A Calculus of Bounded Capacities

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Global Computing and Ambient Intelligence involve scenarios where mobile devices enter and exit domains and networks.

Typical Devices:
Today: Smart Cards, Embedded devs (e.g. in cars), Mobile phones, PDAs, Sat navigators, ...
Tomorrow: PAN, VAN, D-ME, P-COM, ...
The Case for Resource Usage Control

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Requirements:
- **Security:** Authentication, Privacy, Non Repudiation
- **Trust Formation and Management**
- **Context (e.g. Location) Awareness**
- **Dynamic Learning and Adaptability**
- **Policies of Access Control and their Enforcement**
- **Negotiation of Access, Access Rights, Resource Acquisition**
- **Protection of Resource Bounds**
The Case for Resource Usage Control

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Central Notion:
Resource Usage
The Case for Resource Usage Control

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Central Notion: Resource Usage

Our Focus: Capacity Bounds Awareness.
Dimensions, Capacities, Mobility

BoCa: Bounded Capacities

- Subjective Mobility
- Bounded Capacity Ambients
- Space as a linear co-capability.
- Fine control of capacity.

\[
\text{subjective move capability} \quad \text{space co-capability}
\]
Minimal Desiderata

- **Realistic** about space occupation. Bigger processes take more space.

\[ n[\text{in } m. \text{big and fat } P] \mid m[\_] \mid n[\text{in } m. \text{small and slim } P] \]

- **Replication** must be handled appropriately

\[ a[!P] = a[!P \mid P] = a[!P \mid P \mid P] = a[!P \mid P \mid P \mid P] = \ldots \]

Allow an analysis of variation in space occupation

- More precisely, control **process spawning**.

  Computation takes space, dynamically, and we’d like to model it.
A Calculus of Bounded Capacities: Movement

Fundamentals: Space Conscious Movement

\[
\begin{align*}
  &a[\text{in} \, b.P \mid Q] \mid b[\neg \mid R] \Downarrow \neg \mid b[a[P \mid Q] \mid R] \\
  &\neg \mid b[a[\text{out} \, b.P \mid Q] \mid R] \Downarrow a[P \mid Q] \mid b[\neg \mid R]
\end{align*}
\]
A Calculus of Bounded Capacities: Movement

Fundamentals: Space Conscious Movement

\[ a[\text{in} b . P | Q] | b[ - | R] \Downarrow - | b[ a[ P | Q] | R] \]
\[ - | b[ a[\text{out} b . P | Q] | R] \Downarrow a[ P | Q] | b[ - | R] \]

Example: Travelling needs but consumes no space.

\[ a[\text{in} b . \text{in} c . \text{out} c . \text{out} b . 0] | b[ - | c[ -]] \]
\[ \Downarrow \Downarrow - | b[ - | c[ a[ \text{out} c . \text{out} b . 0] ]]] \]
\[ \Downarrow \Downarrow a[ 0] | b[ - | c[ -]] \]
A Calculus of Bounded Capacities: Well-formedness

Fundamentals: Space Conscious Movement

But the size of travellers matters!

\[
a^k \{ \text{in} b \cdot P \mid Q \} \mid b[a \mid \ldots \mid a \mid R] \Downarrow \quad k \times \quad\]
\[
\Downarrow \quad k \times \quad\]
\[
\Downarrow \quad k \times \quad\]

Workshop on GC – pp.6/20
But the size of travellers matters!

What is the $a^k$? A well-formedness annotation measuring the size of $P$.

It counts spaces: $\text{weight}(\_\_\_) = 1$, $\text{weight}(a^k[ P ]) = k$ if $\text{weight}(P) = k$, $\perp$ otherwise.

Reduction only for well-formed terms: (1) weights appear as conditions on reductions; (2) the calculus’ operators make only sense with type annotations.

