf{K}_{\vec{a}}$

Important special cases for substitutions:

$$a \stackrel{def}{=} {}^{a}/_{\emptyset}; \quad a \leftarrow b \stackrel{def}{=} {}^{a}/_{\{b\}}; \quad a \Leftarrow b \stackrel{def}{=} {}^{a}/_{\{a,b\}}.$$

Link Graph Logic (ctd)

LGL

Fixed the transparency predicate τ, $LGL(\mathcal{K}, \tau)$ is

$$BiLog(\mathcal{P}(\Lambda), \uplus, \emptyset, \equiv, \mathcal{K} \cup \{/a, {}^{a}/_{X}\}, \tau).$$

Resource Sharing in LGL

Fixed G and G', let $\vec{b}$ be fresh names (as many as $\vec{a}$).

$$G \overset{\vec{a}}{\otimes} G' \overset{def}{=} [\vec{a} \Leftarrow \vec{b}] \circ (([\vec{b} \leftarrow \vec{a}] \circ G) \otimes G')$$

At the level of formulae, we need a fresh name quantification in the style of Nominal Logic

$$G \models \text{И}\vec{x}.\, A \overset{def}{=} \exists \vec{a} \notin fn(G) \cup fn(A).\, G \models A\{\vec{x} \leftarrow \vec{a}\}$$

using which we can define $\vec{a}$-linked name quantification for fresh names in order to explore names linked to $\vec{a}$:

$$\vec{a}\,\mathbf{L}\,\vec{x}.\, A \overset{def}{=} \text{И}\vec{x}.\, ((\vec{a} \Leftarrow \vec{x}) \otimes \mathbf{id}) \circ A.$$

Finally, we can define separation-upto connective.

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Encoding SGL into LGL

LGL can be seen as a contextual (and multi-edge) version of Spatial Graph Logic (SGL).

$$G ::= \quad nil \quad | \quad a(x,y) \quad | \quad G_1 \mid G_2 \quad | \quad (\nu x)G.$$

$$\phi ::= \quad \mathbf{nil} \quad | \quad a(x,y) \quad | \quad \mathbf{F} \quad | \quad \phi \Rightarrow \psi \quad | \quad \phi \mid \psi$$

Spatial Graphs into two-ported Ground Link Graphs

$$[\![nil]\!]_X \stackrel{def}{=} X \qquad [\![a(x,y)]\!]_X \stackrel{def}{=} K(a)_{x,y} \otimes X \setminus \{x,y\}$$

$$[\![G \mid G']\!]_X \stackrel{def}{=} [\![G]\!]_X \overset{X}{\otimes} [\![G']\!]_X \qquad [\![(\nu x)G]\!]_X \stackrel{def}{=} ((/x \otimes id_{X\setminus\{x\}}) \circ [\![G]\!]_X) \otimes x$$

SGL formulae into LGL formulae

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$$[\![\mathbf{F}]\!]_X \stackrel{def}{=} \mathbf{F} \qquad [\![\phi \Rightarrow \psi]\!]_X \stackrel{def}{=} [\![\phi]\!]_X \Rightarrow [\![\psi]\!]_X$$

$$[\![\phi \mid \psi]\!]_X \stackrel{def}{=} [\![\phi]\!]_X \overset{X}{\otimes} [\![\psi]\!]_X$$

Encoding SGL into LGL (ctd)

Theorem

For all graphs G and formulae ϕ of the propositional fragment of SGL, we have that

$$G \models_{SGL} \phi \quad \textit{if and only if} \quad [\![G]\!]_X \models [\![\phi]\!]_X .$$

Bigraph Logic, finally

- The product monoid $(\mathbb{N} \times \mathcal{P}(\Lambda), \otimes, \epsilon)$ where $\langle m, X \rangle \otimes \langle n, X \rangle \stackrel{def}{=} \langle m + n, X \uplus Y \rangle$ and $\epsilon \stackrel{def}{=} \langle 0, \emptyset \rangle$.
- Θ is the union of place and link graph constructors, but the controls are replaced by the new *discrete ion* constructor $K_{\vec{a}} : 1 \rightarrow \langle 1, \vec{a} \rangle$; this is a prime bigraph containing a single node with ports named $\vec{a}$ and an hole inside.
- $\equiv$ for link graphs is refined with obvious axioms for links, modelling α-conversion and extrusion of closed names.

Theorem

There exists an enconding of the Context Tree Logic into BGL which extends the encoding of STL into PGL.

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Deriving a 'next-step' modality?