Notation. We use $\_\_\_^k$ as a shorthand for $\_\_\_ | \ldots | \_\_\_$. 
A Calculus of Bounded Capacities: Open

Fundamentals: Space Conscious Opening

\[ \text{opn} a \cdot P \mid a^k [ \text{opn} \cdot Q \mid R ] \Downarrow P \mid Q \mid R \]
A Calculus of Bounded Capacities: Open

Fundamentals: Space Conscious Opening

\[
\text{opn} a \cdot P \mid a^k [ \text{opn} \cdot Q \mid R ] \quad \triangleright \quad P \mid Q \mid R
\]

Example: Recovering Mobile Ambients.

\[
\llbracket a \llbracket P \rrbracket \rrbracket \triangleq a^0 [ \text{opn} \mid \llbracket P \rrbracket ]
\]

\[
\llbracket (\nu a) P \rrbracket \triangleq (\nu a^0) \llbracket P \rrbracket
\]

\ldots
A Calculus of Bounded Capacities: Spawning

Fundamentals: Space Conscious Process Activation

\[\bigtriangledown^k P \mid \downarrow^k \equiv P\]
A Calculus of Bounded Capacities: Spawning

Fundamentals: Space Conscious Process Activation

passive process: \( P \)

weighs 0

\( P \) weighs \( k \)

\( k \)
A Calculus of Bounded Capacities: Spawning

Fundamentals: Space Conscious Process Activation

Example: Replication: \(!^k A \triangleq !^k \triangleright^k\)

Types ensure only 0-weighted processes are replicable: One must use spawning, so that replication needs space proportional to the process’ weight.
A Calculus of Bounded Capacities: Spawning

Fundamentals: Space Conscious Process Activation

Example: Replication: !^k \triangleq !\triangleright^k

Types ensure only 0-weighted processes are replicable: One must use spawning, so that replication needs space proportional to the process’ weight.

Example: Recursion (well, almost):

\[
\text{rec}(X^k)P \triangleq (\nu X^k)(!\text{opn} X \cdot \triangleright^k \widehat{P} | X[\_^k]), \quad \text{where} \quad \widehat{P} \triangleq P\{X[\_^k]/X\}
\]
Example: Ambient Spawning

$$spw^k b[ P ] \triangleq \exp^0 [ \text{out } a \cdot \text{opn} \cdot \triangleright^k b[ P ] ]$$

Then,

$$a[ spw^k b[ P ] \mid Q ] \mid \triangleright^k \mid \text{opn} \exp \searrow a[ Q ] \mid b[ P ].$$

The father must provide enough space for the activation, of course.
**BoCa: Examples (Open)**

**Example: Ambient Spawning**

\[
\text{spw}^k b[P] \triangleq \exp^0[\text{out} a \cdot \overline{\text{opn}} \cdot \triangleright^k b[P]]
\]

Then,

\[
a[\text{spw}^k b[P] | Q] | \triangleleft^k | \text{opn} \exp \downarrow a[Q] | b[P].
\]

The father must provide enough space for the activation, of course.

**Example: Ambient Renaming**

\[
a \cdot \text{be} \cdot \text{b}^k \cdot P \triangleq \text{spw}_a b[\triangleleft^k | \text{opn} a] | \text{in} b \cdot \overline{\text{opn}} \cdot P.
\]

Then,

\[
\triangleleft^k | \text{opn} x | a^k[a \cdot \text{be} \cdot \text{b}^k \cdot P | Q] \downarrow b[P | Q] | \triangleleft^k.
\]

Ambient \(a\) needs to **borrow** space to rename itself.
A Calculus of Bounded Capacities: Transfer

Fundamentals: Space Acquisition and Release

\[ a^\hat{.} P \mid \_ \mid a^k \langle \_ \rangle Q \mid R \ \Rightarrow \ \ P \mid a^{k+1} Q \mid \_ \mid R \]

\[ a^{k+1} [ \langle . P \mid \_ \mid S \rangle \mid b^h [ a] \rangle Q \mid R \ \Rightarrow \ a^k [ P \mid S \rangle \mid b^{h+1} [ Q \mid \_ \mid R \]

Example: A Memory Module

memMod, mem[256 M B] \rightarrow mem, malloc, m[\_] mem[free] = malloc & 256 M B mem[\_] free \rightarrow memMod\_malloc \& 256 M B mem[\_] free \rightarrow memMod\_malloc \& 256 M B
A Calculus of Bounded Capacities: Transfer

Fundamentals: Space Acquisition and Release

\[ a \cdot P | \_ | a^k[ \_ . Q | R ] \Downarrow P | a^{k+1}[ Q | \_ | R ] \]

\[ a^{k+1}[ \_ . P | \_ | S ] | b^h[ a] . Q | R ] \Downarrow a^k[ P | S ] | b^{h+1}[ Q | \_ | R ] \]

Transfer from Child:

\[ \text{get\_from\_child} \ a . P \triangleq (\nu n)(\text{open} \ n . P | n[ a].\text{open} ) \]
A Calculus of Bounded Capacities: Transfer

Fundamentals: Space Acquisition and Release

\[ a \cdot P | \_ | a^k [ \_ . Q | R ] \triangleright P | a^{k+1} [ Q | \_ | R ] \]

\[ a^{k+1} [ \_ . P | \_ | S ] | b^h [ a] . Q | R ] \triangleright a^k [ P | S ] | b^{h+1} [ Q | \_ | R ] \]

Transfer from Child:

\[ \text{get\_from\_child } a . P \triangleq (\nu n)(\text{opn } n . P | n[a] \text{\_opn }) \]

Example: A Memory Module

\[ \text{memMod} \triangleq \text{mem} [ \_ ^{256 MB} | !\_ | !\text{free} ] \]

\[ \text{malloc} \triangleq m[ !\text{mem} ] . \text{free} [ \text{out} m . m ] . \_ | !\_ ] \]
A Calculus of Bounded Capacities: Transfer

Fundamentals: Space Acquisition and Release

\[ a^\hat{\cdot} P \mid \perp a^k [ \perp \cdot Q \mid R ] \downarrow P \mid a^{k+1} \{ Q \mid \perp \mid R \} \]

\[ a^{k+1} [ \perp \cdot P \mid \perp \mid S ] \mid b^h [ a\} \cdot Q \mid R ] \downarrow a^k [ P \mid S ] \mid b^{h+1} \{ Q \mid \perp \mid R \} \]

Transfer from Child:

\[ \text{get_from_child } a \cdot P \triangleq (\nu n)(\text{opn} n \cdot P \mid n[ a\} \cdot \text{opn}]) \]

Example: A Memory Module

\[ \text{memMod} \triangleq \text{mem} [ \perp 256MB \mid !\ll | !\text{free} ] \]

\[ \text{malloc} \triangleq m[ !\text{mem} \} . \text{free} [ \text{out} m . m \} . !\} | !\ll ] \]

\[ \text{memMod} \mid \text{malloc} \downarrow 256MB \text{mem} [ !\ll \mid !\text{free} ] \mid m[ \perp 256MB \mid \ldots ] \downarrow 2 \times 256MB \]

\[ \text{mem} [ !\ll \mid !\text{free} ] \mid \text{malloc} \mid \text{free} 256MB [ \perp \mid !\} \downarrow 256MB \text{memMod} \mid \text{malloc} \mid \ldots \]
On the nature of space

An economic vehicle for multiple concepts

- Available space: \( a[\_ | P ] \)

- Occupied space: \( M . \_ \). (Notation: \( M . \spadesuit . \))

- Lost space: \((\nu a)a^k[\_^k \_]\). (Notation: \(0^k\).)

\[
\text{destroy}^k \triangleq (\nu a)\underbracket{a \ldots a}_{k \text{ times}}.0 \mid a^0[\underbracket{\ldots \ldots \ldots \ldots}{k \text{ times}}.0 ]
\]

\[
\text{destroy}^k \mid _{k} \bigtriangleup^k 0^k
\]
A Calculus of Bounded Capabilities: Syntax

\[ P ::= \text{true} \mid \text{false} \mid 0 \mid M \cdot P \mid P \mid P \mid M[ P ] \mid !P \mid \nabla^k P \mid (\nu n : \pi)P \mid (x : \chi)P \mid \langle M \rangle P \]

\[ C ::= \text{in} M \mid \text{out} M \mid \text{opn} M \mid M^{\hat{\cdot}} \mid « \]