- In process algebras the dynamics is often presented by reaction (or rewriting) rules of the form $r \longrightarrow\!\!\triangleright\ r'$. The same happens with bigraphical reactive systems:

$$\mathsf{act}_a[\,]_1 \mid \mathsf{coact}_a[\,]_2 \longrightarrow\!\!\triangleright\ [\,]_1 \mid [\,]_2$$

  No reaction can occur inside the controls act and coact, as they are passive. In general, we have an activeness predicate δ on contexts.
- A ground bigraph g reduces to g' if there is $(R, R') \in S$, a set of names Y, a bigraph D with $\delta(D)$ true, and a ground bigraph d, such that:

$$g \equiv D \circ (R \otimes id_Y) \circ d \quad \text{and} \quad g' \equiv D \circ (R' \otimes id_Y) \circ d.$$

- The next step modality

$$g \models \Diamond A \quad \text{iff} \quad g \longrightarrow\!\!\triangleright\ g' \text{ and } g' \models A.$$

  can then in some circumstances be expressed in BiLog.

Modelling a small CCS

Let's assume there is only a finite set of actions X. The following encoding yields bigraphs with the outer faces $\langle 1, X\rangle$.

$$
\begin{aligned}
[\![\mathbf{0}]\!] &\stackrel{def}{=} X;\\
[\![a.P]\!] &\stackrel{def}{=} (act_a \overset{a}{\otimes} id_X) \circ [\![P]\!];\\
[\![\bar{a}.P]\!] &\stackrel{def}{=} (coact_a \overset{a}{\otimes} id_X) \circ [\![P]\!];\\
[\![P \mid Q]\!] &\stackrel{def}{=} join \circ ([\![P]\!] \overset{X}{\otimes} [\![Q]\!]).
\end{aligned}
$$

We have $P \longrightarrow\!\!\triangleright\ P'$ if and only if $[\![P]\!] \longrightarrow\!\!\triangleright\ [\![P']\!]$.

The logic $\mathcal{L}_{spat}$:

$$A, B ::= \quad 0 \quad \mid \quad A \wedge B \quad \mid \quad A \mid B \quad \mid \quad \neg A \quad \mid \quad A \triangleright B \quad \mid \quad \Diamond A.$$

Theorem

$\mathcal{L}_{spat}$ can be encoded into $BiLog(M, \otimes, \epsilon, \Theta, \equiv, \tau)$ where Θ is standard, with $\mathcal{K} = \{act, coact\}$, and τ is always true.

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The Encoding of $\mathcal{L}_{spat}$ into Bilog

Rules $(R_a, R'_a) = (a.[\,]_1 \mid \bar{a}.[\,]_2 \,,\, [\,]_1 \mid [\,]_2)$, for every $a \in X$.

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\begin{aligned}
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[\![\, A \triangleright B \,]\!] &\stackrel{def}{=} \text{И}X'.\, (((X' \leftarrow X) \otimes id_1) \circ [\![\, A \,]\!]_\epsilon) -\!\!\otimes \\
&\qquad (\mathbf{join} \circ ((X \Leftarrow X') \otimes id_1) \circ\!\!- [\![\, B \,]\!]) \\
[\![\, \Diamond A \,]\!] &\stackrel{def}{=} \bigvee_{a \in X} \tilde{R}_a \circ [(\tilde{R}'_a \circ\!\!- [\![\, A \,]\!]) \wedge (\mathbf{T}_{\langle 1, X \rangle} \otimes \mathbf{T})]
\end{aligned}
$$

Theorem

For every CCS process P,

$$P \models_{spat} A \quad \text{if and only if} \quad [\![\, P \,]\!] \models [\![\, A \,]\!].$$

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[\![A \triangleright B]\!] &\stackrel{def}{=} \text{И} X'. \left(((X' \leftarrow X) \otimes id_1) \circ [\![A]\!]_\epsilon\right) \multimap\!\!\otimes \\
&\qquad\qquad (\mathbf{join} \circ ((X \leftleftarrows X') \otimes id_1) \circ\!\!- [\![B]\!]) \\
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Further work

A metalogical framework for static and dynamic resources, controlling sharing and separation.

- Place Graph Logic, Link Graph Logic, BiGraph Logic
- Spatial Tree Logic, Spatial Graph Logic, Context Tree Logic
- XML and Web Services

To be done:

- The role of transparency.
- Proof theory (not obvious how a completeness result can be obtained).
- Model checking for significant fragments (within the boundaries fixed by existing undecidability results).
- Monadic second order quantification for dynamics.

$$\Diamond A \quad \stackrel{def}{=} \quad \exists X.(Act \wedge X) \circ (R \circ [\,(X \circ R')\, \circ\!\!- A\,])$$