\[ \overline{C} ::= \text{opn} \mid \overline{\cdot} \mid M» \]

\[ M ::= \varepsilon \mid x \mid C \mid \overline{C} \mid M \cdot M \]
A Calculus of Bounded Capabilities: Syntax

\[ P ::= \| | 0 | M \cdot P | P | P | M[ P ] | !P | \text{Diamond}^k P | (\nu n : \pi)P | (x : \chi)P | \langle M \rangle P \]

\[ C ::= \text{in } M | \text{out } M | \text{opn } M | M^\hat{\cdot} | \langle \rangle \]

\[ \overline{C} ::= \text{opn} | \hat{\cdot} | M \rangle \]

\[ M ::= \varepsilon | x | C | \overline{C} | M \cdot M \]

**Structural Congruence:**

\[ (\|, 0) \text{ is a commutative monoid.} \]

\[ (\nu a)(P | Q) \equiv (\nu a)P | Q \quad \text{if } a \not\in \text{fn}(Q) \]

\[ (\nu a)0 \equiv 0 \]

\[ (\nu a)\langle M \rangle P \equiv \langle M \rangle(\nu a)P \quad \text{if } a \not\in \text{fn}(P) \]

\[ (\nu a)(\nu b)P \equiv (\nu b)(\nu a)P \]

\[ a[ (\nu b)P ] \equiv (\nu b)a[ P ] \quad \text{if } a \neq b \]

\[ !P \equiv !P | P \]
BoCa: Reduction Semantics

(enter) \[ a^k [ \text{in} b . P | Q ] | b[\cdot^k | R ] \downarrow b[ a [ P | Q ] | R ] \]

(exit) \[ \cdot^k | b[a^k [ \text{out} b . P | Q ] | R ] \downarrow a^k [ P | Q ] | b[\cdot^k | R ] \]

(open) \[ \text{opn} a . P | a[\text{opn} . Q | R ] \downarrow P | Q | R \]

(tranD) \[ a \hat{\cdot} . P | \cdot | a^k [ \hat{\cdot} . Q | R ] \downarrow P | a^{k+1} [ Q | R ] \]

(trans) \[ a^{k+1} [ \langle . P | \cdot | S ] | b^h [ a \rangle . Q | R ] \downarrow a^k [ P | S ] | b^{h+1} [ Q | R ] \]

(spawn) \[ \triangledown^k P | \cdot^k \downarrow P \]

(comm) \[ (x : \chi)P | \langle M \rangle Q \downarrow P\{x \leftarrow M\} | Q \]
A System of Capacity Types

Capacity Types: $\phi, \ldots$ are pairs of nats $[n, N]$, with $n \leq N$.

Effect Types $\mathcal{E}, \ldots$ are pairs of nats $(d, i)$, representing $\text{dec}$s and $\text{inc}$s.

Exchange Types: $\chi ::= \text{Shn} \mid \text{Amb} \langle \sigma, \chi \rangle \mid \text{Cap} \langle \mathcal{E}, \chi \rangle$

Process and Ambient and Capability Types:

- $a : \text{Amb} \langle \phi, \chi \rangle$ $a$ has no less than $\phi_m$ and no more than $\phi_M$ spaces
- $P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle$ $P$ weighs $k$ and produces the effect $\mathcal{E}$ on ambients
- $C : \text{Cap} \langle \mathcal{E}, \chi \rangle$ $C$ transforms processes adding $\mathcal{E}$ to their effects

Effects and capacities componentwise and are ordered as follows:

$$\sigma < \phi \equiv \phi_m \leq \sigma_m \text{ and } \sigma_M \leq \phi_M,$$
A Typing System: Capabilities

(Axiom)
\[ \Gamma, a : \text{Amb}(\phi, \chi) \vdash a : \text{Amb}(\phi, \chi) \]

(Empty)
\[ \Gamma \vdash \varepsilon : \text{Cap}(\langle 0, 0 \rangle, \chi) \]

(In)
\[ \Gamma \vdash M : \text{Amb}(\phi, \chi') \]
\[ \Gamma \vdash \text{in} M : \text{Cap}(\langle 0, 0 \rangle, \chi) \]

(Out)
\[ \Gamma \vdash M : \text{Amb}(\phi, \chi') \]
\[ \Gamma \vdash \text{out} M : \text{Cap}(\langle 0, 0 \rangle, \chi) \]

(TranD)
\[ \Gamma \vdash M : \text{Amb}(\phi, \chi') \]
\[ \Gamma \vdash \hat{M} : \text{Cap}(\langle 0, 0 \rangle, \chi) \]

(TranS)
\[ \Gamma \vdash \text{«} : \text{Cap}(\langle 1, 0 \rangle, \chi) \]

(Open)
\[ \Gamma \vdash M : \text{Amb}(\langle n, N \rangle, \chi) \]
\[ \Gamma \vdash \text{opn} M : \text{Cap}(\langle N - n, N - n \rangle, \chi) \]
A Typing System: CoCapabilities and Processes

(coTranD)

\[ \Gamma \vdash \tau : \text{Cap} \langle (0, 1), \chi \rangle \]

(coTranS)

\[ \Gamma \vdash M : \text{Amb} \langle \phi, \chi' \rangle \]
\[ \Gamma \vdash M \triangleright : \text{Cap} \langle (0, 1), \chi \rangle \]

(coOpen)

\[ \Gamma \vdash \text{opn} : \text{Cap} \langle (0, 0), \chi \rangle \]

(Composition)

\[ \Gamma \vdash M : \text{Cap} \langle \mathcal{E}, \chi \rangle \]
\[ \Gamma \vdash M' : \text{Cap} \langle \mathcal{E}', \chi \rangle \]
\[ \Gamma \vdash M.M' : \text{Cap} \langle \mathcal{E} + \mathcal{E}', \chi \rangle \]

(Slot)

\[ \Gamma \vdash \_ : \text{Proc} \langle 1, (0, 0), \chi \rangle \]

(Zero)

\[ \Gamma \vdash \text{0} : \text{Proc} \langle 0, (0, 0), \chi \rangle \]

(Input)

\[ \Gamma, x : \chi \vdash P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle \]
\[ \Gamma \vdash (x : \chi)P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle \]

(Output)

\[ \Gamma \vdash M : \chi \]
\[ \Gamma \vdash P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle \]
\[ \Gamma \vdash \langle M \rangle P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle \]
A Typing System: Processes

(Prex)
\[
\Gamma \vdash M : \text{Cap}(\mathcal{E}, \chi) \quad \Gamma \vdash P : \text{Proc}(k, \mathcal{E}', \chi)
\]
\[
\Gamma \vdash M \cdot P : \text{Proc}(k, \mathcal{E} + \mathcal{E}', \chi)
\]

(Replication)
\[
\Gamma \vdash P : \text{Proc}(0, (0, 0), \chi)
\]
\[
\Gamma \vdash !P : \text{Proc}(0, (0, 0), \chi)
\]

(New)
\[
\Gamma, a : \text{Amb}(\phi, \chi) \vdash P : \text{Proc}(k, \mathcal{E}, \chi')
\]
\[
\Gamma \vdash (\nu a : \text{Amb}(\phi, \chi)) P : \text{Proc}(k, \mathcal{E}, \chi')
\]

(Spawn)
\[
\Gamma \vdash P : \text{Proc}(k, \mathcal{E}, \chi)
\]
\[
\Gamma \vdash \triangledown^k P : \text{Proc}(0, \mathcal{E}, \chi)
\]

(Parallel)
\[
\Gamma \vdash P : \text{Proc}(k, \mathcal{E}, \chi) \quad \Gamma \vdash Q : \text{Proc}(k', \mathcal{E}', \chi)
\]
\[
\Gamma \vdash P \mid Q : \text{Proc}(k + k', \mathcal{E} + \mathcal{E}', \chi)
\]

(Ambient)
\[
\Gamma \vdash M : \text{Amb}([n, N], \chi) \quad \Gamma \vdash P : \text{Proc}(k, (d, i), \chi) \quad n \leq k - d \quad k + i \leq N
\]
\[
\Gamma \vdash M^k[P] : \text{Proc}(k, (0, 0), \chi')
\]
Thm: Subject Reduction

If \( \Gamma \vdash P : \text{Proc}\langle k, \mathcal{E}, \chi \rangle \) and \( P \Downarrow Q \) then \( \Gamma \vdash Q : \text{Proc}\langle k, \mathcal{E}', \chi \rangle \) for some \( \mathcal{E}' < \mathcal{E} \).
A Calculus of Bounded Capabilities

Thm: Subject Reduction

If $\Gamma \vdash P : \text{Proc}(k, E, \chi)$ and $P \xrightarrow{\text{p}} Q$ then $\Gamma \vdash Q : \text{Proc}(k, E', \chi)$ for some $E' < E$.

The missing bit:

Grave interferences in the use of spaces

$$a[\mathtt{in} \ b \ ] \mid b[\triangleright P \mid \_ \ ] \mid a[\mathtt{out} \ a \ ] \ ] $$
**A Calculus of Bounded Capabilities**

**Thm: Subject Reduction**

If $\Gamma \vdash P : \text{Proc}(k, \mathcal{E}, \chi)$ and $P \Downarrow Q$ then $\Gamma \vdash Q : \text{Proc}(k, \mathcal{E}', \chi)$ for some $\mathcal{E}' < \mathcal{E}$.

The missing bit:

Grave interferences in the use of spaces

\[
\begin{array}{c}
\text{a[ in b ]} \\
\text{b[ P[\text{\textgreater{}}}]
\end{array}
\]

\[
\begin{array}{c}
\text{a[ out a ]}
\end{array}
\]
**A Calculus of Bounded Capabilities**

**Thm: Subject Reduction**

If $\Gamma \vdash P : \text{Proc}\langle k, E, \chi \rangle$ and $P \Downarrow Q$ then $\Gamma \vdash Q : \text{Proc}\langle k, E', \chi \rangle$ for some $E' < E$.

The missing bit:

Grave interferences in the use of spaces

\[
\text{rec}(X^k)P \triangleq (\nu X^k)(!\text{opn} \cdot X^k \triangleright \hat{P} | X[\_^k])
\]
A Calculus of Bounded Capabilities

Thm: Subject Reduction
If $\Gamma \vdash P : \text{Proc}(k, \mathcal{E}, \chi)$ and $P \xrightarrow{\cdot} Q$ then $\Gamma \vdash Q : \text{Proc}(k, \mathcal{E}', \chi)$ for some $\mathcal{E}' \prec \mathcal{E}$.

The missing bit:

Grave interferences in the use of spaces

\[
\text{rec}(X^K)P \triangleq (\nu X^K)(!\text{opn} X \cdot \triangleleft^K \widehat{P} \mid X[-^K])
\]

\[
\xrightarrow{\cdot} (\nu X^K)(!\text{opn} X \cdot \triangleleft^K \widehat{P} \mid \text{opn} X \cdot \triangleleft^K \widehat{P} \mid X[-^K])
\]
Thm: Subject Reduction
If $\Gamma \vdash P : \text{Proc}\langle k, \mathcal{E}, \chi \rangle$ and $P \downarrow Q$ then $\Gamma \vdash Q : \text{Proc}\langle k, \mathcal{E}', \chi \rangle$ for some $\mathcal{E}' < \mathcal{E}$.

The missing bit:

Grave interferences in the use of spaces

\[
\text{rec}(X^k)P \triangleq (\nu X^k)(!\text{opn } X \Rightarrow^k \hat{P} \mid X[-^k])
\]

\[
\downarrow (\nu X^k)(!\text{opn } X \Rightarrow^k \hat{P} \mid \text{opn } X \Rightarrow^k \hat{P} \mid X[-^k])
\]

\[
\downarrow (\nu X^k)(!\text{opn } X \Rightarrow^k \hat{P} \mid \Rightarrow^k \hat{P} \mid [-^k])
\]
A Calculus of Bounded Capabilities

Thm: Subject Reduction
If $\Gamma \vdash P : \text{Proc} \langle k, \mathcal{E}, \chi \rangle$ and $P \xrightarrow{} Q$ then $\Gamma \vdash Q : \text{Proc} \langle k, \mathcal{E}', \chi \rangle$ for some $\mathcal{E}' < \mathcal{E}$.

The missing bit:

Grave interferences in the use of spaces

\[
\begin{align*}
\text{rec}(X^k)P & \triangleq (\nu X^k)(!\text{opn } X . \triangleright^k \hat{P} \mid X[\neg k]) \\
\triangleright (\nu X^k)(!\text{opn } X . \triangleright^k \hat{P} \mid \text{opn } X . \triangleright^k \hat{P} \mid X[\neg k]) \\
\triangleright (\nu X^k)(!\text{opn } X . \triangleright^k \hat{P} \mid \triangleright^k \hat{P}) \mid \neg^k \quad \text{Oooops}
\end{align*}
\]
Control Space Usage: Named Slots

\[ P ::= \_a \mid a \triangleright^k P \mid \ldots \quad \text{(spawn)} \quad a \triangleright^k P \mid \_a \quad \rightarrow \quad P \]
Control Space Usage: Named Slots

\[ P ::= \text{renaming slots} \]

\[ P ::= \text{spawn} \]

Example: Renaming slots

\[ \{x/y\}_k \cdot P \triangleq y^k (\text{renaming slots} | P) \]

Then,

\[ y^k \cdot \{x/y\}_k \cdot P \rightarrow \text{renaming slots} | P \]
Control Space Usage: Named Slots

\[ P ::= -a \mid a^k P \mid \cdots \quad \text{(spawn)} \quad a^k P \mid a^k \]

Example: Renaming slots

\[ \{x/y\}_k P \triangleq y^k ( -^k_x \mid P) \]

Then,

\[ -^k_y \mid \{x/y\}_k P \Downarrow -^k_x \mid P \]

Example: Recursion (now right):

\[ \text{rec}(X^k) P \triangleq (\nu X)(!X^k \widehat{P} \mid -^k_X), \quad \text{where} \quad \widehat{P} \triangleq P\{ -^k_X / X \} \]
Control Space Usage: Named Slots

\[ P ::= \_a \mid a^{k}P \mid \cdots \quad \text{(spawn)} \quad a^{k}P \mid \_a \quad \vdash \quad P \]

Example: Renaming slots

\[ \{x/y\}_k \cdot P \triangleq y^{k}(\_x \mid P) \]

Then,

\[ \_y^k \mid \{x/y\}_k \cdot P \vdash \_x^k \mid P \]

Example: Recursion (now right):

\[ \text{rec}(X^k)P \triangleq (\nu X)(!X^{k}\widehat{P} \mid \_X^k), \quad \text{where} \quad \widehat{P} \triangleq P\{\_X^k/X\} \]

Example: Deriving Named Slots

\[ \_a \triangleq a[ \_ \mid \_ ] \]

\[ a^{k}P \triangleq (\nu n)(n[a^k] \cdot \opn^k \cdot P] \mid \opn n) \]
Conclusions

Typed Barbed Congruence:

\[ P \downarrow_b \text{ if } P \equiv (\nu x)b[\_ | Q'] | Q'', \text{ where } b \notin \bar{x} \]

\[ P \Downarrow_b \text{ if } P \downarrow^* b \]

This is sufficient to capture important differences.

Labelled Transition System: Easy enough.
Conclusions

Typed Barbed Congruence:

\[ P \downarrow_b \quad \text{if} \quad P \equiv (\nu \bar{x})b[ \quad \dashv \quad | \quad Q' \quad ] \quad | \quad Q'' \quad , \quad \text{where} \quad b \notin \bar{x} \]

\[ P \downarrow_p \quad \text{if} \quad P \xrightarrow{*} \downarrow_b \]

This is sufficient to capture important differences.

Labelled Transition System: Easy enough.

Yet to be done:

- In the large: Resource bounds negotiation and enforcement in GC.
- In the small: Expressiveness of BoCa; Equational theory; Smarter types; . . .
- In general: A lot to be done. . .